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Profit Efficiency Evaluation: A composed Approach of DEA And multi- objective programming

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Abstract

Data Envelopment Analysis (DEA) is a nonparametric method for evaluating the relative efficiency of decision making units (DMUs) described by multiple inputs and multiple outputs. The issue of measuring the cost, revenue and profit efficiency in manufacturing and economic systems is one of the most important issues for managers. In this research, using Data envelopment analysis and multi-objective programming an attempt is made to provide a model for evaluating profit efficiency of banking industry. We apply data envelopment analysis (DEA) and multi-objective programming (MOP) models to measure profit efficiency as cost and revenue scores are as close as possible to their best scores and as far away as possible to their worst scores. The results showed that composing these two models, can directly affect the result and also findings of research distinguished the differences between the efficient DMUs from the point of view of DEA. In this study, Profit efficiency score has been obtained from a fairer perspective than the previous models. A numerical example of Iranian banking industry is used to illustrate the proposed model.

Keywords:
Data Envelopment Analysis (DEA)
Profit Efficiency
Multi-objective Programming

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INTRODUCTION

Data Envelopment Analysis (DEA) has been originated for measuring the relative efficiencies of a set of homogeneous decision making units (DMUs) that applies multiple inputs to generate multiple outputs. Currently, there has been a growing interest among decision makers in application of non-parametric techniques like DEA which extends markedly beyond the task of evaluating cost and revenue efficiency. Cost efficiency was first pioneered by Farrell (1957) and then extended by Fare et.al (1985). The following approach for modeling cost efficiency goes back to Camanho and Dyson (2005). The authors developed the traditional cost efficiency model into two various situations including precise known prices and incomplete price situations. Their model estimated upper and lower bound for cost efficiency evaluation in presence of price uncertainty. Jahanshahloo et.al (2008) continued their debate and refined the model with reducing the number of constraints and variables. In this respect, Jahanshahloo et.al (2011) offered an interpretation of cost models and introduced an alternative model foe assessment of cost efficiency assuming that the input prices of each DMU are accessible. In this framework, Amirteimoori et.al (2006) improved the cost efficiency interval of a DMU by adjusting its observed inputs and outputs. Camanho and Dyson (2005) contributed to this topic by obtaining cost efficiency from optimistic and pessimistic viewpoints including uncertain price. Considering uncertain price, Toloo and Ertay (2014) applied an alternative cost efficiency model based on DEA approach posits finding the most efficient unit. The concept of revenue efficiency was first debated in Fare et.al (1985). In their 1994 study [6], the authors improved the overall output price with the goal of maximizing revenue. As another instance, Fukuyama and Matousek (2017) expanded the environmental revenue function based on directional distance function in two-stage network structures. Another recent studies in revenue efficiency, Mogaddas and Vaez Ghasemi (2022) applied DEA approach to compute a specific set of weights to evaluate cost efficiency in a two-stage network system. A review of the DEA literature demonstrates that Fare et.al (2004) studies are widely recognized as a seminal reference in profit efficiency research. Fare et.al (2004) investigated two sources of inefficiency in assessing profit efficiency, including technical inefficiency and allocative inefficiency. Portela and Thanassoulis (2007) highlighted the drawbacks of existing approaches in the literature and suggested another measure of profit efficiency which is grounded in the geometric mean of input/output adjustments to achieve maximum profitability. Several researchers have proposed methods to address profit efficiency. A new indicator of profit inefficiency was suggested by Fukuyama and Weber (2008) emphasizing the choices made by decision-makers regarding the allocation of funds to inputs and the revenue derived from outputs, rather than the physical measurement of input and output quantities. Park and Cho (2011) introduced a linear programming model, for the evaluation of profit efficiency. The main focus of their paper was on approximation of profit efficiency in the absence of price information. Aparicio et.al (2013) illustrated the utility of DEA for measuring and decomposing revenue inefficiency. Their study considered all sources of technical waste with a specific emphasis on the Spanish quality waste sector. In a current study of all industries, the utilization of profit efficiency is becoming increasingly crucial especially in bank branch activities. In analyzing the literature, it is evident that several studies have advocated profit efficiency from the optimistic perspective. With respect to non-parametric Data Envelopment Analysis (DEA) models, the attention of this study has been given to both optimistic and pessimistic standpoint. This study examines the reasonable and equitable amount for profit regarding to costs incurred and generated revenues. The remainder of this study is organized as follows. Section 2 provides a brief overview of cost, revenue and profit efficiency. Then Section 3 formulates an alternative model for assessing profit efficiency as a Multi-Objective Programming (MOP) task. To clarify the details of the proposed method, a real case in banking sector is given in Section 4. Eventually, Section 5 concludes the paper.

