Modeling of Nonlinear Systems with Friction Structure Using Multivariable Taylor Series Expansion

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Abstract

The major aim of this article is modeling of nonlinear systems with friction structure that, this method is essentially extended based on taylore expansion polynomial. So in this study, the taylore expansion was extended in the generalized form for the differential equations of the state-space form. The proposed structure is based on multi independent variables taylore extended. According to the proposed method, in order to extract state-space form modeling, it is needed to some samples of the measured value of system states and/or their combination. Then, the matrix of measured values and response vectors of derived states were forming by these measured value of system states. By use of the presented equations and the measured values, the coefficients of polynomials were calculated as the model of state-space of studied nonlinear system. The result of proposed simulation using proposed algorithm show, that it is importable having an independent information system under test to system identification. A few suggestion were proposed to decrease of the dependence of the measured samples, the results of algorithm with different parameters show that proposed algorithm perform better. Of course in during the evaluation of algorithm, the results of different experimental were used.

Keywords: Modeling, Nonlinear systems, Taylore extended, Flexibility robot

1- Introduction

Many engineering applications need a compact and accurate description of the dynamic behavior of the system under consideration. This is especially true of automatic control applications such as SFLs [1]. And alternative way of building models is by system identification. In system identification, the aim is to estimate dynamic models directly from observed input and output data [2]. An important step in system identification is the determination of the type of model to be used. This decision is based on knowledge of the system under consideration, and on the properties of the model. Certain types of models can be used to approximate the input-output behavior of a smooth nonlinear dynamical system up to arbitrary accuracy. Such models have the so-called universal approximation capability. An example of a universal approximate is the neural network [3].

The drawback of these models is that are complex, and difficult to estimate and analyze. Therefore, other model structures have received considerable attention over the years. The linear time-invariant model is the most popular one. It has been used successfully in many engineering applications, and a considerable body of theory exists for system identification and automatic control of linear systems. Attractive methods for linear system identification are the subspace methods developed [4,5,6]. These are numerically robust methods that can easily deal with systems having multiple inputs and outputs and are non iterative, unlike many other identification methods. Although linear models are attractive for several reasons, they also have their limitations. Most real-life systems show nonlinear dynamic behavior, a linear model can only describe such a system for a small range of input and output values[7]. where systems are pushed to the limits of their performance and accurate descriptions over large input and output ranges are needed, linear models are often not satisfactory any more. Therefore, a considerable interest in identification methods for nonlinear systems has risen [8]. The overall goals in this paper are worked to be achieved through modeling nonlinear systems with nonlinear friction by the help of Taylor expansion. And we are going to implement our proposed algorithm on a real system (a nonlinear FJR laboratory robot system). Materials presented in this article were arranged as follows:

In the second part we will describe the model structure and in third part the results of investigated laboratory system will be expressed. In fourth part results of simulation will be studied. In section 5 we will present conclusion and then add references.

2- Model Structure

Equations dominating nonlinear system could be presupposed by following equations that can define all nonlinear systems:

$$\begin{cases} \dot{x} = f(x,u) \\ y = g(x,u) \end{cases} \tag{1}$$

Where $x \in R^n$ is the states of the system, $u \in R^m$ the number of system inputs, $y \in R^L$ the measureable outputs. Our aim in this paper is nonlinear input - output systems.

These equations can be represented as follows based on Taylor expansion:

$$\begin{cases} \dot{x}_1 = \begin{bmatrix} x^{[1]} & x^{[2]} & x^{[3]} & \cdots & x^{[k]} \end{bmatrix} \theta_1 \\ \dot{x}_2 = \begin{bmatrix} x^{[1]} & x^{[2]} & x^{[3]} & \cdots & x^{[k]} \end{bmatrix} \theta_2 \\ \vdots \\ \dot{x}_n = \begin{bmatrix} x^{[1]} & x^{[2]} & x^{[3]} & \cdots & x^{[k]} \end{bmatrix} \theta_n \\ y = \begin{bmatrix} x^{[1]} & x^{[2]} & x^{[3]} & \cdots & x^{[k]} \end{bmatrix} \theta_y \end{cases} \tag{2}$$

