Geoid Determination Based on Log Sigmoid Function of Artificial Neural Networks: (A case Study: Iran)

Omid Memarian Sorkhabi

M.S.C., Department of Civil Engineering, Ahar Branch, Islamic Azad University, Ahar, Iran
E-mail: omidmemaryan@gmail.com

ABSTRACT

A Back Propagation Artificial Neural Network (BPANN) is a well-known learning algorithm predicated on a gradient descent method that minimizes the square error involving the network output and the goal of output values. In this study, 261 GPS/Leveling and 8869 gravity intensity values of Iran were selected, then the geoid with three methods "ellipsoidal stokes integral", "BPANN", and "collocation" were evaluated. Finally obtained results were compared and best the method was introduced. In Iran, the consequences showed that "BPANN" has been superior than other methods. Root Mean Square Error of this algorithm was less than ±0.292 m. Therefore, we concluded that BPANN can be used for geoid determination as an excellent alternative to the classic methods.

KEYWORDS: Geoid, Collocation, Ellipsoidal stokes integral, Artificial Neural Networks.

1.INTRODUCTION

Geoid determination can be divided into two basic methods, the geometric and the gravimetric. The geometric method means to use the known "geoid heights" at some points, which are derived from collocated GPS derived heights and leveled heights. The gravimetric method means to determine a geoid model using gravity measurements. In this study, both methods are used for geoid determination and comparison [2,3].

There are many researches available about geoid model construction using the GPS/Leveling method; e.g., Kiamehr and Sjöberg [8], Nunez et al. [12], Lin [7], Abromzic et al., [1]. The artificial neural network (ANN) has been applied in different fields of geodesy and geo-science e.g., Gullu et al, [4]. The main goal of this study is to evaluate a back propagation artificial neural network (BPANN) for modeling GPS/Leveling geoid undulations as an alternative method of collocation. In this research, the geoid undulations are estimated from BPANN and ellipsoidal stokes integral. Then collocation is compared to the geoid undulations based on GPS/Leveling measurements in terms of root mean square error (RMSE) of the undulation differences. This study was done in Iran.

2. GPS/LEVELING

The GPS/Leveling geoid undulations are calculated by Heiskanen and Moritz [5] by:

$$N = h - H \tag{1}$$

Where, N denotes the geoid undulation, h denotes the ellipsoidal height and H denotes the orthometric height.

Practically, it is extremely hard to compute geoid undulation for every point on the Earth. Therefore, an analytical geoid surface is created by utilizing the points that best exhibit the geoid in regions with precisely determined ellipsoidal and orthometric heights. Therefore, the geoid undulations for the mediate points encountered great difficulty in practice [6].

3. ARTIFICIAL NEURAL NETWORKS

Focus on artificial neural networks, generally called "neural networks", has been inspired from its inception by the identification that human brain computes in a completely different way from the routine digital computers. The brain is a very intricate, nonlinear, and parallel computer. It can systematize its structural components, called neurons, to be able to perform certain calculations faster than the fastest digital computer available today [9,10].

We recognize three basic components of the neural model: a set of synapses or connecting links; all of that will be characterized with a weight of its own, an adder for adding the input signals; weighted by the corresponding synapses of neurons, and an activation function called squashing function in a way that its squashes allowed amplitude array of the output signal with a finite value. The activation function employed for ANN could

be the sigmoid function, described by equation (2).

$$f(z) = \frac{1}{1+e^{-z}} \tag{2}$$

Where, z is the input information of the neuron and f(z) is activation function, between (0, 1). The proposed ANN for estimating the geoid undulations is trained utilizing the back propagation algorithm with a well–known ability as function approximators e.g., Pandya and Macy [13].

3.1 Back Propagation Artificial Neural Network

BPANN is a well-known learning algorithm predicated on a gradient descent method that minimizes the square error involving the network output and the goal of output values. The error is consequently propagated back through the weights of the multi layered networks before the desired error threshold is reached.