PRELIMINARIES

According to Farrell [4] efficiency consist of two elements: technical efficiency (TE) and allocative efficiency (AE). TE refers to production where the best available technologies are applied and AE refers to allocation of inputs and products to different producers. Together, these efficiencies are named the economic efficiency, (EE defined. The (EE) is expressed in). The (EE) is expressed is different manners, depending on how the best available production technology is terms of cost minimization, revenue maximization or profit maximization. If cost minimization is assumed, The (EE) is expressed as (CE). In this case, CE constitutes a combination of inputs that generates the minimum possible cost. In a similar manner, the (EE) is expressed as (RE), RE constitutes a combination of outputs that generates the maximum possible revenue and if maximization of profit is of concern, the (EE) is expressed as profit efficiency (PE), that is, the amount of output that maximizes profit (Seyedboveir et.al, 2018).

Cost Efficiency

Suppose that there is a set of n decision-making units DMU_j $(j = 1. \dots . n)$. Let m, s be the numbers of inputs and outputs respectively. The term $x_{ij} \in R^+ (i = 1, \dots, m ; j = 1, \dots, n)$ is applied in the input resource i to DMU_j to produce the output$y_{rj} \in R^+ (r = 1, \dots, s ; j = 1, \dots, n)$, that is, the output product r fromDMU_j. Also let the unit price of all input be known, and $c_{ij} \in R^+$ shows the price of inputi fromDMU_j. Given these assumptions, the cost efficiency model can be written as follows:

$$
\begin{aligned}
& min \quad \sum_{i=1}^{m} c_{io}\, \bar{x}_{io} \\
& s.t. \\
& \sum_{j=1}^{n} \lambda_j x_{ij} \le \bar{x}_{io} \qquad i = 1, \dots, m \qquad (1) \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} \qquad r = 1, \dots, s; \\
& \lambda_j, \bar{x}_{io} \ge 0 \, .
\end{aligned}
$$

Model (1) is a constant return to scale (CRS), the observed cost obtained through DMU_o is presented as $\sum_{i=1}^{m} c_{io}\, x_{io}$. The cost efficiency of DMU_o (CE_o) is measured through :

$$CE_o = \frac{\sum_{i=1}^{m} c_{io}\, \bar{x}_{io}^*}{\sum_{i=1}^{m} c_{io} x_{io}}$$

In which $\bar{x}_{io}{}^*$ is the optimal solution of model (1).

Revenue Efficiency

Let p_{ro} be the price of the under evaluated unit (DMU_o) output r, then DEA model of revenue maximization is:

$$
\begin{aligned}
& R_o^* = max \quad \sum_{r=1}^{s} p_{ro} \bar{y}_{ro} \\
& s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{io} \qquad i = 1, \dots, m, \qquad (2) \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \ge \bar{y}_{ro} \qquad r = 1, \dots, s, \\
& \lambda_j , \bar{y}_{ro} \ge 0
\end{aligned}
$$

Model (2) is a constant return to scale (CRS). The revenue obtained through the DMU_o is equal to $\sum_{r=1}^{s} p_{ro} y_{ro}$. The revenue efficiency of DMU_o (RE_o) is measured through:

$$RE_o = \frac{\sum_{r=1}^{s} p_{ro} y_{ro}}{\sum_{r=1}^{s} p_{ro}\, \bar{y}_{ro}^*}$$

