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Comprehensive Performance Evaluation of Iranian Banking Groups: A Group Two-Stages Data Envelopment Analysis

Mohammad Sajjad Shahbazifar[1], Reza Kazemi Matin[2], Mohsen Khounsiavash[1]* and Fereshteh Koushki[1]

[1]Department of Mathematics, Qazvin Branch, Islamic Azad University, Qazvin, Iran.
[2] Department of Mathematics, Karaj Branch, Islamic Azad University, Karaj, Iran.

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Abstract

This paper focuses on the application of Group Network efficiency evaluation (GNE) to conduct a comparative analysis of Iranian banking branches. The objective is to evaluate the efficiency of these branches and identify the factors contributing to their performance. The proposed method utilizes two-stages analysis to evaluate the banking branches, taking into account the relative efficiency scores of each unit within its respective group. The evaluation system provides insights into the strengths and weaknesses of individual branches and allows for comparison and benchmarking among the different banks. The results of this study contribute to enhancing the efficiency and performance of the banking sector by identifying areas for improvement and best practices. The findings can be utilized by banking institutions, policymakers, and regulators to make informed decisions and implement strategies for achieving higher levels of efficiency and competitiveness in the banking industry. In an empirical study, we compared 72 banking branches belonging to three different banking groups with each other.

Keywords:
Banking evaluation
Network data envelopment analysis
Group-Ranking

***Correspondence E-mail**: mfsiavash@gmail.com

INTRODUCTION

Nowadays, the business environment has become highly competitive, to the point where only strong and powerful institutions can survive and thrive in an efficient and effective manner. Success in a competitive market requires a high level of performance achieved through operational improvements and real learning. Managers, in order to enhance their competitive advantage, need to be aware of their strengths, weaknesses, threats, and opportunities compared to other similar institutions and their past performance. Today, these managers are in search of a comprehensive, reliable, and flexible solution to evaluate the performance of their organizations. They seek to obtain accurate and sufficient information about their current position among competitors so that they can not only ensure the implementation of their strategies but also promote and improve their organizations with a future-oriented approach. The banking industry is one of those businesses where banks compete with each other in a highly competitive environment. Bank evaluation is a critical aspect of assessing the performance and efficiency of banking institutions. In recent years, there has been a growing interest in utilizing data envelopment analysis (DEA) introduced by Charnes et al. [1] as a powerful tool for evaluating banks and their branches. DEA provides a comprehensive framework for measuring the relative efficiency of banks by considering multiple input and output variables. Numerous studies have been conducted in the field of bank evaluation using DEA, highlighting its effectiveness in assessing the performance of banking institutions. Berger et al. [2] provides an international survey on the efficiency of financial institutions and outlines directions for future research in this field. Arora et al. [3] provide a comprehensive and systematic review of the literature on banking efficiency. they synthesize existing knowledge, identifies gaps in research, and proposes future directions for further investigation. Zhao et al. [4] introduce the network slacks-based measure (NSBM) as a tool to evaluate bank efficiency. This method considers the interdependence and shared resources among different banking activities, providing a more comprehensive and realistic assessment of efficiency. Zhou et al. [5] proposed a multi-period Data Envelopment Analysis (DEA) model to assess the efficiency of banking systems considering uncertainty. Phung et al. [6] introduced a mixed network Data Envelopment Analysis (DEA) model that considers shared resources in the assessment of performance for the banking industry. Wang et al. [7] explored the efficiency measures of the Chinese commercial banking system by employing an additive two-stage Data Envelopment Analysis (DEA) model. The study aims to assess the efficiency and productivity of Chinese banks while considering both input and output variables. Titko et al. [8] applied Data Envelopment Analysis (DEA) to compare the efficiency of the Latvian and Lithuanian banking sectors. The study aims to benchmark the performance of these two countries' banking systems by evaluating their efficiency levels.

Kraidi et al.[9] introduced a weight-restricted approach to constant returns to scale DEA models, applied to Turkey's internet banking sector from 2006 to 2018. This model optimizes efficiency measurement by assigning strictly positive weights, reducing inflated efficiency scores, and serving as a reliable benchmark for decision-making and financial planning in the banking industry. Amirteimoori et al. [10] introduced a method to enhance the technical efficiency of firms deemed efficient by Data Envelopment Analysis (DEA). They identified a gap between actual and estimated efficiency and proposed an inverse DEA model (IDEA) to improve efficient decision-making units (DMUs). The approach was validated using data from 106 Chinese banks in 2021.

Repkova et al. [11] examined the efficiency of the Czech banking sector using the DEA analysis approach. The study assesses the performance of banks over a specific time period and identifies their efficiency trends. By using DEA, Kotrim Henriques et al. [12] evaluated the efficiency of Brazilian banks in the period 2012 to 2016.

