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An Improved Gap Metric and Stability Margin-Based Analysis to Control Heating, Ventilating, and Air Conditioning Systems Based on Multiple-Model and Model Order Reduction

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Abstract

To address computational complexity in heating, ventilating, and air conditioning systems, two general reduced multiple model control designs based on gap metric, stability margin, and model order reduction are proposed. The difference between two designs lies in the sequence of implementing model order reduction and multiple model techniques, resulting in distinct control approaches. As the number and location of reduced multiple models are not necessarily the same in two cases, the selected models will also be different. This could make one approach preferable to another in terms of closed-loop performance. Therefore, we introduce a model selection criterion to predict the most suitable approach for improving indoor thermal comfort and air quality in considered system. This criterion is based on maximum gap metric, maximum stability margin, and number of nominal models. Finally, two new approaches called OR-MM and MM-OR and a new criterion called MSC are proposed. To validate the effectiveness of our method, we conduct computer simulations that demonstrate their achievements.

Keywords: Heating, ventilating, and air conditioning (HVAC) system, Air quality, Indoor environment, Outdoor environment, Model order reduction, Multiple models.
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1. Introduction

The control of HVAC systems has gained significant attentions in the recent decades due to their high order and nonlinearity [1]-[3].Although much efforts have been made to overcome the challenges in the field of the high order and nonlinearity, few researchers have tackled both challenges simultaneously.

In general, there are four alternative solutions for dealing with the high order nonlinear HVAC systems: nonlinear controller, a single linear controller, multiple linear controllers, and reduced multiple controllers (Figure 1). Although some solutions are commonly used, their drawbacks cannot be ignored. According to the first view, nonlinear controllers are directly designed based on high order nonlinear HVAC systems (Figure 1A) [4], [5]. Variety of approaches might be employed in this regard, however, the difficulty in design, analysis, and implementation would not be deniable. Indeed, the computational load in optimization problem and the complexity of design, analysis, and implementation are the main disadvantages of nonlinear controllers.

The second view aims to simplify the control design procedure. The simplification is based on just linearization. The high order nonlinear HVAC systems are linearized around the most common operating point and then a single linear controller is designed (Figure 1B) [6]. Despite the simplicity, the approach is not an appropriate when there are extreme weather changes.

The drawback of employing a single linear controller encourages the researchers to select more operating points. Indeed, MM approach is adopted to describe high order nonlinear HVAC systems (Figure 1C) [7]. A linear controller is designed for each linear model and then multiple linear controllers are constructed. Although wide operating space is included in control design procedure, no preparations are made for the order of model.

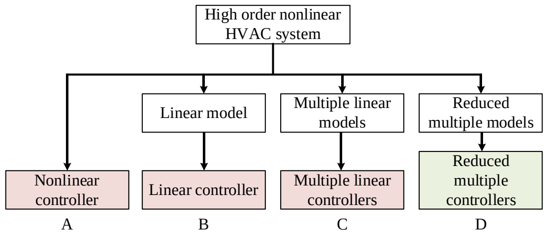

Fig. 1. A) Nonlinear controller, B) linear controller, C) multiple linear controllers, and D) reduced multiple controllers.

To deal with existing disadvantages, MM and model order reduction techniques are employed simultaneously (Figure 1D). Then, reduced multiple controllers are designed to control the high order nonlinear HVAC system. Hence, both the high order and the wide operating space of the model have been considered. Combination of MM and OR is a great idea to cope with complexity in considered systems.

As stated, MM could be an appropriate approach for simplification in nonlinear processes which not only operate in wide range of operating space, but also have event-based set point [8]. This approach includes two phases: decomposition and combination. In decomposition phase, a complex nonlinear system is represented by some nominal linear models so that each model only characterizes a particular range of operating space. The number and location of these models are to be determined. Decomposition can be performed based on physical components, physical and chemical phenomena, and operating conditions [9]. All these methods depend on experience and a priori knowledge. The gap metric and maximum stability margin criteria are two attractive tools lead to effective decomposition [10]. In gap metric criterion, distances between linear models are calculated and then the maximum stability margin is used to select the nominal linear models. Finally for each nominal linear model a linear controller is designed. In combination phase, the designed controllers are combined via HS [11] or SS [12] to construct the global controller.

Another challenge facing high order nonlinear HVAC systems lies in complexity in controller design. As previously mentioned, OR can be a pretty good option in this regard. OR includes mathematical techniques that replace the original model by an approximating one with a smaller dimension. These techniques are divided into two general categories called SVD-based [13] and Krylov-based [14] methods. The balanced truncation, as a member of the SVD category, is the most common approach that transforms a model to a basis where all states are ordered according to the energy that they transfer from input to output. Then, the reduced model is obtained by truncating those states which contribute little from input to output.

The main contribution of present work is to combine MM and OR approaches to deal with the complexity in high order nonlinear HVAC systems so that reduced multiple control design will be attained. Moreover, as MPC algorithms are used by many researchers to control such a complex system [2], [15], MMPC will be able to improve air quality in wide operating conditions in the presence of disturbances and constraints [16]. In addition, sequence of combination could be performed in either OR-MM or MM-OR, where different results could be obtained [17]. The preferable scheme is decided by a newly defined tool in this paper called MSC based on the maximum gap metric and stability margin. Indeed, OR-MM and MM-OR approaches are proposed to deal with the complexity in high order nonlinear HVAC systems. As one of them would be more appropriate in presence of extreme weather condition, MSC is proposed to predict the one which produces the better closed-loop performance.

The proposed combination eases controller design, simplifies analysis, makes implementation cheap and easy, reduces computational loads, reduces computational complexity, and is especially more fit HVAC systems.

The structure of the paper is as follows. First, the basic preliminaries including the multiple models, gap metric, stability margin, OR, and MPC formulation are summarized in Section 2. Section 3 is dedicated to three innovative algorithms. Two comparative reduced multiple controller algorithms (OR-MM and MM-OR) are proposed to simplify the control design procedure for high order nonlinear HVAC systems. The best reduced multiple controller in the closed-loop sense is predicted using a newly defined criterion MSC whereby the more preferable algorithm is determined before the control design procedure. In Section 4, the proposed methods are verified through HVAC system which has been investigated in both tropical and temperate climate zones. This includes implementation of OR-MM, MM-OR, and MSC algorithms to demonstrate the effectiveness of the proposed procedures. Finally, Section 5 summarizes the study.

2. Preliminaries and problem formulation

A high order nonlinear dynamical system could be represented by

$$\dot{x} = f(x,u), y = g(x) \tag{1}$$

where $y \in \mathbb{R}$, $u \in \mathbb{R}$, $x \in \mathbb{R}^n$ are output, control input, and state vectors, respectively. $f: \mathbb{R}^{n\times 1} \to \mathbb{R}^n$ and $g: \mathbb{R}^{n\times 1} \to \mathbb{R}$ are known nonlinear differentiable vector-valued functions and f is Lipschitz function.