Profit Efficiency

According to the assumptions of the previous two parts, the profit maximization problem is solved as follows:

$$\begin{aligned} max \quad & \sum_{r=1}^{r_k} p_{ro}\bar{y}_{ro} - \sum_{i=1}^{m_k} c_{io}\bar{x}_{io} \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \le \bar{x}_{io} \qquad i = 1, \dots, m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \ge \bar{y}_{ro} \qquad r = 1, \dots, s, \qquad (3) \\ & \bar{x}_{io} \le x_{io}, \bar{y}_{ro} \ge y_{ro} \\ & \lambda_j \ge 0 . \end{aligned}$$

Where c_{io} is the price of input i and p_{ro} is the price of output r of DMU_o the profit obtained by the DMU_o is $\sum_{r=1}^{r_k} p_{ro}y_{ro} - \sum_{i=1}^{m_k} c_{io}x_{io}$ and profit efficiency (PE_o) of is measure as follows:

$$PE_o = \frac{\sum_{r=1}^{r_k} p_{ro}y_{ro} - \sum_{i=1}^{m_k} c_{io}x_{io}}{\sum_{r=1}^{r_k} p_{ro}\,\bar{y}_{ro}^* - \sum_{i=1}^{m_k} c_{io}\,\bar{x}_{io}^*}$$

PROPOSED METHODS

Surveying the previous studies on profit efficiency concept, the optimistic point of view is highlighted in their evaluation. However, the pessimistic perspective was disregarded. Although, the existing studies has its merits but addressing the fairest and the most appropriate amount of profit continues to elicit questions. To obtain reliable results and improve applicability, a modifications appears warranted. Hence, for achieving the best profit to the decision maker, this paper aims to determine the most appropriate amount of cost and revenue, simultaneously. Toward this end, the combination of data envelopment analysis (DEA) and multi-objective programming (MOP) is becoming increasingly crucial as it utilizes mutual attributes of each model. Models (1) and (2) discussed in the previous sections, solely provide minimum cost (C_o^+) and maximum revenue (R_o^+) as the best values of cost and revenue. As mentioned before, both models focus on optimistic points of view. In order to consider both optimistic and pessimistic perspectives in a linear model, it is logical to obtain the worst values, i.e., maximum cost (C_o^-) and minimum revenue(R_o^-). For this modification, it seems sufficient to maximize model (1) and minimize model (2) respectively. Considering the concept of multi-objective programming (MOP), dual perspectives are replaced in the proposed model. To achieve the fair and reasonable values of cost and revenue the multi-objective model is applied. The proposed model ensures the values as close as possible to the most optimistic values C_o^+ and R_o^+ and sufficiently far from values C_o^- and R_o^-. Again assume that there are n decision-making units DMU_j $(j = 1, \dots, n)$, the data for DMU_k on vectors of inputs and outputs are represented $x_{ij} \in R^+ (i = 1, \dots, m ; j = 1, \dots, n)$ and $y_{rj} \in R^+ (r = 1, \dots, s ; j = 1, \dots, n)$, respectively. Applying the modified objective function, the objective function of model (3) can be rewritten as a bi-objective function as follows:

$$\begin{aligned} & Max \quad -\sum_{i=1}^{m} c_{io}\,\bar{x}_{io} \\ & \qquad\qquad\qquad\qquad (4) \\ & Max \quad \sum_{r=1}^{s} p_{ro}\bar{y}_{ro} \end{aligned}$$

According to the amount of cost and revenue and employing the objective function (4), the fairest amount of profit can be achieved. Among different approach to solve a MOP, the Min-Max weighted format is adopted for our goal. In the following, we develop the proposed model with aggregating the two objective functions, which transforms the bi-objective function in a single weighted objective function. Aggregating the objective function (4) along with the constraints of Model (3), the following single-objective program is proposed.

