A data-based comparison of the two undesirable output disposability approaches in environmental efficiency

Abstract:

Data envelopment analysis (DEA) was originally developed to evaluate entities that carry out production processes in which multiple inputs are converted into multiple outputs. Subsequent advancement in DEA literature also points out the production of undesirable outputs. The most common approach which has attracted the researcher attention was the assumption of weak disposability. This axiom makes the proportional reduction for desirable and undesirable outputs in underlying technology. Linking to individual-proportion reduction of bad outputs, this study aims to reveal the contribution of non-disposed inputs or the idle inputs in production process. Toward this end, a simple modification is presented. The modified model demonstrates the effect of pointless inputs in reduction undesirable outputs and implies that fading their effect may improve the efficiency measure and optimize the performance. A real example compares the results and illuminate the model practicality.

Keywords: Undesirable outputs, Data Envelopment Analysis (DEA), Decision Making Unit (DMU), Weak disposability, Environmental assessment, performance analysis.

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1. Introduction

The present era has been witnessed an expanding interest in the realm of economic development and productivity. The economic development refers to the improvement of living standards in a society from a low quality to affluence. A key factor contributing in this process is the existence of effective manufacturing. In the literature of productive efficiency analysis, the seminal paper of Charnes et.al (1978) and the extended work by Banker et.al (1984) have been mostly implemented to evaluate the relative efficiency of comparable decision-making units (DMUs) which consume multiple inputs to produce multiple outputs. The original Data Envelopment Analysis (DEA) model assumed that all outputs are desirable and decision makers would like to increase production of these good outputs and to decrease the level of inputs. However, the production process often results in undesirable outputs which the decision makers tend to reduce these factors. Modeling undesirable outputs such as water pollution, waste, greenhouse gas emission (a mixture of carbon dioxide, methane, nitrous oxide, water vapor, and ozone), noise pollution, and etc. has attracted considerable attention in DEA literature. In the environmental performance analysis, one common approach for treating undesirable outputs is assuming the weak disposability of undesirable output. This assumption which implies to the reduction of undesirable outputs by decreasing the level of production activity goes back to Shepard (1970). The weak disposability conditions establishes the essential principle in the production nature, i.e., the more production of desirable output inevitably accompanies the additional generation of undesirable output. This is because desirable and undesirable outputs cannot be produced independently, i.e., one is necessarily linked with the other. Fare and Grosskopf (2003, 2004) was the first to propose DEA-based approach by imposing a single abatement factor for all units. However, the use of this single abatement factor affects the production set. Kuosmanen (2005) showed that employing the uniform abatement factor tend to overestimate the technical efficiency and the benchmark for inefficient units maybe technically inefficient. Podinovski and Kuosmanen (2011) followed the debate of weak disposability with a further light to modeling undesirable outputs under relaxed convexity assumption. . Linking to weak disposable axioms in most DEA studies, the abatement factor posits that a proportional reduction in the level of undesirable outputs can be achieved if accompanied by a reduction in desirable outputs in the same proportion. Several researchers have proposed methods to address undesirable outputs. Amirteimoori et.al (2017) proposed an additive definition of weak disposability based on an axiomatic foundation to construct an alternative production technology set. The proposed model can be justified assuming the fact that zero undesirable (desirable) outputs do not necessarily require zero desirable (undesirable) outputs. Roshdi et al. (2018) introduced a new concept of exponential weak disposability assumption for undesirable outputs, allowing for different types of trade-offs between desirable and undesirable outputs. Implementing three axioms (concavity, linearity, and convexity), a piecewise Cobb-Douglas environmental technology was derived. Based on this technology, radial and non-radial functions were extracted to measure environmental performance. As another effort, Mehdiloo and Podinovski (2019) argued that the disposability assumption may not be suitable and could lead to meaning less efficiency scores when inputs or outputs are overlapping or strongly correlated. To tackle with this deficiency, they developed a production technology in which groups of closely related inputs and outputs are only jointly weakly disposable. Mehdiloozad and Podinovski (2018) concluded that the input weak disposability assumption based on Shepard technology can cause problematic side effects, such as congestion measurement. To address this issue, the authors developed a suitable technology that incorporates weak input disposability. Then, based on progressively relaxed convexity assumptions, various ranges of technologies were also investigated. Pham and Zelenyuk (2019) discussed the use of single or multiple scaling factors in different scenarios and revealed the link between various returns to scale and weak disposability of desirable and undesirable outputs. Another contribution of their study was the construction of a comprehensive taxonomy of reference technology sets for activity analysis models with various return to scale assumptions. Shen-Ren Piao et.al (2019) proposed three DEA models considering weak disposability, strong disposability and distinguishing weak and strong disposability of undesirable outputs, respectively. Qiang Cui (2021) examined five disposability approach for undesirable outputs. The main contribution of the study is to attempt a data-based comparison of these five approaches, which is an effective supplement of the theoretical comparison in literature. The author contributed detailed models for these five approaches based on range adjusted measure (RAM) model proposed. The data of 29 international airlines from 2008 to 2014 are applied to compare the disposability approaches in airline environmental efficiency. A review of the DEA literature also reveals numerous DEA models for modeling undesirable inputs using the concept of weak disposability. Monzeli et al. (2020)