Definition 1. (x_e, u_e, y_e) is called an equilibrium point if $f(x_e, u_e) = 0$, $y_e = g(x_e)$.

Definition 2. $\{(x_e, u_e, y_e) | f(x_e, u_e) = 0, y_e = g(x_e)\}$ is named equilibrium manifold constructed from the set of equilibrium points of the system.

A set of available variables such as states, inputs, outputs, and/or disturbances are chosen as scheduling variables noted by θ. The selected variables should properly characterize the nonlinear behavior of the considered system. Noting the global operating space of the system by Ψ, the scheduling variables could assume values in this specific space. It is assumed that Ψ is decomposed into N_s operating subspaces ψ_i such that $\Psi = \cup_{i=1}^{N_s} \psi_i$.

At this stage, only linearization will be performed. Let assume that Ψ is gridded into N operating points (x_{ei}, u_{ei}, y_{ei}), $i = 1, \dots, N$ and linearize system (1) around these points. The following high order linear models are obtained:

$$\begin{aligned} \delta\dot{x}_i &= A_i \delta x_i + B_i \delta u_i \\ \delta y_i &= C_i \delta x_i \end{aligned} \tag{2}$$

where $A_i = \partial f / \partial x|_{(x_{ei}, u_{ei})}$, $B_i = \partial f / \partial u|_{(x_{ei}, u_{ei})}$, $C_i = \partial g / \partial x|_{(x_{ei}, u_{ei})}$, $\delta x_i = x - x_{ei}$, $\delta u_i = u - u_{ei}$, and $\delta y_i = y - y_{ei}$.

Remark 1. $N_s \le N$, where N and N_s represent number of local and nominal linear models, respectively.

When range of the operating space is wide, a single nominal linear model would not be enough ($N_s \ne 1$) to control the nonlinear system. In this case, use of multiple model approach could be a reasonable choice.

A) Multiple model method

MM method is a technique to deal with the systems facing strong nonlinearity. In this technique, Ψ is gridded into N points. Thus, N local linear models are obtained. Then Ψ decomposes into N_s number of ψ_i. Indeed, all N local linear models are grouped together and N_s nominal linear models are found (decomposition phase). Finally nominal linear models are combined (combination phase) as

$$\begin{aligned} \dot{x} &= \sum_{i=1}^{N_s} \omega_i (A_i \delta x_i + B_i \delta u_i) \\ y &= \sum_{i=1}^{N_s} \omega_i \big(\delta y_i + g(x_{ei})\big) \end{aligned} \tag{3}$$

where ω_i is time-dependent weighting function. HS or SS can be used to weight the nominal models. As stated, the global operating space decomposes into N_s subspaces. In each subspace, one nominal linear model is determined such that the weighted combination of these N_s nominal linear models could represent the high order nonlinear system described in (1), adequately. However, how to select the nominal models is a question that has to be answered.

Gap metric:If two linear models behave similarly in open-loop sense, it does not mean that they behave similarly in closed-loop sense as well [18]. Similarity of two systems in closed-loop sense was always the problem left to be solved. Although norm metric was used to compare the systems, it was not an appropriate criterion in the closed-loop sense. Gap metric was introduced by Zames and El-Sakkary, in 1980 [18], and has been employed to compare two LTI systems. This metric reveals that a similar behavior of two linear models in the open-loop sense does not necessarily imply a similar behavior in the closed-loop sense as well [19].

Let $\mathcal{M}(s)$ and $\mathcal{N}(s)$ represent normalized left coprime factorization of the transfer function $P(s)$ i.e. $P(s) = \mathcal{M}^{-1}(s)\mathcal{N}(s)$ such that $\mathcal{M}(s)\widetilde{\mathcal{M}}(s) + \mathcal{N}(s)\widetilde{\mathcal{N}}(s) = I$, where $\widetilde{\mathcal{M}}(s) = \mathcal{M}^T(-s)$ and $\widetilde{\mathcal{N}}(s) = \mathcal{N}^T(-s)$ and $(\tilde{.})$ denotes complex conjugate [19]. Then gap between two LTI systems $P_1(s)$ and $P_2(s)$ can be computed as [20]

$$\delta(P_1, P_2) = \max\left(\inf_{Q \in \mathrm{H}_\infty} \left\| \begin{bmatrix} \mathcal{M}_1 \\ \mathcal{N}_1 \end{bmatrix} - \begin{bmatrix} \mathcal{M}_2 \\ \mathcal{N}_2 \end{bmatrix} Q \right\|_\infty, \inf_{Q \in \mathrm{H}_\infty} \left\| \begin{bmatrix} \mathcal{M}_2 \\ \mathcal{N}_2 \end{bmatrix} - \begin{bmatrix} \mathcal{M}_1 \\ \mathcal{N}_1 \end{bmatrix} Q \right\|_\infty \right) \tag{4}$$

where $0 < \delta(P_1, P_2) \le 1$. In the closed-loop sense, two linear systems behave similarly, if $\delta(P_1, P_2)$ is close to zero and behave dissimilarly if $\delta(P_1, P_2)$ is close to one. In fact, a single controller is able to stabilize similar linear systems. On the other words, a single controller is sufficient for similar systems to guarantee the closed-loop stability [21].

Stability margin:How to select N_s nominal linear model(s) from N local linear models in MM approach is a challenge that could be handled based on the stability margin.

Theorem 1. [21] Suppose that controller K stabilizes transfer function P. If $\mathcal{P} \triangleq \{P_\Delta | \ \delta(P, P_\Delta) < \delta_P\}$, then K could robustly stabilize all $P_\Delta \in \mathcal{P}$ if and only if

$$\delta(P, P_\Delta) < \delta_P \le b_{\text{opt}}(P) \tag{5}$$

where $b_{\text{opt}}(P)$ is maximum stability margin calculated as

$$\begin{aligned} b_{\text{opt}}(P) &= \left\{ \inf_{K \text{stablizing}} \left\| \begin{bmatrix} I \\ K \end{bmatrix} (I + PK)^{-1} [I \quad P] \right\|_\infty \right\}^{-1} \\ &= \sqrt{1 - \|[\mathcal{N} \quad \mathcal{M}]\|_{\mathrm{H}}^2} < 1 \end{aligned} \tag{6}$$

$\|\cdot\|_{\mathrm{H}}$ is the Hankel norm.

Equation (5), called robust stability condition, has been employed as a valuable tool to select the nominal model(s) [11], [22].