Equations based on Taylor expansion are up to K degree. In these equations n-dimensional vector is equal to $x = [x_1 \ \cdots \ x_n]^T$ and the rest of vectors are defined as follows:

$$\begin{aligned} x^{[0]} &= 1 \\ x^{[1]} &= x \\ x^{[k]} &= [x_1{}^k \quad x_1{}^{k-1} x_2 \quad \cdots \ x_1{}^{k-1} x_n \\ &\qquad x_1{}^{k-2} x_2{}^2 \quad x_1{}^{k-1} x_2 x_3 \quad \cdots \\ &\qquad x_1{}^{k-2} x_2 x_n \quad x_n{}^k]^T, k > 1 \end{aligned} \tag{3}$$

Where in equation 2, θ_1 to θ_n and θ_y are coefficient vectors for $x^{[1]}, x^{[2]}, \ldots x^{[k]}$ that have the same sizes and could be defined as follows:

$$\begin{cases} \theta_1 = \left[a_{1-1}, \ldots, a_{1-L}\right] \\ \theta_2 = \left[a_{2-1}, \ldots, a_{2-L}\right] \\ \vdots \\ \theta_n = \left[a_{n-1}, \ldots, a_{n-L}\right] \\ \theta_y = \left[a_{y-1}, \ldots, a_{y-L}\right] \end{cases} \quad (4)$$

Where:

$$L = (n+1) + c(n+1,2) + c(n+1,3) + \cdots + c(n+1,k) \quad (5)$$

To model a nonlinear system it is enough to calculate and find the amounts of θ_1 to θ_n and θ_y in a way that equations (2) representing a nonlinear system are created.

Lem1: for differential equation i of state space equations we have:

$$\left[x^{[1]}, x^{[2]}, x^{[3]}, \cdots, x^{[k]}\right]\theta_i = \dot{x}_i \quad (6)$$

This lem will be used to calculate θ_i. In this equation the vectors $x^{[1]}, x^{[2]}, \ldots x^{[k]}$ and also $\dot{x}_i$ are supplied based on states measurement and system input and thus they will be vectors with fixed coefficients. θ_i has L members that are unknown and we should calculate them for system modeling. Regarding equation (6) we can see that for **L** unknowns we have an equation. This means that it can have an infinite number of answers. To achieve a unique response it is necessary to increase the number of equations to equal the same unknowns **L** times. The only solution is to use equation (6) in different times of **L**. That is equation (6) should be modified as follows:

$$x_{t_L}.\theta_i = \begin{bmatrix} \dot{x}_i(t_0) \\ \dot{x}_i(t_1) \\ \vdots \\ \dot{x}_i(t_L) \end{bmatrix}$$

$$x_{t_L} = \begin{bmatrix} x^{[1]}(t_0) & x^{[2]}(t_0) & \cdots & x^{[k]}(t_0) \\ x^{[1]}(t_1) & x^{[2]}(t_1) & \cdots & x^{[k]}(t_1) \\ \vdots & & & \\ x^{[1]}(t_L) & x^{[2]}(t_L) & \cdots & x^{[k]}(t_L) \end{bmatrix} \quad (7)$$

t_0 to t_L are sampling times that can be selected randomly or with the same intervals. If we consider equation (7) as follows:

$$A.\theta_i = \underline{\dot{x}_i} \quad (8)$$

And if $|A| \neq 0$ we can calculate θ_i as follows:

$$\theta_i = A^{-1}.\underline{\dot{x}_i} \quad (9)$$

Accordingly we can use the following equation to calculate θ_y:

$$\theta_y = A^{-1}.\underline{y} \quad (10)$$

In a way that:

$$\underline{y} = \left[y(t_0) \quad y(t_1) \quad \cdots \quad y(t_L)\right]^T \quad (11)$$

A weak point of this method is the small amount of |**A**| that can happen due to bad selection of data related to states and input during different times. In this case some raw in matrix **A** will resemble each other a lot and this will result in |**A**| approaching zero. This can cause estimation error increases for θ_i and θ_y.