BPANN is commonly utilized in many fields, particularly in engineering due to its high learning capacity and simple algorithm. This algorithm aims to lessen errors backwards, from input to output. BPANN is a supply forward and supervised learning network. Generally, BPANN includes an input layer, an output layer, and a couple of intermediate hidden layers. Each layer contains different quantities of neurons related with the situation involved [16,17].

A network with one hidden layer utilizing a sigmoid activation function can approximate any continuous functions given a sufficient quantity of hidden neurons. Fig 1 shows the architecture of BPANN. The delta rule predicated on squared error minimization is useful for BPANN training procedure.
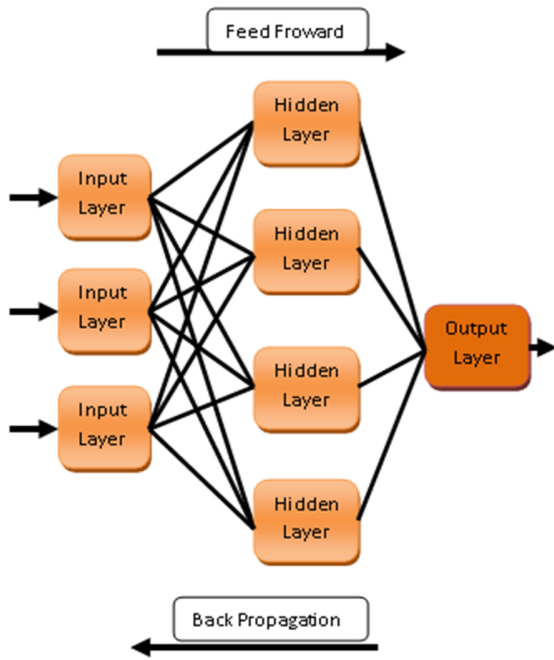
Fig1. The BPANN architecture

In the training process, the weights involving the hidden layers and the output layer are adjusted based on the data set that comprises the known input and output parameters. This iterative procedure adjusted the weights to be able to reduce the residuals (difference involving the estimated output and the actual output) of the output of the neural network (Gullu et al., [4]). The training protocol includes two main steps: Feed-forward and back-propagation.

4. ELLIPSOIDAL STOKES INTEGRAL (ESI)

The ellipsoidal Stokes integral (Martinec and Grafarend, 1997) were described by equation (3).

$$N(b_0, \Omega) = \frac{b_0}{4\pi\gamma} \iint_{\Omega'}^0 f(\Omega') \big[S(x) - e_0^2 S^{ell}(\Omega, \Omega') \big] d\Omega' \qquad (3)$$

Where, x is the angular distance between directions Ω and Ω', S(x) is the spherical and ellipsoidal Stokes functions and, $S^{ell}(\Omega, \Omega')$ is the geoidal heights $N(b_0, \Omega)$. Due to the lack of gravity anomaly $f(\Omega')$ on some parts of the globe, the integral is split into to the near-zone and the far-zone contributions described by:

$$N(b_0, \Omega) = N^{x_0}(b_0, \Omega) + N^{\pi - x_0}(b_0, \Omega) \qquad (4)$$

$N(b_0, \Omega)$ is the near-zone contribution and $N^{x_0}(b_0, \Omega)$ is the far-zone contribution $N^{\pi - x_0}(b_0, \Omega)$. Computing the near-zone contribution of N, we have equation (5).

$$N^{x_0}(b_0, \Omega) = \frac{b_0}{4\pi\gamma} \int_0^{x_0} \int_0^{2\pi} f(\Omega') \big[S(x) - e_0^2 S^{ell}(\Omega, \Omega') \big] d\Omega' \qquad (5)$$

Computing the geoid heights of far-zone contribution considering equation (3), we have

$$N^{\pi - x_0}(b_0, \Omega) = \frac{b_0}{4\pi\gamma} \int_{x_0}^{\pi} \int_0^{2\pi} f(\Omega') \big[S(x) - e_0^2 S^{ell}(\Omega, \Omega') \big] \sin x dx \, d\Omega' \qquad (6)$$