Grounded on Theorem 1, the maximum stability margin of the local linear models is used as a criterion to select nominal model(s). Thus, local stability is guaranteed [9]. Though the local stability does not guarantee the global stability of the high order nonlinear system, increasing the number of nominal linear models could enhance the global stability condition. On the other hand, increasing the number of linear models requires more effort to design the controllers. Here, designers need to make a trade-off between the stability enhancement and computation. Analysis of global stability requires further study which is not conducted in this work.

B) Model order reduction

"Why OR?" is a question that leads to study approximation techniques and their applications. To scape computational loads and complicated controllers in nonlinear large-scale dynamic systems, all approximation techniques could be a brilliant decision. Via OR methods, the obtained reduced order models behave similarly as the high order models in response to the same input(s).

Balanced realization is an approach that utilizes in SVD-based OR methods. At first, a balanced representation of state space is obtained. Then, some states are discarded according to controllability and observability Gramians [23]. Therefore, stability, controllability, and observability of high order model are preserved. In Krylov-based OR methods, however, an optimization problem is solved, which could complicate the process in comparison with SVD-based OR ones [24].

To approximate high order linear models via SVD-based OR, (2) is rewritten as

$$\begin{bmatrix}\delta\dot{x}_i^1\\ \delta\dot{x}_i^2\end{bmatrix} = \begin{bmatrix}A_i^{11} & A_i^{12}\\ A_i^{21} & A_i^{22}\end{bmatrix}\begin{bmatrix}\delta x_i^1\\ \delta x_i^2\end{bmatrix} + \begin{bmatrix}B_i^1\\ B_i^2\end{bmatrix}\delta u_i$$
$$\delta y_i = \begin{bmatrix}C_i^1 & C_i^2\end{bmatrix}\begin{bmatrix}\delta x_i^1\\ \delta x_i^2\end{bmatrix} \tag{7}$$

where $x_i^1 \in \mathbb{R}^{n_\mathrm{r}}$ and $x_i^2 \in \mathbb{R}^{n_\mathrm{d}}$ are remained and discarded state vector(s), respectively. Indeed, $x_i \in \mathbb{R}^n$ $(n = n_r + n_d)$. Thus, the reduced order models are constructed as

$$\delta\dot{x}_i^1 = A_i^{11}\delta x_i^1 + B_i^1\delta u_i$$
$$\delta y_i = \hat{C}_i\delta x_i^1 \tag{8}$$

Since $n_r < n$, the simplification of the high order models is successfully done in (8).

C) Model predictive control

MPC is a feedback control algorithm that has become popular in control engineering. Since its inception, model predictive control (MPC) has been one of the prospective solutions for HVAC management systems to reduce both costs and energy usage [25]. The controller could be served as a powerful tool for slow dynamics such as HVAC. In an MPC algorithm, it is possible to not only optimize a cost function but also introduce input(s) and/or output(s) constraints.

The discrete time state space representation of the nominal linear reduced models can be expressed by

$$x_i(k+1) = A_{di}x_i(k) + B_{di}u_i(k)$$
$$y_i(k) = C_{di}x_i(k) \tag{9}$$

By introducing the control action increment, $\Delta u_i(k) = u_i(k) - u_i(k-1)$, the augmented state space representation is obtained as

$$x_{ai}(k+1) = \underbrace{\begin{bmatrix}A_{di} & B_{di}\\ 0_{m\times n_i} & I_{m\times m}\end{bmatrix}}_{A_{ai}} x_{ai}(k) + \underbrace{\begin{bmatrix}B_{di}\\ I_{m\times m}\end{bmatrix}}_{B_{ai}}\Delta u_i(k))$$
$$y_i(k) = \underbrace{\begin{bmatrix}C_{di} & 0_{q\times m}\end{bmatrix}}_{C_{ai}} x_{ai}(k) \tag{10}$$

where $x_{ai}(k) = [x_i(k)^T \;\; u_i(k-1)]^T$ and n_i, m, and q are the order of i^{th} nominal linear model, input number, and output number, respectively.

According to (10), the output predictions could be derived as

$$y_i(k+1) = C_{ai}A_{ai}x_{ai}(k) + C_{ai}B_{ai}\Delta u_i(k)$$
$$y_i(k+2) = C_{ai}A_{ai}^2x_{ai}(k) + C_{ai}A_{ai}B_{ai}\Delta u_i(k) + C_{ai}B_{ai}\Delta u_i(k+1)$$
$$\vdots$$
$$y_i(k+n_p^i) = C_{ai}A_{ai}^{n_p^i}x_a(k) + C_{ai}A_{ai}^{n_p^i-1}B_{ai}\Delta u_i(k) + \cdots + C_{ai}A_{ai}^{n_p^i-n_c^i}B_{ai}\Delta u_i(k+n_c^i-1) \tag{11}$$

where n_c^i and n_p^i are the control and prediction horizons for the i^{th} nominal linear model. Then, the prediction model is obtained as follows

$$Y_i(k+1) = \begin{bmatrix}C_{ai}A_{ai}\\ C_{ai}A_{ai}^2\\ C_{ai}A_{ai}^3\\ \vdots\\ C_{ai}A_{ai}^{n_p^i}\end{bmatrix}x_{ai}(k) + \begin{bmatrix}C_{ai}B_{ai} & 0 & \cdots & 0\\ C_{ai}A_{ai}B_{ai} & C_{ai}B_{ai} & \cdots & \vdots\\ C_{ai}A_{ai}^2B_{ai} & C_{ai}A_{ai}B_{ai} & \cdots & C_{ai}B_{ai}\\ \vdots & \vdots & \ddots & \vdots\\ C_{ai}A_{ai}^{n_p^i-1}B_{ai} & C_{ai}A_{ai}^{n_p^i-2}B_{ai} & \cdots & C_{ai}A_{ai}^{n_p^i-n_c^i}B_{ai}\end{bmatrix}\Delta U_i(k) \tag{12}$$

where $Y_i(k+1) = \left[y_i(k+1) \cdots y_i(k+n_p^i)\right]^T$ and $\Delta U_i(k) = \left[\Delta u_i(k) \cdots \Delta u_i(k+n_c^i-1)\right]^T$. ΔU_i will be adjusted by optimizing a cost function, which is generally defined by

$$\min_{\Delta U_i}\left\{J_i = \sum_{j=1}^{n_p^i}\left\|Q_i\left(Y_{di}(k+j) - Y_i(k+j|k)\right)\right\|^2 + \sum_{j=1}^{n_c^i}\|R_i\Delta U_i(k+j|k)\|^2\right\}, \tag{13}$$

subject to

$$\underline{u}_i \le u_i(k) \le \overline{u}_i,\ \Delta\underline{u}_i \le \Delta u_i(k) \le \Delta\overline{u}_i,\ \underline{y}_i \le y_i(k) \le \overline{y}_i, \tag{14}$$

where $y_{di}(k)$ is the desired output trajectory. $Q_i \ge 0$ and $R_i > 0$ are weights on error between $Y_{di}(k)$ and $Y_i(k)$ $(e(k+j) = Y_{di}(k+j) - Y_i(k+j|k))$ and control action increment, respectively. There are some techniques to adjust proper weights in the cost function [26]. $\underline{u}_i$, $\Delta\underline{u}_i$, $\underline{y}_i$, and $\overline{u}_i$, $\Delta\overline{u}_i$, and $\overline{y}_i$ are lower and upper bands constraints on control action, control action increments, and output, respectively. The cost function is in charge of optimizing the error and control action.