3- The Laboratory System Investigated

A nonlinear FJR robot laboratory system made of single joint single-arm is presupposed whose dynamic will be explained as below [9]:

$$\begin{aligned} & I\ddot{\theta}_1 + mgl\sin(\theta_1) + f_v\dot{\theta}_1 = U_c \\ & U_c = K(\theta_2 - \theta_1) + D(\dot{\theta}_2 - \dot{\theta}_1) \\ & J\ddot{\theta}_2 + K_{MB}\dot{\theta}_2 + U_c = K_m.I_a \\ & \dot{I}_a = V_m - K_b\dot{\theta}_2 + R.I_a \\ & \quad :K_{MB} = \left(B + \frac{K_m K_b}{R} \right) \end{aligned} \tag{12}$$

By selecting:

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{bmatrix} = \begin{bmatrix} \theta_1 & \dot{\theta}_1 & \theta_2 & \dot{\theta}_2 & I_a \end{bmatrix} \tag{13}$$

This system has five state variable and one input (m=5, n=1). Practical results on laboratory robot using model structure or the proposed algorithm by choosing will be:

$$x^{[1]} = \begin{bmatrix} x_1 \\ \vdots \\ x_5 \\ v_m \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \\ I_a \\ v_m \end{bmatrix} \tag{14}$$

And also model approximation up to k = 3 of equations related to modeling dynamics will solely include $x^{[1]}$ and $x^{[3]}$ which can be defined as follows:

$$x^{[3]} = \begin{bmatrix} x_1{}^3 & x_1^2 x_2 & \dots & x_5{}^3 & \dots v_m{}^3 \end{bmatrix}^T \tag{15}$$

Equations for this electro-mechanic system will entail only phrases with odd powers due to symmetry around base. This means that the coefficients $x^{[2]}$ and $x^{[2k]}$ will be equal to zero, on the whole. Then we would have the following:

$$\begin{cases} \dot{x}_1 = \begin{bmatrix} x^{[1]} \\ x^{[3]} \end{bmatrix}^T \psi_1 & \dot{x}_2 = \begin{bmatrix} x^{[1]} \\ x^{[3]} \end{bmatrix}^T \psi_2 \\ \dot{x}_3 = \begin{bmatrix} x^{[1]} \\ x^{[3]} \end{bmatrix}^T \psi_3 & \dot{x}_4 = \begin{bmatrix} x^{[1]} \\ x^{[3]} \end{bmatrix}^T \psi_4 \\ \dot{x}_5 = \begin{bmatrix} x^{[1]} \\ x^{[3]} \end{bmatrix}^T \psi_5 \end{cases} \tag{16}$$

Where:

$$\begin{cases} \psi_1 = [a_{1-1}, \dots, a_{1-62}] \\ \psi_2 = [a_{2-1}, \dots, a_{2-62}] \\ \psi_3 = [a_{3-1}, \dots, a_{3-62}] \\ \psi_4 = [a_{4-1}, \dots, a_{4-62}] \\ \psi_5 = [a_{5-1}, \dots, a_{5-62}] \end{cases} \tag{17}$$

Also the matrix **A** dimension is equal to $L \times L$ where L is calculated as follows:

$$\begin{aligned} & L = (n+m) + c(n+m, k) \\ & L = (6) + c(6,3) = 6 + 56 = 62 \end{aligned} \tag{18}$$

To create matrix **A** it is necessary to have L=62 times for sampling. Thus matrix **A** would have a structure like what follows:

$$A = \begin{bmatrix} x_1[k_1] & x_1[k_2] & \dots & x_1[k_{62}] \\ \vdots & \vdots & \dots & \vdots \\ x_6[k_1] & x_6[k_2] & \dots & x_6[k_{62}] \\ x_1^3[k_1] & x_1^3[k_2] & \dots & x_1^3[k_{62}] \\ x_1^2 x_2[k_1] & x_1^2 x_2[k_2] & \dots & x_1^2 x_2[k_{62}] \\ \vdots & \vdots & \dots & \vdots \\ x_6^3[k_1] & x_6^3[k_2] & \dots & x_6^3[k_{62}] \end{bmatrix}^T \tag{19}$$

k_1 to k_{62} will represent measurement samples, respectively. Of course, there is no need to have a fixed sequence for them. Of course, there is a necessary and sufficient condition and that is, raw and columns in matrix A should be independent to be able to calculate the inverse of this matrix. Practically the more distance between determinant **A** and zero will result in coefficients calculated to approach reality more. Then we can calculate ψ_1 to ψ_5 as follows:

$$\begin{cases} \psi_1 = A^{-1}.\underline{\dot{x}_1} \\ \psi_2 = A^{-1}.\underline{\dot{x}_2} \\ \psi_3 = A^{-1}.\underline{\dot{x}_3} \\ \psi_4 = A^{-1}.\underline{\dot{x}_4} \\ \psi_5 = A^{-1}.\underline{\dot{x}_5} \end{cases} \quad (20)$$