determined efficiency measurements in the presence of undesirable inputs and outputs using a three-step approach: first, an appropriate production possibility set was defined based on problem assumptions; second, the undesirable effects in DMUs were modeled by considering the weak disposability assumption; and third, the efficiency of DMUs was calculated using a radial DEA model. Jo et.al (2023) incorporated undesirable output satisfying weak disposable axiom in the slack-based measure (SBM) model then combined it with the bootstrapping technique. Two issues has been investigated in the study. With constructing the SBM–DEA-based bootstrapping model, some DMUs were placed out of frontier which leads to negative slack problems. The concept of super-efficiency were solved this issue. The second issue was according to the characteristics of data employed. The authors employed a revised data driven method to handle this deficiency. Kao and Hwang (2020) proposed a common-proportion model for determining the minimum level of the undesirable outputs that cannot be avoided based on the assertion of output weak disposable. In their approach, a common abatement reduction was employed for all units to show the amount of undesirable output that should be allowed to generate. Compared with the conventional models in the literature, their findings admitted that the idea of common-proportion abatement factor related to the effect of generating excessive amounts of undesirable outputs obtains more logical results. Kao and Hwang (2023) showed that their corresponding technology is not correct. The first deficiency was back to ignoring the consumed inputs in generating undesirable outputs and the second was concerned with return to scale axiom which assumed to be constant, while in subsequent technology was assumed to be variable. Rectifying these shortcomings, the proposed individual-proportion model allows for the reduction factor to be different for each DMU. The comparison was stated that the proposed individual - proportional model was able to separate the effect of producing undesirable outputs from the inefficiency of producing the desirable outputs in measuring the efficiency of a production unit. Maghbouli et.al (2024) proposed a non-radial model grounded on a non-uniform abatement factor. The application of this proposed model anticipates a suitable quantity for the decreasing of undesirable outputs. Concurrently, the model ensures a corresponding and satiable amount for reduction in undesirable outputs. Implementing weak disposability axioms and with reference to units' potential, a reliable abatement factor can be determined.