3. Model selection criterion and its application

We proposed use of MM and OR to simplify complexity involved in high order nonlinear HVAC systems. Order of using these methods provides different results. There are two ways to recognize the preferable approach:

1- Implement both approaches and compare the closed-loop responses.

2- Introduce a criterion to predict the preferable approach.

The first idea is not reasonable in terms of computational load and time wasting. We introduce a criterion to cope with the second idea.

A) Reduced multiple model control algorithms

Two main characteristics of strong nonlinearity and high order encourage the researchers to call MM and OR techniques to provide better performance as well as simple controller design and implementation. "How to prioritize the techniques" is a challenge that leads to generate two creative approaches: OR-MM and MM-OR. In the first approach (OR-MM), the priority is to call OR technique and construct reduced order model bank. Then MM technique is summoned. This order is performed vice versa in the second approach (MM-OR).

Figures 2A and 2B, respectively, illustrate OR-MM and MM-OR approaches. In both cases the high order nonlinear HVAC model is decomposed into HOLLMs: P_i^{HL}, $i = 1, \dots, N$. However, they act differently in next steps. In OR-MM, HOLLMs are reduced by OR to make ROLLMs: $P_i^{\mathrm{RL}}, i = 1, \dots, N$. Then, RONLMs: P_i^{RN}, $i = 1, \dots, N_s$ are selected via MM method. While, in MM-OR, HOLLMs are categorized by MM to select HONLMs: P_i^{HN}, $i = 1, \dots, N_s$. Then, HONLMs are reduced to form RONLMs i.e., P_i^{RN}, $i = 1, \dots, N_s$. The remaining steps (design of nominal controllers $K_i, i = 1, \dots, N_s$ for RONLMs and combine them via weighing functions ω_i, $i = 1, \dots, N_s$ to construct the global controller) are the same for both approaches.

Priority in calling MM and OR provides two simple procedures to design the reduced multiple controller. For accurate comprehension, these approaches are described in two following subsections.

Algorithm 1: OR-MM approach

A1.1. Finding HOLLMs: Choose the scheduling variable (θ) and grid it to find N operating points. Then linearize the given model around the operating points and obtain N HOLLMs ($P_i^{\mathrm{HL}}, i = 1, \dots, N$).

A1.2. Finding ROLLMs: Reduce HOLLMs and obtain ROLLMs (P_i^{RL}, $i = 1, \dots, N$) via OR. In this way ROMB is constructed.

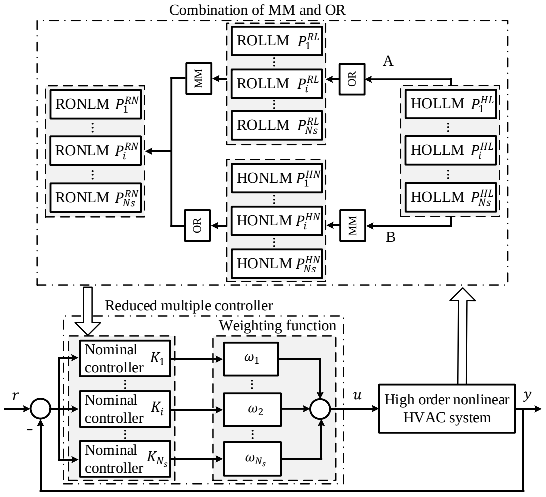

Fig. 2. Two proposed algorithms: A) OR-MM and B) MM-OR.

A1.3. Finding RONLMs: Select RONLMs from the ROMB according to (5) (P_i^{RN}, $i = 1, \dots, N_s^{\mathrm{OR-MM}}$). Due to the priority (first OR, next MM), this algorithm is referred to by OR-MM.

A1.4. Designing MPCs: Design MPC for each RONLM according to (13).

A1.5. Constructing global controller: Combine the MPCs as a global controller using soft/hard switching.

Algorithm 2: MM-OR approach

A2.1. Finding HOLLMs: Choose the scheduling variable (θ) and grid it to find N operating points. Then linearize the given model around the operating points and obtain N HOLLMs (P_i^{HL}, $i = 1, \dots, N$). In this way the High Order Models Bank (HOMB) is constructed.

A2.2. Finding HONLMs: Select HONLMs from the HOMB according to (5) (P_i^{HN}, $i = 1, \dots, N_s^{\mathrm{MM-OR}}$).

A2.3. Finding RONLMs: Reduce HONLMs via OR and obtain RONLMs (P_i^{RN}, $i = 1, \dots, N_s^{\mathrm{MM-OR}}$). Due to the priority (first MM, next OR), this algorithm is referred to by MM-RO.

A2.4. Designing MPCs: Design MPC for each RONLM according to (13).

A2.5. Constructing global controller: Combine the MPCs as a global controller employing soft/hard switching.

Remark 2. $N_s^{\mathrm{OR-MM}}$ and $N_s^{\mathrm{MM-OR}}$ and location of RONLMs are not necessarily the same in the above algorithms.

Remark 3. Quadratic GPC is implemented in this paper. However, other variants of MPCs could be studied as well. As already stated, P_i^{HL} and P_i^{RL} are high and reduced order local linear models, respectively. P_i^{HN} and P_i^{RN} are high and reduced order nominal linear models. Due to the fact explained in Remark 2, there will be a rightful demand to introduce a criterion so that it can predict the preferable approach in advance of implementing Algorithms 1 or 2, completely.

B) Model selection criterion

One of the proposed approaches (OR-MM and MM-OR) studied in this paper would provide better performance on HVAC systems.. MSC is a prediction tool to discover which approach will be more efficient. To compare the approaches, first the biggest gap $\delta_{\max}(i),\ i=1,\dots,N_s$ between nominal linear model(s) and local linear models in each sub region (ψ_i) is computed based on (4). Then, the maximum stability margin of nominal linear models $b_{\text{opt}}(i),\ i=1,\dots,N_s$ is determined according to (6). Finally, the following relation as a representative for nonlinearity measure (NM) [27] is provided

$$\frac{\delta_{\max}(i)}{b_{\text{opt}}(i)} \le 1,\ i=1,\dots,N_s. \tag{15}$$

Remark 4. Note that the nominal and local linear models are low order $(P^{\text{RN}}, P^{\text{RL}})$ in OR-MM and high order $(P^{\text{HN}}, P^{\text{HL}})$ in MM-OR.