In these equations $\underline{\dot{x}_1}$ to $\underline{\dot{x}_5}$ have been defined as follows:

$$\underline{\dot{x}_i} = \begin{bmatrix} \dot{x}_i[k_1] \\ \dot{x}_i[k_2] \\ \vdots \\ \dot{x}_i[k_{62}] \end{bmatrix} \qquad i = 1,2,3,4,5 \quad (21)$$

The schematic of created laboratory FJR has been shown in figure 1.

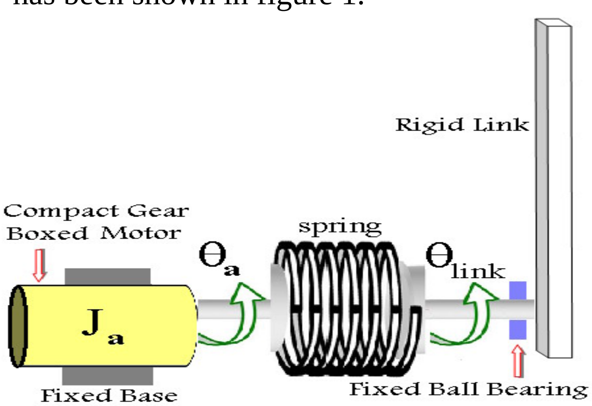

Fig.1. Single-armed FJR schematic

4- Simulation Results

In result section of the proposed algorithm administration, some parameters are taken into consideration to apply the algorithm and they also affect modeling error reduction. The following stages have been used to achieve the aims:

1. Sampling start point effect (Start)
2. Control signal structure effect (Test)
3. Sampling distance effect (Sample)
4. Random distance in sampling effect

To study the effect of parameters we have used 3 experiments in different conditions on a laboratory robot. Results of test 1 have been shown in figures 2 and 3.

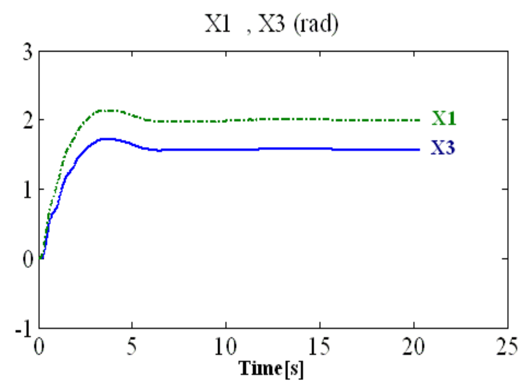

Fig.2. Arm position (x_1) and driver (x_3) in a number of experiments

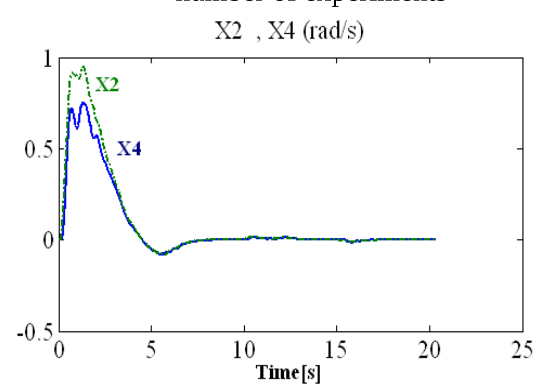

Fig.3. Arm speed (x_2) and driver (x_4) in a number of experiments

The validity measurement of modeling is carried out solely based on position and speed of arms and driver. Results of test 2 have been shown in figures 4 and 5.