Although each approach in the literature has its merits and different perspectives are determined, the application of the weak disposability axiom in activity analysis continues to elicit questions. Surveying the existing studies reveals that solving DEA-based models with full reduction of all undesirable outputs leads to incorrect and unreliable results because the performance evaluation is based on the production technology where all the outputs must be produced by all the inputs. With respect to weak disposability, the most attention has been given to undesirable outputs reduction, while the production pertains to the consumption of inputs. However, the contribution of inputs in production process are not adequately distinguished. This means that some inputs may be idle or non- disposed in producing outputs. For example, in a brick-kiln production process, the coal and labors are used as inputs to produce the final product brick. In busy days of working all capacities of inputs are employed, but in leisure days, some of labors (as inputs) are out of work and held idle. This example shows that in some circumstances, the idle inputs integrate in producing both desirable and undesirable outputs. However, their contribution are scaled down with employing the non -negative weights. Now the question arise here. If the inputs are held idle, why their contribution are regarded non -negative as the disposed inputs. What if, the share of idle inputs are analyzed as a free variable in the production process. In this case we should not expect to have the same efficient frontiers as those defined by other approaches. What's more, a firm may be inefficient according to our approach, while it is efficient according to another approach. By targeting this questions and linking to weak disposable axiom, this study analyze the impact of contribution of both non-disposed and disposed inputs. The reminder of the paper is organized as follows. Section 2 provides a brief overview of weak output disposability axioms, followed by a modification of weak output disposability in Section 3. Finally, conclusion will end the paper.

2. Weak Disposable Technology

The conventional DEA models emphasize on maximizing outputs and minimizing inputs to improve the efficiency measure. However, in many real contexts, there are undesirable factors such as emission of harmful substances in air, energy wasted in power plant, and it is crucial to minimize these factors. Suppose that there are K DMUs and for DMU_k $(k=1,...,K)$ data on the vectors of inputs, desirable and undesirable outputs are presented $x_k=(x_{1k},...,x_{Nk})\geq 0, v_k=(v_{1k},...,v_{Mk})\geq 0$ and $w_k=(w_{1k},...,w_{Jk})\geq 0$, respectively. Further assume that $x_k\neq 0, v_k\neq 0$ and $w_k\neq 0$. The production possibility set is characterized by the set

$$P(x)=\{(x,v,w) \mid x \text{ can produce } (v,w),\ x\in\mathbb{R}_N^+\}$$

According to Shepard (1970), outputs (desirable and undesirable) are weakly disposable if and only if $(v,w)\in P(x)$ and $0\le\theta\le1$ imply that $(\theta v,\theta w)\in P(x), x\in\mathbb{R}_N^+$.

The multiplier θ in the definition above refers to the abatement factor, which allows for proportional decrease in outputs according to the conditions $0\le\theta\le1$ presented. Although, the level of undesirable outputs would be equal to zero if and only if $\theta=0$ and in this case, the level of desirable outputs is also equal to zero. Fare and Grosskopf (2003) proposed the following technology under variable return to scale satisfying weak-disposability assumption:

$$\widehat{Y}_s=\Big\{(x,v,w)\Big|\sum_{k=1}^{K} z^k x_n^k \le x_n\ ,\ n=1,...,N$$
$$\sum_{k=1}^{K}\theta z^k v_m^k \ge v_m,\ m=1,...,M$$
$$\sum_{k=1}^{K}\theta z^k w_j^k = w_j,\ \ j=1,...,J \qquad (1)$$
$$\sum_{k=1}^{K} z^k =1$$
$$z^k\ge 0,$$
$$0\le\theta\le1\Big\}$$

The abatement parameter θ in the formulation (1) corresponds to Shepard's definition of weak disposability. This parameter allows for common proportion reduction for desirable and undesirable outputs. The variable $z=(z^1,z^2,...,z^k)$ is characterized as intensity variable. Kuosmanen (2005) employed the individual-proportion or non-uniform abatement factor and proposed the following production technology:

$$\widehat{Y}_K=\Big\{(x,v,w)\Big|\sum_{k=1}^{K} z^k x_n^k \le x_n\ ,\ n=1,...,N$$
$$\sum_{k=1}^{K}\theta^k z^k v_m^k \ge v_m \ge 0\ ,\ m=1,...,M$$
$$\sum_{k=1}^{K}\theta^k z^k w_j^k = w_j\ ,\quad j=1,...,J \qquad (2)$$
$$\sum_{k=1}^{K} z^k =1$$
$$z^k\ge 0,$$
$$0\le\theta^k\le1\Big\}$$