$$\min\left[\max\left\{\mu_1\left(C_o^+ - \sum_{i=1}^{m_k} c_{io}x_{io}\right).\mu_2\left(R_o^+ - \sum_{r=1}^{r_k} p_{ro}y_{ro}\right)\right\}\right]$$

$$\begin{aligned} s.t. \quad & \sum_{j=1}^{n}\lambda_j x_{ij} \le \bar{x}_{io} \qquad i = 1,\dots,m_k, \\ & \sum_{j=1}^{n}\lambda_j y_{rj} \ge \bar{y}_{ro} \qquad r = 1,\dots,r_k \qquad (5) \\ & \bar{x}_{io} \le x_{io}, \bar{y}_{ro} \ge y_{ro} \\ & \lambda_j \ge 0\,. \end{aligned}$$

It should be noted that, the weights (μ_1, μ_2) defined in Model (5) are positive parameters. Also, solving the weighted Min-max formulation (5) means searching for a solution to obtain the most appropriate cost and revenue. In this sense, the above model (5) aligns more effectively with the concept of “the best” and “the worst” in the assessments. Getting advantages of employing the dual perspectives, the weights can be defined as follows:

$$\mu_1 = \frac{1}{C_o^+ - C_o^-} \qquad (6)$$
$$\mu_2 = \frac{1}{R_o^+ - R_o^-}$$

The proposed Model (5) transforms to Model (7) using the substitution of the above weights μ_1 and μ_2. The modified Model (7) has the following format:

$$\begin{aligned} & \min \delta \\ & s.t \\ & \frac{C_o^+ - \sum_{i=1}^{m_k} c_{io}x_{io}}{C_o^+ - C_o^-} \le \delta \\ & \frac{R_o^+ - \sum_{r=1}^{r_k} p_{ro}y_{ro}}{R_o^+ - R_o^-} \le \delta \\ & \sum_{j=1}^{n}\lambda_j x_{ij} \le \bar{x}_{io} \qquad i = 1,\dots,m_k, \qquad (7) \\ & \sum_{j=1}^{n}\lambda_j y_{rj} \ge \bar{y}_{ro} \qquad r = 1,\dots,r_k, \\ & \bar{x}_{io} \le x_{io}, \bar{y}_{ro} \ge y_{ro} \\ & \lambda_j \ge 0 \end{aligned}$$

Upon close examination, all the constraints within the modified Model (7), support the idea of achieving the fairest amount of profit. The first and second constraints in Model (7) guarantee a fair cost and revenue that is as close as possible to the most optimistic C_o^+ and R_o^+ and sufficiently far from C_o^- and R_o^-. Therewith, Model (7) addresses some issues. First, two optimistic and pessimistic perspectives are integrated in a combined model. Second, defining appropriate weights μ_1 and μ_2 in the objective function of Model (7) provide the most appropriate and fair amount of profit for each unit. The last but not least, as the objective function denotes, parameter δ, minimizes the deviation between the best value and the worst value. The rest of constraints are same as in Model (3). Considering model (7) searches for a solution where the deviation are equal and minimized. The main advantages of Model (7) over Model (3) is that it provides a suitable criterion for determining the difference between efficient DMUs. It can be easily demonstrated that Model (7) is always feasible.

Definition 1: The unit $\mathrm{DMU_o}$ in the evaluation with model (7) is considered efficient if the optimal value of the model is equal to 1.

NUMERICAL EXAMPLES

The applicability of the proposed Model (7) is demonstrated using a real data set including ten units. The data set are related to one of the Iranian bank. Input and output indicators have been considered according to past researches and experts' opinions. There are three inputs characterized by deposits(x_1), operating expenses (x_2) and facilities(x_3). The three outputs are reported by revenue from commissions(y_1), annual net profit (y_2) and transactions(y_3). The input and output data are given in Table 1.