This integral can be viewed as a spherical Stokes integration extended by the term linked to ellipsoidal contribution. Then we divided this integral as follows:

$$N^{\pi - x_0}(b_0, \Omega) = \frac{b_0}{4\pi\gamma} \int_{x_0}^{\pi} \int_0^{2\pi} f(\Omega') S(x) \sin x dx \, d\alpha - \frac{b_0}{4\pi\gamma} \int_{x_0}^{\pi} \int_0^{2\pi} f(\Omega') e_0^2 S^{ell}(\Omega, \Omega') \sin x dx \, d\alpha \qquad (7)$$

Since the magnitude of the second part of equation (7) is small, we approximate the far-

zone contribution by just taking the first part of the right-hand side of equation (8) into account. According to Heiskanen and Moritz [5] we have:

$$N^{\pi-x_0}(b_0, \Omega) = \frac{b_0}{2\gamma} \sum_{j=2}^{\infty} Q_j(x_0) \sum_{m=-j}^{j} f_{jm} Y_{jm}(\Omega) \qquad (8)$$

Where, $N^{\pi-x_0}(b_0, \Omega)$ are the geoidal heights of the far-zone contribution, $Q_j(x_0)$ are the Molodenkij truncation coefficients [11], f_{jm} can be determined by a Global Geo-potential Model.

5. LEAST SQUARES COLLOCATION

Least-squares collocation (LSC) is a really generalized estimation method that has been applied successfully to the interpolation of potential field anomalies and to answer varied problems in physical geodesy. LSC could be generalized to arbitrary data as a strictly analytical approximation method. Recently, LSC has been used to estimate crustal deformation fields from GPS measurements [14] [15]. LSC is predicated for minimization of the mean squared error (MMSE). An important rule that is to be obeyed is the information required to be centered prior to the collocation. In other words, trend needs to be taken from the raw data in a way that mean of the data could be corresponding to zero.This trend removal process could be accomplished by making use of various trend models to the raw data; For example mean removal, first order polynomial fit, second order polynomial fit

(in this study second order polynomial fit has been used).

Determination of the covariance function model and its parameters is really a prerequisite for composition of the covariance matrices. In this study, covariance function has been described by:

$$C\_s(r) = C\_0(1 + r^2 / D^2)^{(-1/2)} \qquad (9)$$

Where, C_0 is signal variance and D is the distinctive distance. Signal prediction has been performed by the Wiener-Kolmogorov formula [15] described by equation (10):

$$(S\_p) = -C\_(S\_p S)(C\_S + C\_V)^{(-1)} l^0 \qquad (10)$$

Where, $\widehat{S_p}$ is predicted signal, $C_{S_p S}$ is the cross-covariance matrix between the predicted and observed signal, C_S is covariance matrix of the signal, C_V is covariance matrix of the noise and l^0 is vector of observations. In order to make error estimation, error covariance matrix $C_{\hat{S}}$ of the estimated signal was described by:

$$[\![C]\!]\_\hat{S} = C\_S(C\_S + C\_V)^{(-1)} C\_S \qquad (11)$$

6. STUDY AREA AND NUMERICAL TEST

In this section, outcomes of our case study in the construction of the geoid of Iran are demonstrated. In this study, the estimates of the geoid undulations were performed over a study area that is located in the province of Iran within the geographical boundaries: $25.5 < \varphi < 40$ and $44 < \lambda < 63$. The geodetic

coordinates of the points were determined by the static GPS surveying method and the orthometric heights of the points were calculated by the geometric leveling method using a digital level from two points whose orthometric heights were already known.

The 261 GPS/Leveling Distribution and shuttle radar topography model (SRTM) were shown in Fig 2. We will use the gravity intensity values (for stokes integral) for the test area (Fig 3) 0 Geoid height determined by BPANN, ellipsoidal stokes integral and LSC. Relative difference N shown in Fig 4 and Properties of statistics, obtained from proposed algorithm, is shown in Table 1.