Kuosmanen (2005) stated that the non-linear technology (2) can be expressed in its equivalent linear form by substituting the intensity variable $z^k=\lambda^k+\mu^k$, where λ^k represents weights of inputs actively used in production, whereas, μ^k captures the weights of inputs that are held idle. In other words the vector $(\mu v,\mu w)$ is abated using scale-down property of weak disposability. The linear technology has the following format:

$$\widehat{Y}_K^{(L)} = \left\{ (x,v,w) \middle| \sum_{k=1}^{K} (\lambda^k + \mu^k) x_n^k \le x_n \ , \ n = 1,...,N \right.$$
$$\sum_{k=1}^{K} \lambda^k v_m^k \ge v_m \ , \ m = 1,...,M$$
$$\sum_{k=1}^{K} \lambda^k w_j^k = w_j \ , \ j = 1,...,J \qquad (3)$$
$$\sum_{k=1}^{K} (\lambda^k + \mu^k) = 1$$
$$\lambda^k \ge 0,$$
$$\left. \mu^k \ge 0 \right\}$$

Using this notation $z^k = \lambda^k + \mu^k$, the linear model of evaluating the efficiency of DMU_o is stated as follows:

$$\text{Min } \theta$$
$$\text{s.t.}$$
$$\sum_{k=1}^{K} (\lambda^k + \mu^k) x_n^k \le x_n^o \quad , n = 1,...,N$$
$$\sum_{k=1}^{K} \lambda^k v_m^k \ge v_m^o \ , \ m = 1,...,M$$
$$\sum_{k=1}^{K} \lambda^k w_j^k = \theta w_j^o \quad , j = 1,...,J \qquad (4)$$
$$\sum_{k=1}^{K} (\lambda^k + \mu^k) = 1$$
$$\lambda^k \ge 0, \ \ \mu^k \ge 0$$

In this linear formulation, $\lambda^k = \theta^k z^k$ represents the disposed part of inputs used actively in production, whereas, $\mu^k = (1-\theta^k) z^k$ presents the weights of non-disposed or idle inputs. Both variable are non-negative.

3. Methodology

The abatement factor, θ^k are discussed in the previously mentioned technologies, belongs to the closed interval $[0,1]$. Also, the definition of weak disposability is given in a multiplier form. In this sense, the level of undesirable outputs would be equal to zero if and only if $\theta = 0$ and in this case, the level of desirable output is also equal to zero. However, the contribution of inputs in producing desirable and undesirable outputs are not clarified. In other words, it may be possible to produce the output vectors when some of inputs are not disposed. For example, in leisure time of a Brick-kiln production, some labors are put aside or held idle. In this sense, the outputs are produced with disposed inputs, whilst, the non-disposed input are present in the system and are not removed from the production process. That is to say, the non-disposed inputs are present in the system but they do not contribute in the production. As a matter of fact, in some cases, the underlying technology addresses to deal with the maximum reduction with the aim of performance measurement with both parts of inputs. To obtain reliable result and improve applicability, a modification appears warranted. Furthermore, determining the contribution of inputs is in line with the underlying technology with reference weak disposability assumption. Model (4), discussed in the previous section, and solely focuses on decreasing the undesirable outputs with non-negative contribution of both input parts. This perspective may lead to different efficiency measures and, in some cases, deviate from reality. Consequently, it is logical to modify the model to not only support the reduction of undesirable output but also encourage the satiable share of both disposed and non-disposed inputs linking to scaling property of weak disposability. This modification may develop approaches aimed at addressing the problem in the presence of undesirable outputs. In order to clarify the share of inputs in output generation, linking to concept of weak disposability, the non-negative variable of μ^k which represents the weights of non-disposed inputs are replaced with an unrestricted variable instead. Since, the related inputs are idle and do not cooperate in production process. Using these changes of variables, the modified model may lead to the reduction of bad outputs regarding to satiable share of disposed inputs as expected in the production process and underlying technology. Again, assume that, there are K DMUs and for DMU_k $(k=1,...,K)$ data on the vectors of inputs, desirable and undesirable outputs are presented $x_k=(x_{1k},...,x_{Nk})\geq 0, v_k=(v_{1k},...,v_{Mk})\geq 0$ and $w_k=(w_{1k},...,w_{Jk})\geq 0$, respectively. Further assume that $x_k\neq 0, v_k\neq 0$ and $w_k\neq 0$. To evaluate the efficiency of DMU_k $(k=1,...,K)$ with the aim of integrating input contributions, Model (3) can be modified accordingly:

$$\begin{aligned}
&\pi = \text{Min } \rho+\varepsilon\sum_{k=1}^{K}\mu^k x_n^k \\
&\text{s.t.} \\
&\sum_{k=1}^{K}(\lambda^k+\mu^k)x_n^k \leq x_n^o \quad ,n=1,...,N \\
&\sum_{k=1}^{K}\lambda^k v_m^k-\sum_{k=1}^{K}\mu^k \geq v_m^o \ , \ m=1,...,M, k=1,...,K \\
&\sum_{k=1}^{K}\lambda^k w_j^k-\sum_{k=1}^{K}\mu^k = \rho w_j^o \quad , j=1,...,J \ ,k=1,...,K \qquad (5) \\
&(\lambda^k+\mu^k)x_n^k \geq \mu^k, \ \ n=1,...,N, k=1,...,K \\
&\lambda^k \geq 0, \mu^k \ \text{ is free in sign}
\end{aligned}$$

Upon close examination, all constraints within the modified model support the idea of DEA weak output disposability. It can be easily seen that the Model (5) is feasible and it always holds that $\rho^*\leq 1$. The first constraint of the Model (5) captures both disposed and non-disposed inputs in production and are as usual weak disposable constraint in Model (4). The requirement for remaining idle of non-disposed input are held with employing the term $-\sum_{k=1}^{K}\mu^k$ in second and third constraints. The last constraint ensures that the weights are used actively in producing outputs and the weights of inputs which are held idle. In terms of efficiency measurement, we scope on minimizing the potential change of each unit in the observed data set. The objective function of Model (5) represents an abatement in undesirable outputs that stems from the transformation of non-uniform abatement factor for all units in the first term. The second term in the objective function, $\varepsilon\sum_{k=1}^{K}\mu^k x_n^k$ points out the decline of pointless input's effect on the production process. The existence of non-Archimedean ε in the second term confirms the priority of idle input reduction concerning the reduction of undesirable outputs. When evaluating using Model (5), DMU_o is said to be efficient if and only if the corresponding optimal value of the objective function is equal to one.

4. Numerical Example

The applicability of the proposed approach is demonstrated using a real data set consisting of thirty units. The data set origins from Kao and Hwang (2020). The data set consist of thirty paper mills along the HUAI River in Anhui Province, China. Each unit employs two sets of inputs to produce two categories of outputs: two desirable and one undesirable output. The data set are depicted in Table1.

Table 1: Data set for thirty paper mills

DMU	Input1 x_1	Input2 x_2	D.Output2 v_1	D.Output1 v_2	Und. Output w_1
1	437	1438	2015	14667	665
2	884	1061	3452	2822	491
3	1160	9171	2276	2484	417
4	626	10151	953	16434	302
5	374	8416	2578	19715	229
6	597	3038	3003	20743	1083
7	870	3342	1860	20494	1053

8	685	9984	3338	17126	740
9	582	8877	2859	9548	845
10	763	2829	1889	18683	517
11	689	6057	2583	15732	664
12	355	1609	1096	13104	313
13	851	2352	3924	3723	1206
14	926	1222	1107	13095	377
15	203	9698	2440	15588	792
16	1109	7141	4366	10550	524
17	861	4391	2601	5258	307
18	249	7856	1788	15869	1449
19	652	3173	793	12383	1131
20	364	3314	3456	18010	826
21	670	5422	3336	17568	1357
22	1023	4338	3791	20560	1089
23	1049	3665	4797	16524	652
24	1164	8549	2161	3907	999
25	1012	5162	812	10985	526
26	464	10504	4403	21532	218
27	406	9365	1825	21378	1339
28	1132	9958	2990	14905	231
29	593	3552	4019	3854	1431
30	262	6211	815	17440	965