Although NM represents a measure of nonlinearity (and therefore complexity), number of nominal models is also decisive. The number of nominal models denotes the model simplicity (MS). As a result, MSC should contain both NM and MS. The first term uses maximum gap metric and stability margin to measure the system nonlinearity and the second one uses number of nominal linear model(s) to check MS. Then, MSC is defined as follows

$$\text{MSC} = \sum_{i=1}^{N_s} \frac{\delta_{\max}(i)}{b_{\text{opt}}(i)} + \frac{1}{N_s}\log N_s,\ i=1,\dots,N_s \tag{16}$$

We propose Algorithm 3 to decide the preferred approach.

Algorithm 3: MSC and its application

A3.1. Select $P^{\text{RL}}(i),\ i=1,\dots,N$ and $P^{\text{RN}}(i),\ i=1,\dots,N_s$ in OR-MM via Algorithm 1 and $P^{\text{HL}}(i),\ i=1,\dots,N$ and $P^{\text{HN}}(i),\ i=1,\dots,N_s$ in MM-OR via Algorithms 2.

A3.2. Compute the maximum gap $\delta_{\max}(i)$ between $P^{\text{RL}}(i)$ and $P^{\text{RN}}(i)$ and between $P^{\text{HL}}(i)$ and $P^{\text{HN}}(i)$ for $i=1,\dots,N_s$ according to (4).

A3.3. Calculate the maximum stability margins of $P^{\text{RN}}(i)$ and $P^{\text{HN}}(i)$ $(b_{\text{opt}}(P^{\text{RN}}(i)), b_{\text{opt}}(P^{\text{HN}}(i))),\ i=1,\dots,N_s$ according to (6).

A3.4. Calculate MSC according to (16).

The preferred approach will have smaller MSC. Algorithm 3 is implemented to ensure which approach is better in the closed-loop sense, while the controller has not been designed yet.For additional description, the flow chart presenting the proposed criterion is depicted in Figure 3.

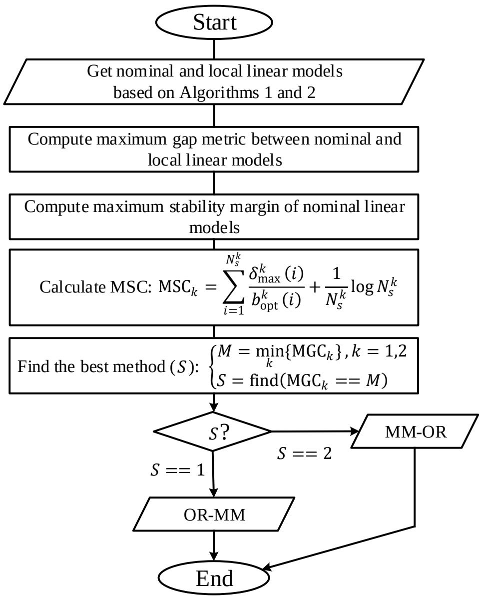

Fig. 3. Flow chart of MSC algorithm.

4. Simulation

HVAC as one of popular complex systems is equipped with two boxes: AHU and VAV. AHU box manipulates supply air temperature and VAV box contains a damper to control the mass flow rate. Three factors affect the system as disturbance: people, equipment, and extreme weather conditions. People and equipment like lights are the IDL, while weather conditions are the EDL. Opening the door and windows can act upon air temperature as EDL. The system diagram for single zone is depicted in Figure 4. It is important to manage the inside temperature in the presence of IDL and EDL that are uncontrolled inputs.

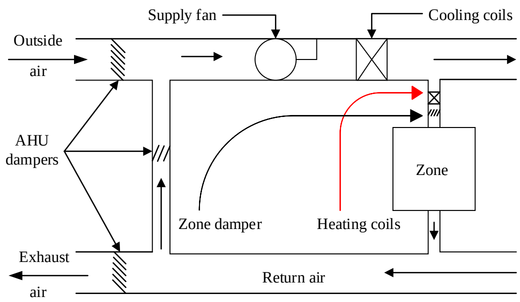

Fig. 4. HVAC diagram.

The following model is frequently used to represents an HVAC dynamic. The zone (T_z), south wall (T_{sw}), north wall (T_{nw}), east wall (T_{ew}), west wall (T_{ww}), and roof wall (T_r) temperatures are six state variables. f_{sa} and T_z are system input and output, respectively. T_{sa} is supply air temperature. It is assumed that T_{sa} is constant [2]. $q(t)$ denotes IDL and $T_o(t)$, outside air temperature, represents EDL. All parameters are described in Table 1.

$$\begin{cases} c_z \dfrac{dT_z}{dt} = f_{sa} c_{pa}(T_{sa} - T_z) + U_{sw}A_{sw}(T_{sw} - T_z) + U_{nw}A_{nw}(T_{nw} - T_z) + \\ \quad U_{ew}A_{ew}(T_{ew} - T_z) + U_{ww}A_{ww}(T_{ww} - T_z) + U_r A_r (T_r - T_z) + q(t) \\ c_s \dfrac{dT_{sw}}{dt} = U_{sw}A_{sw}(T_z - T_{sw}) + U_{sw}A_{sw}(T_o - T_{sw}) \\ c_n \dfrac{dT_{nw}}{dt} = U_{nw}A_{nw}(T_z - T_{nw}) + U_{nw}A_{nw}(T_o - T_{nw}) \\ c_e \dfrac{dT_{EW}}{dt} = U_{ew}A_{ew}(T_z - T_{ew}) + U_{ew}A_{ew}(T_o - T_{ew}) \\ c_w \dfrac{dT_{ww}}{dt} = U_{ww}A_{ww}(T_z - T_{ww}) + U_{ww}A_{ww}(T_o - T_{ww}) \\ c_r \dfrac{dT_r}{dt} = U_r A_r (T_z - T_r) + U_r A_r (T_o - T_r) \end{cases} \quad (17)$$

To optimize energy usage, two levels are considered to track the desired zone temperature (T_{zd}). If the zone, like conference hall, is empty, it will be reasonable to select the LL temperature. However, if the hall is occupied by participants, then the controller is responsible for the HL temperature. Thus, the zone temperature is in charge of tracking two levels: LL ($T_{zd}^{\text{LL}} = 16$ °C) and HL ($T_{zd}^{\text{HL}} = 23$ °C). The supply air temperature is assumed to be constant, $T_{sa}^{\text{LL}} = 15$°C for cooling ($T_{zd}^{\text{LL}} = 16$ °C)) and $T_{sa}^{\text{HL}} = 24$ °C for heating ($T_{zd}^{\text{HL}} = 23$ °C). Number of participants plays a significant role in setting the low or high levels.