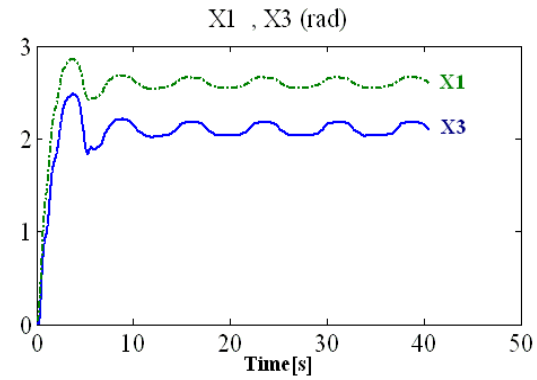

Fig.4. Arm position (x_1) and driver (x_3) in test 2

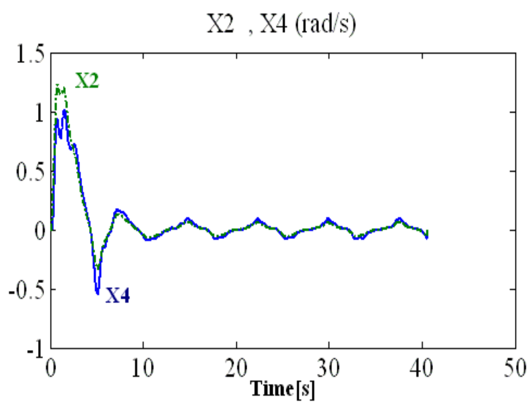

Fig.5. Arm speed (x_2) and driver (x_4) in test 2

Results of test 3 have been shown in figures 6 and 7.

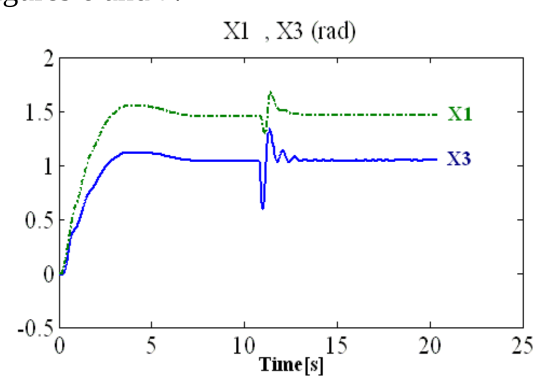

Fig.6. Arm position (x_1) and driver (x_3) in test 3

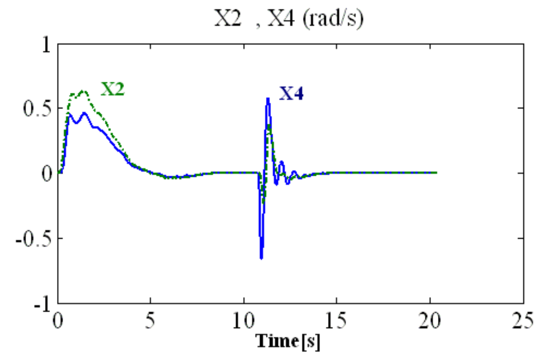

Fig.7. Arm speed (x_2) and driver (x_4) in test 3

One of parameters being considered in applying algorithm was to choose sampling start point (start) and |**A**| matrix determinant. If sampling is started at k =1, usually due to system delays the stored vector data may be shown as approaching zero which results in a singular |**A**| matrix. Therefore, it seems that the selection of start point can be important for sampling. Results of administering the proposed algorithm for 2 states can be seen in figures 8 and 9. Due to the fact that x1 and x2 are the position and speed of the arm and they are considered as the main exit of this system, we can carry out validity measurement for modeling based on relevant dynamic equations.