Table 1: Properties of Statistics in this research (meter)

Methods	Minimum	Maximum	Mean	RMSE
BPANN	-36.227	24.384	-8.001	0.292
ESI	-36.118	24.447	-7.505	0.321
LSC	-35.651	25.769	-7.015	0.359
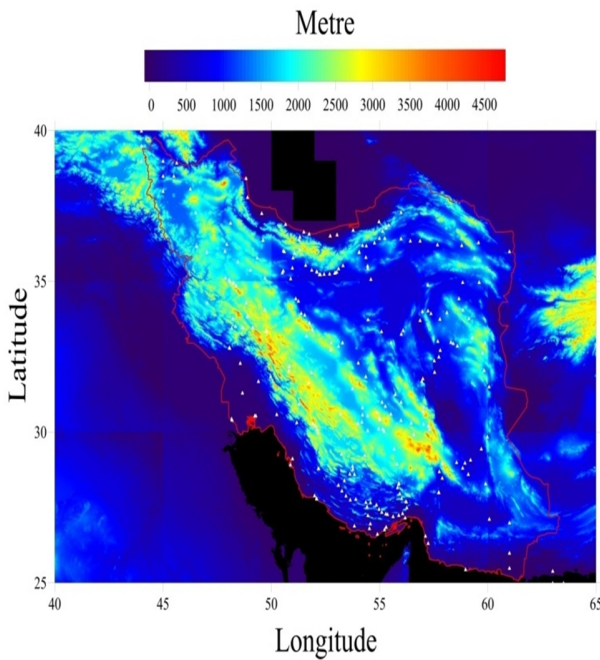
Fig.2. The 261 GPS/Leveling distribution and SRTM
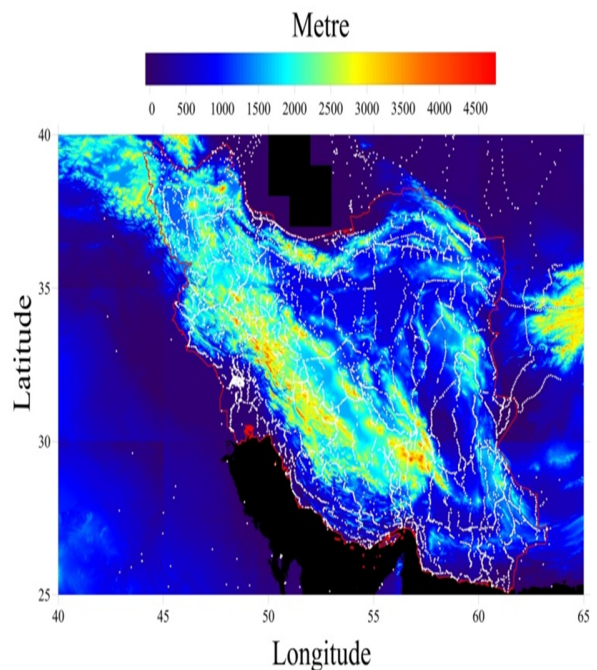

Fig.3. Coverage map of 8869 gravity intensity stations in Iran (from BGI database) and SRTM

7. CONCLUSION

In this study, the estimations of the geoid undulations were performed over a study area that is located in the province of Iran within the geographical boundaries of: $25.5 < \varphi < 40$ and $44 < \lambda < 63$. The 261 GPS/Leveling and 8869 gravity intensity values of Iran are selected, the geoid with three methods "BPANN", "ESI" and "collocation" are evaluated and compared.