Table.1.
HVAC parameters.

Parameter	***Description***	***Value***
U_{sw}	Heat transfer coefficient of the south wall	0.6 (W /m² °C)
U_{nw}	Heat transfer coefficient of the north wall	0.1 (W /m² °C)
U_{ew}	Heat transfer coefficient of the east wall	0.6 (W /m² °C)
U_{ww}	Heat transfer coefficient of the west wall	0.1 (W /m² °C)
U_r	Heat transfer coefficient of the roof	1 (W/m² °C)
A_{sw}	Area of the south wall	200 (m²)
A_{nw}	Area of the north wall	200 (m²)
A_{ew}	Area of the east wall	100 (m²)
A_{ww}	Area of the west wall	100 (m²)
A_r	Area of the roof	200 (m²)
c_z	Thermal capacitance of the zone	47.1 (J/°C)
c_s	Thermal capacitance of the south wall	60 (J/°C)
c_n	Thermal capacitance of the north wall	60 (J/°C)
c_e	Thermal capacitance of the east wall	70 (J/°C)
c_w	Thermal capacitance of the west wall	70 (J/°C)
c_r	Thermal capacitance of the roof	80 (J/°C)
c_{pa}	specific heat of air	1.005 (J /kg °C)

Remark 5. IDL is measurable to determine if the hall is occupied or not.

Remark 6. Number of people in the conference hall is a decision maker to adjust T_{sa} and T_{zd}.

Remark 7. The reference signal is continuous and differentiable during day and night.

Remark 8. The presence of air conditioners and heaters is necessary to optimize indoor air quality, when disturbances change rapidly.

The following settings are considered for supply air and desired zone temperatures.

$$T_{sa} = \begin{cases} \text{LL: } 15\text{°C}, & \text{IDL} \leq 0.01 \\ \text{HL: } 24\text{ °C}, & \text{IDL} > 0.01 \end{cases}$$
$$T_{zd} = \begin{cases} \text{LL: } 16\text{°C}, & \text{IDL} \leq 0.01 \\ \text{HL: } 23\text{ °C}, & \text{IDL} > 0.01 \end{cases} \quad (18)$$

The linearized model of HVAC system in (17) has the following matrices.

$$A_i = \begin{bmatrix} A_{i11} & \frac{U_{sw}A_{sw}}{c_z} & \frac{U_{nw}A_{nw}}{c_z} & \frac{U_{ew}A_{ew}}{c_z} & \frac{U_{ww}A_{ww}}{c_z} & \frac{U_r A_r}{c_z} \\ \frac{U_{sw}A_{sw}}{c_s} & -\frac{2U_{sw}A_{sw}}{c_s} & 0 & 0 & 0 & 0 \\ \frac{U_{nw}A_{nw}}{c_n} & 0 & -\frac{2U_{ew}A_{ew}}{c_n} & 0 & 0 & 0 \\ \frac{U_{ew}A_{ew}}{c_E} & 0 & 0 & -\frac{2U_{ew}A_{ew}}{c_e} & 0 & 0 \\ \frac{U_{ww}A_{ww}}{C_w} & 0 & 0 & 0 & -\frac{2U_{ww}A_{ww}}{C_w} & 0 \\ \frac{U_r A_r}{c_r} & 0 & 0 & 0 & 0 & -\frac{2U_r A_r}{c_r} \end{bmatrix}, \quad (19)$$

$$A_{i11} = -\frac{u(i)c_{pa} + U_{sw}A_{sw} + U_{nw}A_{nw} + U_{ew}A_{ew} + U_{ww}A_{ww} + U_r A_r}{c_z}$$

$$B_i = \begin{bmatrix} \frac{c_{pa}(T_{sa} - T_z(i))}{c_z} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, C = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0].$$

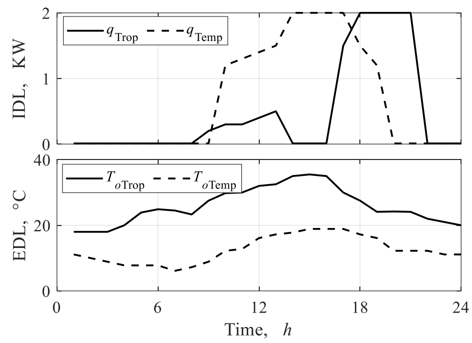

Fig. 5. The difference between internal and external load.

Since the HVAC system is affected by extreme EDL, two zones with different weather conditions, tropical and temperate climate zones, are considered. Figure 5 shows IDL and EDL considered in this paper for both tropical and temperate climate zones [2]. q_{Trop} and q_{Temp} denote IDL and $T_{o\text{Trop}}$ and $T_{o\text{Temp}}$ represent EDL of tropical and temperate climate zones, respectively. More information about these climate zones is provided in two following subsections.

A) Tropical climate zones

Stopping activities during some hours of day and night is fairly typical in tropical climate zones. Extreme hot weather causes to close any places like conference hall. Since the maximum outside temperature occurred during $t = 2$ pm to $t = 4$ pm (Figure 5), the conference hall would be empty ($q_{\text{Trop}} = 0$) in this period. Meanwhile, as Figure 5 shows the hall is closed during $t = 10$ pm to $t = 8$ am. Then, opening hours in tropical climate zones are $t = 9$ am to $t = 1$ pm in the morning shift and $t = 5$ pm to $t = 9$ pm in the evening shift.

According to the opening hours, T_{sa} and T_{zd} could be adjusted based on (18), i.e. when the hall is empty ($t = 10$ pm to $t = 8$ am and $t = 2$ pm to $t = 4$ pm), $T_{sa}^{\text{LL}} = 15$°C and $T_{zd}^{\text{LL}} = 16$°C and when it is occupied ($t = 9$ am to $t = 1$ pm and $t = 5$ pm to $t = 9$ pm), $T_{sa}^{\text{HL}} = 24$°C, and $T_{zd}^{\text{HL}} = 23$°C.

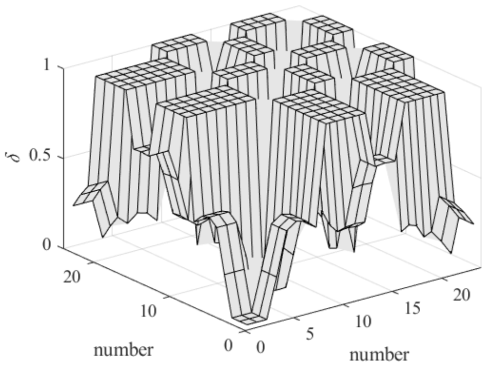

Fig. 6. Distance between ROLLMs of the HVAC system in tropical climate zones.