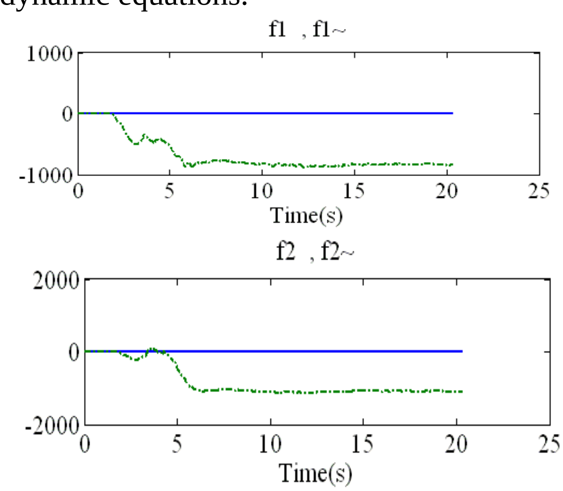

Fig.8. St=1 Sa=1 Start=1 Test=1 |A|=2.26e-193

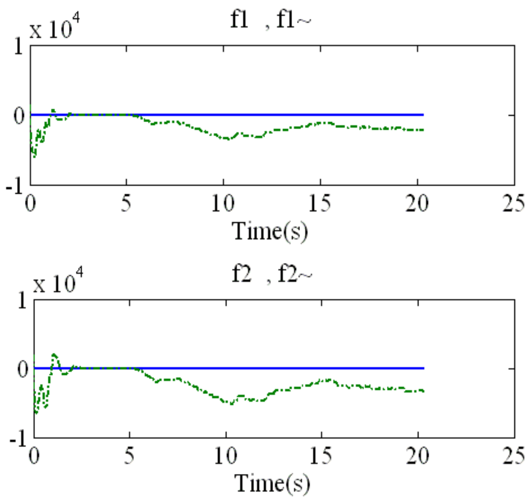

Fig.9. St=1 Sa=1 Start=100 Test=1 |A|= -9.46e-288

$$\tilde{f}_1 \xleftarrow{=} \dot{x}_1 = \begin{bmatrix} x^{[1]} \\ x^{[3]} \end{bmatrix}^T \Psi_1$$
$$\tilde{f}_2 \xleftarrow{=} \dot{x}_2 = \begin{bmatrix} x^{[1]} \\ x^{[3]} \end{bmatrix}^T \Psi_2$$
$$\tilde{f}_3 \xleftarrow{=} \dot{x}_3 = \begin{bmatrix} x^{[1]} \\ x^{[3]} \end{bmatrix}^T \Psi_3 \qquad (22)$$
$$\tilde{f}_4 \xleftarrow{=} \dot{x}_4 = \begin{bmatrix} x^{[1]} \\ x^{[3]} \end{bmatrix}^T \Psi_4$$
$$\tilde{f}_5 \xleftarrow{=} \dot{x}_5 = \begin{bmatrix} x^{[1]} \\ x^{[3]} \end{bmatrix}^T \Psi_5$$

In above figures $f_1 = \dot{x}_1$ and $f_2 = \dot{x}_2$ and $\tilde{f}_1 = \dot{\tilde{x}}_1$ and $\tilde{f}_2 = \dot{\tilde{x}}_2$:

$$f_1 \xleftarrow{=} \dot{x}_1 = x_2$$
$$f_2 \xleftarrow{=} \dot{x}_2 = -I^{-1}(mgl\sin(x_1) + f_v.x_2) + I^{-1}(K(x_3 - x_1) + D(x_4 - x_2))$$
$$f_3 \xleftarrow{=} \dot{x}_3 = x_4 \qquad (23)$$
$$f_4 \xleftarrow{=} \dot{x}_4 = -J^{-1}(K_{MB}x_4 + K(x_3 - x_1) + D(x_4 - x_2)) + \frac{K_m}{J}x_5$$
$$f_5 \xleftarrow{=} \dot{x}_5 = V_m - K_b x_4 + R.x_5$$

By comparing figures 8 and 9 the first conclusion is that the selection of sampling start point start = 100 will result in a reduction in |**A**|. Also the range estimations for f1 and f2 has been greater than the state of start=1. Results gained showed that starting point cannot be highly important in itself. In these experiments confirm the results obtained in violation of this concept.

In parameter the control signal structure effect (Test), using random signals to identify systems has been one of primary and approved issues. But the problem we encounter practically is lack of responding by physical systems to such signals, because due to the certain behavior of friction forces and looseness in such systems, often even random inputs cannot stimulate the system. Thus, in this article the effect of two control signals should be compared. In these tests results gained reinforced that this hypothesis has been correct.

To study this parameter we have used the comparison between results of test 1 and test2. Results of administering the proposed algorithms for two states were shown in figures 10 and 11. By comparing figures 10 and 11 we can clearly see that using input signal with changes, even in periodic, should be identified better.