In the BPANN method, RMSE was calculated as ±0.292m, ESI as ±0.321 and LSC as ±0.359m. The main advantages of ANN are learning networks, parallel processing and computation flexibility. The disadvantage of neural network is a disability of algorithm to interpret the output and how to select the training data. The direct numerical computation of the integral includes revealing a comparatively long and wasting time process considering the singularity of spherical and ellipsoidal Stokes functions.
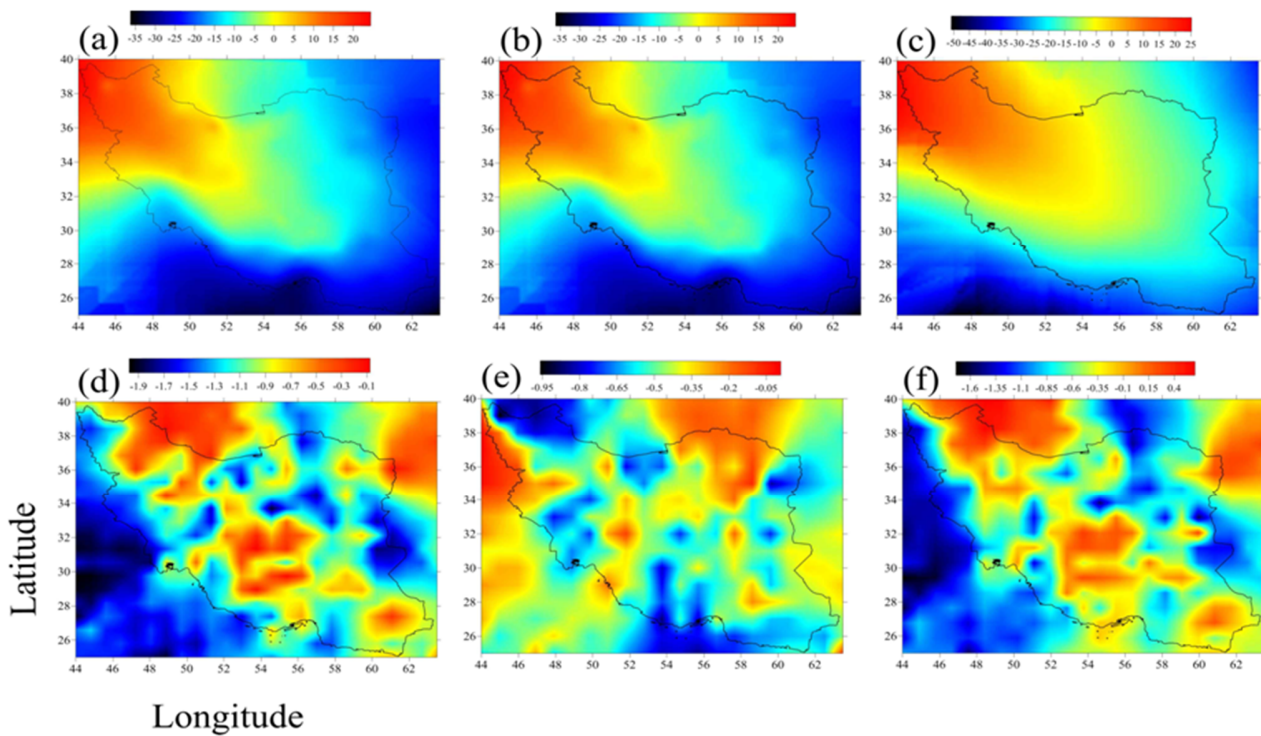
Fig .4. a) N determined by BPANN (m) b) N determined by EIS (m) c) N determined by LSC (m) d) BPANN and EIS Relative difference N (m) e) BPANN and LSC Relative difference N (m) f) EIS and LSC Relative difference N (m)

Collocation methods have disadvantages depending on how the results of the covariance function can be defined (precision of the covariance function and correlation function). BPANN can be used for geoid undulation modeling as an

alternative to the classic methods. Unfortunately, unlike other engineering sciences, artificial neural networks are not well known in geodesy and so it is recommended in other areas such as geodetic point velocity. Finally BPANN is utilized and results are compared with other methods. We concluded that BPANN can be used for geoid determination as an excellent alternative to the classic methods.

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