Supply, outside, and desired air temperatures are known (T_{sa}, T_o, T_{zd}). System output is defined as scheduling variable (θ). Then, the operating points can be calculated in twenty-four hours a day. HOLLMs (P_i^{HL}, $i = 1, \ldots, 24$) are obtained for day and night. According to OR-MM approach, ROLLMs are calculated based on (8). The gap metric values between $N = 24$ ROLLMs are computed and plotted in Figure 6. The maximum gap metric ($\delta_{\max} = 1$) shows that one nominal model is obviously not enough ($N_s^{\text{OR−MM}} \neq 1$). Besides, the maximum stability margin of ROLLMs is determined based on (6). Next, RONLMs are selected via (5). Three nominal models are required ($N_s^{\text{OR−MM}} = 3$) to satisfy the robust stability, two nominal models for occupied region and one nominal model for empty region. MM-OR strategy results in $N_s^{\text{MM−OR}} = 3$. As noted in Remark 2, the location of RONLMs is not necessarily the same in two proposed methods. Results of employing Algorithms 1 and 2 presented in Tables 2 and 3 indicate different locations in the proposed approaches. MSM represents the maximum stability margin in the tables.

Constrained MPC is scheduled for both OR-MM and MM-OR strategies and the closed-loop responses are shown in Figure 7. The cooling and heating constraints are given by $-20 \leq u_{\text{empty}} \leq 20$ and $-80 \leq u_{\text{occupied}} \leq 80$, respectively. Outputs in tropical climate zones are denoted by $y_{\text{OR−MM−Trop}}$ and $y_{\text{MM−OR−Trop}}$. In addition, the control inputs of OR-MM and MM-OR are expressed as $u_{\text{OR−MM−Trop}}$ and $u_{\text{MM−OR−Trop}}$, respectively. Since three nominal models have been considered, three weighting functions are used (HS in this paper). Figure 8 shows how such functions work. These functions have been used to combine the nominal controllers. To get more insight, four integral indices are computed. These indices are IAE, ISE, ITAE, and ITSE. Table 4 has listed all integral errors for two proposed approaches.

Table.2.
Results of OR-MM on HVAC in tropical climate zones.

Sub-region	**1st**	**2nd**	**3rd**
Included local models	22-8, 14-16	9-13	17-21
Operating point (x_{1e}, u_e)	15th $(16, 3.1821$	9th $(23, -0.73$	19th $(23, -0.181$
Reduced order	$n_r = 1$	$n_r = 2$	$n_r = 2$
$\delta_{\max}$	$\delta_{\max}(P_1^*) = 0.6946$	$\delta_{\max}(P_2^*) = 0.438$	$\delta_{\max}(P_3^*) = 0.6936$
MSM	$b_{opt}(P_1^*) = 0.9903$	$b_{opt}(P_2^*) = 0.5445$	$b_{opt}(P_3^*) = 0.7424$
Controller parameter (N_p, N_c, Q, R)	$(10, 1, 5000,$	$(10, 1, 5000$	$(10, 1, 50, 0.0$

Table.3.
Results of MM-OR on HVAC in tropical climate zones.

Sub-region	***1st***	***2nd***	***3rd***
Included local models	22-8, 14-16	9-13	17-21
Operating point (x_{1e}, u_e)	1st $(16, 0.3264$	9th $(23, -0.73$	19th $(23, -0.181$
Reduced order	$n_r = 1$	$n_r = 2$	$n_r = 2$
$\delta_{\max}$	$\delta_{\max}(P_1^*) = 0.7346$	$\delta_{\max}(P_2^*) = 0.438$	$\delta_{\max}(P_3^*) = 0.6936$
MSM	$b_{opt}(P_1^*) = 0.8728$	$b_{opt}(P_2^*) = 0.5445$	$b_{opt}(P_3^*) = 0.7424$
Controller parameter (N_p, N_c, Q, R)	$(15, 1, 100, 0.$	$(15, 1, 100, ($	$(15, 7, 10, 0.0$

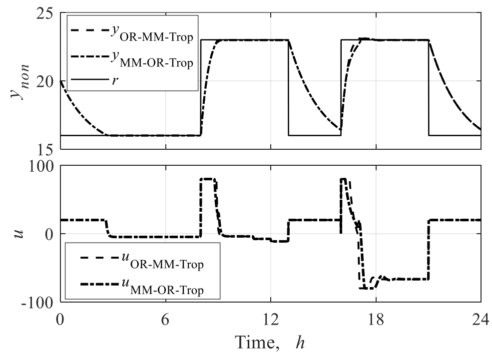

Fig. 7. Closed-loop response of the HVAC under the proposed approaches in tropical climate zones.

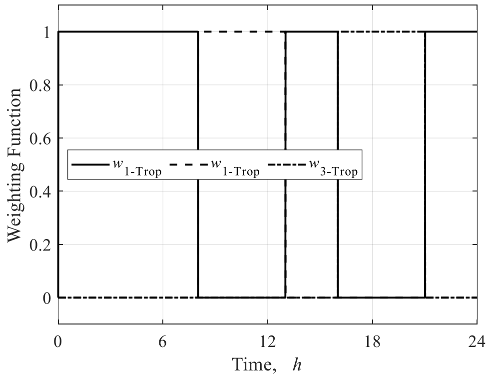

Fig. 8. Switching functions of HVAC in tropical climate zones.

Table.4.
Integral indices of HVAC system in tropical climate zones.

Strategy	OR-MM	MM-OR
IAE	25.674	26.0981
ISE	96.3266	96.6403
ITAE	363.6099	369.8497
ITSE	1431.4	1436.2

The integral indices indicate that OR-MM strategy performs better. By implementing Algorithm 3, MSCs are calculated as follows

$$\begin{cases} \mathrm{MSC}_1 = \frac{0.6946}{0.9903} + \frac{0.438}{0.5445} + \frac{0.6936}{0.7424} + \frac{1}{3}\log 3 = 2.5991 \\ \mathrm{MSC}_2 = \frac{0.7364}{0.8728} + \frac{0.438}{0.5445} + \frac{0.6936}{0.7424} + \frac{1}{3}\log 3 = 2.7394 \end{cases} \quad (20)$$

The most preferred approach can be identified by the following equation

$$\begin{aligned} &M = \min\{\mathrm{MSC}_1, \mathrm{MSC}_2\} = \min\{2.5991, 2.7394\} = 2.5991 \\ &S = \mathrm{find}(\{\mathrm{MSC}_1, \mathrm{MSC}_2\} == M) \to S = 1 \end{aligned} \quad (21)$$

The flow chart in Figure 3 states that $S = 1$ indicates OR-MM is the succeeded approach. The result of Algorithm 3 is consistent with the closed-loop results in Figure 7 and Table 4.