Evaluations of these units with Model (4) and the proposed Model (5) are recorded under the heading of Model (4) and Model (5) in Table 2. As the Table 2 shows there are five efficient DMUs out of thirty units in evaluating with Model (4). On the other hand, the proposed Model (5) evaluates one efficient units. The quantity of non-Archimedean ε in Model (5) is regarded $\varepsilon = 0.00001$.The average of efficiencies are recorded in the last row of Table 2. From the statistical point of view, the average efficiency of proposed Model (5) is significantly lower than of Model (4) with values 0.272 and 0.172, respectively.

Table2: Efficiency Score of Models (3) and (4)

DMU	θ^* Model(4)	π^* Model(5)	$\mu^{k*} > 0$
1	1	0.88	#15
2	1	0.10	#15
3	0.14	0.14	#15
4	0.08	0.08	#15,#25
5	0.64	0.62	#12,#25
6	0.08	0.07	#15,#25,#26
7	0.02	0.02	#15,#25,#26
8	0.10	0.10	#15,#25,#26
9	0.07	0.07	#15,#25,#26
10	0.08	0.08	#15,#25,#26
11	0.14	0.14	#15,#25,#26
12	1	1	#15,#25,#26
13	0.16	0.16	#15,#25,#26
14	0.13	0.13	#15,#25,#26
15	1	0.12	#15,#25,#26
16	0.29	0.29	#15,#25,#26
17	0.08	0.07	#15,#25,#26
18	0.01	0.01	#15,#25,#26
19	0.12	0.12	#15,#25,#26
20	0.30	0.30	#15,#25,#26
21	0.02	0.02	#15,#25,#26
22	0.04	0.04	#15,#25,#26
23	0.10	0.10	#15,#25,#26
24	0.04	0.04	#15,#25,#26
25	0.15	0.15	#15,#25,#26
26	0.22	0.22	#12,#25,#26
27	0.02	0.02	#5,#15,#25,#26
28	0.09	0.07	#5,#25,#26
29	0.06	0.06	#2,#5,#15,#25,#26
30	1	0.13	#2,#5,#15,#25,#26
Average	0.272	0.172	

Interestingly enough, the efficiency scores obtained by Model (5), are not greater than the efficiency scores of Model (4) formulated by the same technology. In a nutshell, Model (5) has been making an effort to make a fair contribution of disposed and non-disposed amount of input in producing the outputs utilizing the same technology. To do so, the positive quantity of free variable μ^{k*}, the share of non-disposed inputs, are recorded in the last column of Table2. Units #15, 26 and 25 have the most repetition in referring positive values for μ^{k*} that advocate the contribution of idle inputs for evaluated units. As the results show, the only efficient unit in Model (5) is unit#12. The results identify the contribution of pointless inputs which may deteriorate the efficiency score. As the results identify, for unit#30, the contribution of these inputs make the efficiency score meet the value of 0.13, whereas, in the original Model (4) it is efficient. This difference shows that the unit cannot meet its real efficiency score if the pointless inputs contributes in evaluation. In a nutshell, employing Model (5) highlights the contribution of pointless input and can pave the way for reflecting the degree of importance of effective inputs in efficiency measurement.

5. Conclusion

The classical definition of weak disposability abates desirable and undesirable output with the same proportion. Employing the non-uniform abatement factor within DEA research was one of the common approaches for addressing the continuing challenge in the realm of productivity and efficiency evaluation. This study aims to highlight the role of idle inputs in producing outputs. Since, the aim is reducing the bad outputs, the contribution of inputs are fade. While, outputs are generated with the contribution of disposed and non-disposed inputs. The proposed model identified the role of non-disposed inputs in the production. Also, the model offered that the contribution of idle inputs may deteriorate the efficiency score. A real example accompanied with data-based comparison illuminated the negative effect of pointless inputs in performance analysis.

6. References

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