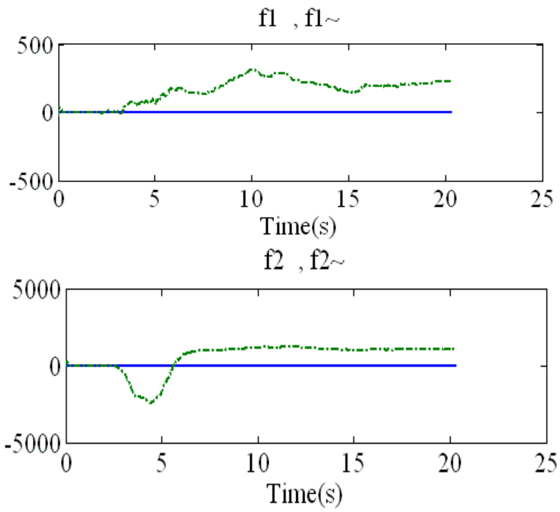

Fig.10. St=1 Sa=1 Start=10 Test=1 |A|=1.47e-195

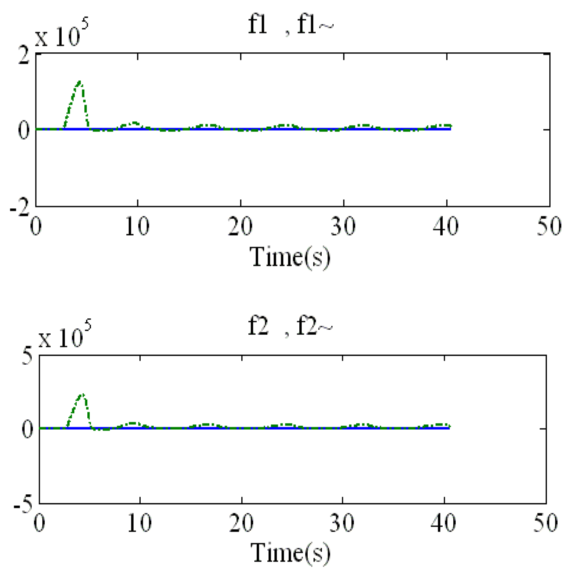

Fig.11. St=1 Sa=1 Start=10 Test=2 |A|=1.78e-179

Even the reduction in |**A**| in figure 11 approves this item because the range of estimations gained for f1 and f2 in most areas been less compared to outputs in figure 10. Results gained showed that the type of input signal to identify can have a determinative importance.

In sampling distance effect parameter (Sample), using saved signals to identify offline of systems will be required and necessary. But practically we should use required equations having independence characteristics. Otherwise it will result in weak identification. In this paper we presuppose that successive samples can have lots of similarities towards each other. To investigate this hypothesis, the proposed algorithm for sampling a few examples that we repeat the exercise. Results approve the correctness of this theory. To study about this parameter we have used results of test 2. Results of this test have been shown in 4 and 5 figures.

Results of implementing the proposed algorithm for 2 states are seen in figures 12 and 13. By comparing figures 12 and 13 we could easily see that using samples with greater distances can result in better identify of system. Even much reduction in |**A**| both in figure 13 showed this condition because the range of estimates gained for f1 and f2 has been much lower in all areas. Results have shown that sampling by greater distance for identification can result in a real modeling.

In parameter the effect of random distance in sampling the results gained for a state through which the signals have been accompanying chaos, it was hypothesized that if the selected samples were random instead of being successive, in a way that even it is include the chaotic area, the resulting state has emphasized on a better modeling.