Table 1: Data Set of Inputs and Outputs

DMU	x_1	x_2	x_3	y_1	y_2	y_3
1	0.948	1	0.337	0.879	0.437	0.537
2	1.330	0.993	0.180	0.538	0.282	0.280
3	0.621	0.675	0.198	0.911	0.098	0.658
4	1.783	0.897	0.491	0.570	0.391	0.461
5	1.892	1.290	0.372	1.086	0.472	0.372
6	0.990	0.856	0.253	0.722	0.263	0.153
7	0.151	0.987	0.241	0.509	0.131	0.441
8	0.108	0.203	0.097	0.619	0.097	0.267
9	1.364	0.432	0.380	1.023	0.380	0.470
10	1.992	0.956	0.178	0.769	0.176	0.288

By implementing the cost and revenue efficiency models, namely Models (1) and (2), the minimum cost (C_o^+) and maximum revenue (R_o^+) values are shown in the second and third columns of Table 2. In order to have the worst amount for cost and revenue, Models (1) and(2) are re-solved with maximization and minimization objective functions, respectively. The results of the maximum cost and minimum revenue are indicated by C_o^- and R_o^-. The results of Models (1) and (2) are reported under the heading of (C_o^+) and (R_o^+) in Table 2. The fourth and fifth column of Table 2 are reporting the amount of C_o^- and R_o^-, respectively. Applying Model (3) on the data set of Table 1 are represented in the sixth column of Table 2. The last two columns of Table 2 Show the amount of profit efficiency and the value of parameter δ. As the seventh column of Table 2 under the heading of "New Profit(Model(7)" shows, there are five units ,namely units# 3, 6, 7, 8 and 9 have the efficiency equal to one. In other words, in evaluation with Model (7), these units are efficient. On the other hand, Model (3) estimates four efficient units, units # 3, 7, 8 and 9 have been evaluated as efficient. As concerns the results obtained by the min-max Model (7), one can see that the profit efficiency score are greater than the counterpart Model (3). The fact that unit six, show the profit efficiency as unity, whilst Model (3) presents the amount of -0.201. What's more, existence of the deviation parameter δ in Model (7) advocates that the proposed Model (7) outperforms its counterpart Model (3). That is to say, the proposed Model (7) performs effectively among efficient units than Model (3) does. The effect of integrating the smallest distance between C_o^+ and C_o^- and between R_o^+ and R_o^-, are reported as parameter δ in the last column of Table 2. The value of parameter δ in Model (7) are reported as positive number except for the unit #8. It is revealed that Unit#8 has better performance among efficient units. In a nutshell, Model (7) outperforms Model (3) in this real example.

CONCLUSION

In spite of numerous advancement about profit efficiency in DEA, this study contributes the continuing debate in the realm of profit efficiency. To be specific, the existing studies consider the optimistic perspective in profit evaluation. Equipped with multi-objective models, a combined model was introduced for evaluation the best amount of profit efficiency. The main advantages of the proposed model was integrating dual perspectives in one model. Some issues are addressed in the proposed model. First, the proposed model aimed to minimize the distance between the minimum cost and the maximum cost, at the same time, the distance between the maximum income and the minimum income is also minimized. Second, according to the amount of cost and income, the proposed model can identify units with the best performance among profit efficient units. Additionally, the proposed model may elucidate the factors of profit inefficiency and provide valuable insights into challenges posed by inefficiency.

Table 2**:** Results of Numerical Example

DMU	C_o^+	R_o^+	C_o^-	R_o^-	Profit Model(3)	New Profit (Model7)	δ
1	0.921	8.602	1.501	0	-.039	0.698	0.07
2	0.523	4.595	0.802	0	-0.835	0.497	0.12
3	0.934	5.054	13.386	0	1	1	0.06
4	1.231	10.941	11.932	0	-.0695	0.995	0.08
5	1.069	9.496	9.628	0	-.0411	0.937	0.06
6	0.720	6.458	3.960	0	-0.201	1	0.09
`7	0.604	3.462	11.414	0	1	1	0.07
8	0.432	2.476	6.911	0	1	1	0.00
9	1.053	5.269	12.165	0	1	1	0.05
10	0.625	4.544	7.454	0	-1.230	0.567	0.07

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