B) Temperate climate zones

In temperate climate zones, temperature is not high enough to stop activates in the middle of the day. As is shown in Figure 5, in contrast to tropical climate zones, the hall is not empty between $t =$ 2 pm and $t = 4$ pm. Indeed, the hall is occupied ($q_{\mathrm{Trop}} \neq 0$) between $t = 9$ am and $t = 6$ pm and is empty rest of day and night. Once the opening hours are specified, operating points and HOLLMs (P_i^{HL}, $i = 1, \ldots, 24$) could be determined. The gap metric values between all pairs of 24 local linear models and maximum stability margin of the models are calculated based on (4) and (6), respectively. The maximum gap metric ($\delta_{\max} = 1$) is larger than the maximum stability margin ($b_{opt} = 0.9746$). Then, it does not suffice to consider one single nominal model for empty and occupied regions.

Table.5.
Results of OR-MM on HVAC in temperate climate zones.

Sub-region	***1st***	***2nd***
Included local models	22-8	9-18
Operating point (x_{1e}, u_e)	19th $(16, -0.6165)$	9th $(23, 1.7578)$
Reduced order	$n_r = 2$	$n_r = 1$
$\delta_{\max}$	$\delta_{\max}(P_1^*) = 0.5395$	$\delta_{\max}(P_2^*) = 0.3541$
MSM	$b_{opt}(P_1^*) = 0.5862$	$b_{opt}(P_2^*) = 0.9202$
Controller parameter (N_p, N_c, Q, R)	$(10, 5, 5000, 0.01)$	$(10, 5, 5000, 0.01)$

Table.6.
Results of MM-OR on HVAC in temperate climate zones.

Sub-region	***1st***	***2nd***
Included local models	22-8	9-18
Operating point (x_{1e}, u_e)	19th $(16, -0.6165)$	14th $(23, 0.6693)$
Reduced order	$n_r = 2$	$n_r = 1$
$\delta_{\max}$	$\delta_{\max}(P_2^*) = 0.5395$	$\delta_{\max}(P_1^*) = 0.3863$
MSM	$b_{opt}(P_2^*) = 0.5862$	$b_{opt}(P_1^*) = 0.9746$
Controller parameter (N_p, N_c, Q, R)	$(10, 1, 100, 0.01)$	$(10, 1, 100, 0.01)$

OR-MM and MM-OR approaches have been implemented and results have been summarized in Tables 5 and 6, respectively. Like the other zone, a single nominal model cannot describe and control the nonlinear systems based on the robust stability condition. In this case, two nominal models ($N_S^{\mathrm{OR-MM}} = 2$) are required. The non-closure of conference hall has led to select one nominal model for empty region and the other one for occupied region.

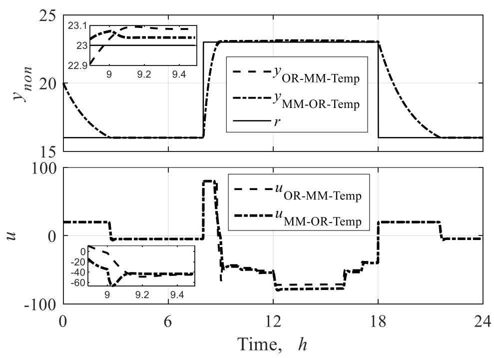

Fig. 9. Closed-loop response of the HVAC under the proposed approaches in temperate climate zones.

The summarized results in Tables 5 and 6 confirm Remark 2. The nominal model locations are not the same in two approaches. The closed-loop responses of HVAC system using OR-MM and MM-OR strategies in temperate climate zones are illustrated in Figure 9. $y_{\text{OR-MM-Temp}}$ and $u_{\text{OR-MM-Temp}}$ and $y_{\text{MM-OR-Temp}}$ and $u_{\text{MM-OR-Temp}}$ denote the output and input based on OR-MM and MM-OR, respectively. Table 7 indicates the integral indices of these structures.

Algorithm 3 is implemented and MSCs are calculated as follows.

$$\begin{cases}\text{MSC}_1 = \dfrac{0.5395}{0.5862} + \dfrac{0.3541}{0.9202} + \dfrac{1}{2}\log 2 = 1.4342 \\ \text{MSC}_2 = \dfrac{0.5395}{0.5862} + \dfrac{0.3863}{0.9746} + \dfrac{1}{2}\log 2 = 1.4907\end{cases} \quad (22)$$

Table.7.
Integral indices of HVAC system in temperate climate zones.

Strategy	OR-MM	MM-OR
IAE	15.3453	15.9856
ISE	53.7537	54.2509
ITAE	190.8926	199.7218
ITSE	731.9977	740.7008

Then, the preferred approach is decided by

$$\begin{cases}M = \min\{\text{MSC}_1, \text{MSC}_2\} = \min\{1.4342, 1.4907\} = 1.4342 \\ S = \text{find}(\{\text{MSC}_1, \text{MSC}_2\} == M) \rightarrow S = 1\end{cases} \quad (23)$$

where $S = 1$ indicates that OR-MM performs better. In occupied region, the nominal models are different. Hence, the closed-loop responses of two proposed structures are not the same in the region. As shown in Figure 9, MM-OR needs more control effort than OR-MM. Although two structures can track the reference signal, more energy saving and more comfort are provided by OR-MM.

5. Conclusion

In this paper, two new approaches called OR-MM and MM-OR, and a new criterion called MSC are proposed. Use of OR-MM and MM-OR approaches could provide simplicity in design and implementation of controllers for high order nonlinear HVAC systems. The gap metric, maximum stability margin, and model order reduction methods are utilized in these approaches. First model order reduction and then gap metric and maximum stability margin are summoned in OR-MM, whereas gap metric and maximum stability margin are utilized before the model order reduction methods in MM-OR.

Two proposed approaches are compared based on MSC to select the one providing better performance. This selection is done prior to enter the controller design stage. The maximum gap metric, maximum stability margin, and number of nominal models are used to determine this criterion value. By applying the proposed methods, simplicity could be definitely obtained. This is while the difficulty in controller design will be one of the undeniable problems if we have directly designed the controller for high order nonlinear systems.

The proposed methods and criterion are investigated on HVAC system. The closed-loop simulations demonstrate that the proposed approaches could perform well when the system is not only high order but also highly nonlinear. Since MM technique is suitable for systems with wide operating space and OR simplifies high order systems, such a combination could help one to find simple controller from design and implementation points of view. Although both OR-MM and MM-OR satisfy the requirements well enough, one could perform better. The less MSC represents the preferred approach.

In addition, the local stability does not guarantee the global stability of the high-order nonlinear system, increasing the number of nominally linear models can increase the global stability condition. On the other hand, increasing the number of linear models requires more effort to design controllers. Therefore, designers must make a trade-off between increasing stability and calculations, which can be suggested as a future work.

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