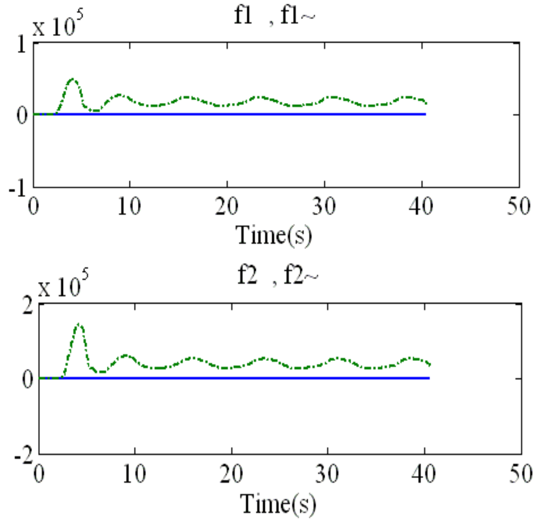

Fig.12. St=1 Sa=1 Start=1 Test=2 |A|=3.67e-173

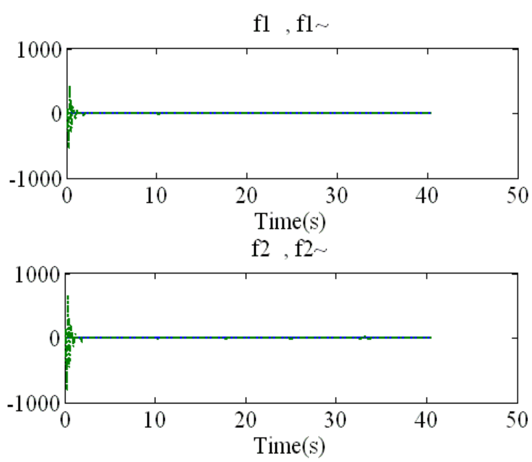

Fig.13. St=1 Sa=5 Start=1 Test=2 |A|=1.74e-106

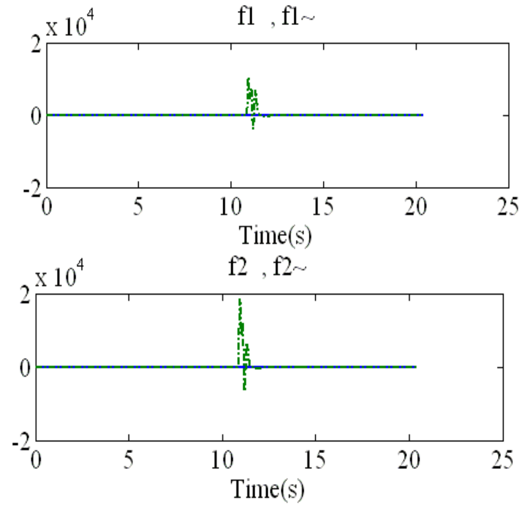

Fig.14. St=3 Sa=5 Start=1 Test=3 |A|=-2.3e-202

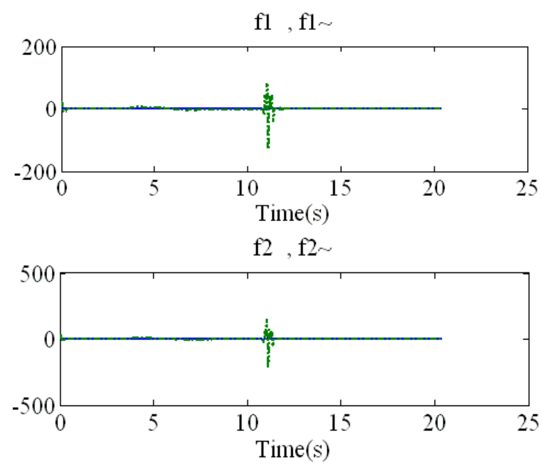

Fig.15. St=3 Sa=rand() Start=1 Test=3 |A|=-2.30e-138

Thus, in this part we have used samples which were chaotic from the start of the period for modeling. Results gained showed a significant difference compared to first state. To study about this parameter we have used test results in 3. Results of carrying out a test have been shown in figures 6 and 7.

Results of implementing the proposed algorithm for 2 states have been shown in figures 14 and 15. By comparing figures 14 and 15 we could observe that exactly according to prediction, modeling result have been affected by samples selection places. Of course, the amount resulted for matrix **A** determinant approves it. The range of estimates gained for f1 and f2, in random (or perhaps smart) samples was much less than first state. Thus, we can conclude that even chaos may be useful. Results gained shows that samples selection should be carried out in a way that it results in highest amounts of a matrix determinant to minimize modeling error.

5- Conclusion

The selection of starting point of sampling resulted in a reduction in |**A**|. Results showed that start point solely cannot be highly important. Using input signals with changes, although periodic, will result in a better identification. Results showed that the type of input signal to identification can have a determinative importance. Using samples with longer distances resulted in a better identification of system because range of estimates has been carried out by less error in most areas. Results showed that sampling with longer distances to identification can cause a more real modeling. Result of modeling is affected by sample's selection place. Estimation range error gained at random selection (or probably smart) of random was very much lower. Thus, we can conclude that chaos may even affect and be useful. Results showed that the sample should be arranged in a way that it represents the highest amount of a matrix determinant production and minimizes modeling error.

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