Izadikhah [13] suggested method to improve the Banks shareholder long term values by using DEA. Kaffash et al. [14] by using DEA employed two widely used selections of inputs and outputs to estimate the efficiency scores for a sample of banks operating in Persian Gulf Council Countries (GCC) over the period of 2002–2011. Valami [15] introduced a group evaluation method to evaluate the performance of Iranian commercial banks with using geometric mean of the output distance function of DMUs from the frontier of PPS corresponding to the group in the output space. Ho et al. [16]explored the application of data envelopment analysis (DEA) and principal component analysis (PCA) for evaluating the performance of online banking. Mahmoudabadi et al. [17] proposed a three-stage slack-based measure (SBM) approach to carry out a comprehensive performance evaluation of banking branches. Fukuyama et al. [18] introduced a dynamic network data envelopment analysis (DEA) approach with a sequential structure and behavioral-causal analysis to evaluate the performance of the Chinese banking industry. Yannick et al. [19] conducted a study with the aim of assessing the technical efficiency of the banking sector in Cote d'Ivoire using data envelopment analysis (DEA). Sahin et al. [20]investigated the impact of the global crisis on the Turkish banking sector and to assess its performance using data envelopment analysis (DEA). The aforementioned studies have primarily focused on evaluating individual Decision-Making Units (DMUs). However, in practical scenarios, these individual DMUs often belong to multiple distinct groups. For instance, consider the units within a bank, which operate independently but are supervised by a single management team aiming to achieve the common objectives of the group. In such cases, it becomes crucial for the bank's management to assess the group's performance rather than solely focusing on individual unit performance. In the context of the banking industry, precise evaluation of group performance for a bank and comparison with competing banking groups holds significant importance. Senior bank managers require detailed information on the individual and group performance of branches, particularly during crisis situations, to make informed decisions. Ang et al. [21] proposed a group efficiency evaluation approach that considers two perspectives: average performance and weakest performance. In the average performance case, each effective member positively contributes to the overall group performance, while ineffective members have a detrimental impact. On the other hand, when evaluating a group based on the weakest performance criterion, the overall group performance is determined by the performance of the worst-performing member. To address this issue, Shahbazifar et al. [22] introduced a novel approach in group network data envelopment analysis, employing a two-stage group network structure to rank groups.

This paper demonstrates the implementation of the aforementioned method in the evaluation of 72 bank units belonging to three distinct banking groups in Gilan province. The primary aim is to compare and assess the performance of these banking groups. Moreover, the data utilized in this study are authentic and derived from actual observations. The proposed method not only compares the performance of the groups under evaluation using group performance evaluation methods, but also takes into account the intermediate processes. By considering these intermediate processes, the study offers compensatory solutions to the managers of the evaluated groups, enabling them to improve their processes and achieve higher efficiency. The inclusion of intermediate processes in the evaluation provides a more comprehensive perspective on the overall performance of the banking groups. It acknowledges that these processes play a critical role in shaping the final outcomes and can significantly impact the overall effectiveness and productivity of the groups. By identifying the strengths and weaknesses of the intermediary processes, this study empowers managers to make informed decisions regarding process improvements. The proposed compensatory solutions provide actionable strategies for the managers to optimize their operations and enhance their overall performance. This method enables more accurate evaluation of assessed groups and provides reliable results. It is important to note that these studies do not focus on determining the criteria for grouping, but rather on evaluating pre-grouped units using the proposed models.

The remainder of the paper is organized as follows. In Section 2, the methodology and the utilized approach are introduced. Section 3 presents the data analysis and research findings. In Section 4, the conclusions are drawn, and potential future directions are suggested.

THEORETICAL FOUNDATIONS AND FRAMEWORK OF RESEARCH

In this section, we present the practical tools utilized in this research, namely the two-stage efficiency evaluation and group efficiency evaluation. Additionally, we introduce the concept of network group efficiency (NGE) proposed by Shahbazifar et al. [22] to further expand upon the evaluation methods.

Two-Stages network DEA

In this approach, the efficiency evaluation is conducted in a two-stage process, where the efficiency of each stage is assessed separately. The first stage involves measuring the efficiency of each DMU in terms of input-output relationships. The outputs of the first stage are then treated as inputs in the second stage, where further efficiency analysis is conducted. By decomposing the overall efficiency score into stage-specific scores, this method provides a more detailed understanding of the factors contributing to the overall efficiency performance of each unit. By utilizing this approach, researchers gain insights into the efficiency of each stage of the production process and the interdependencies between them. This decomposition enables a more comprehensive analysis and identification of potential areas for improvement in each stage, ultimately leading to enhanced overall efficiency and productivity. Considering the two-stage structure, in the first stage, branches use financial loans X_1 and current operations X_2 as inputs to obtain deposits Z. In the second stage, these deposits are utilized to generate loan income Y_1 and other revenues Y_2. This structure helps assess the overall performance of branches by focusing on the efficient use of financial resources and revenue generation. [25] (Fig.1)

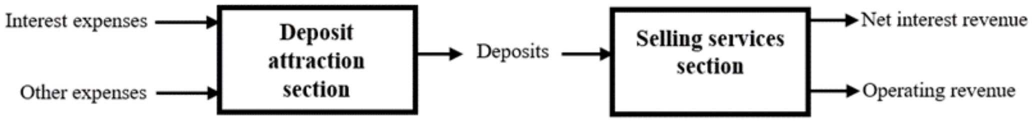

Fig.1. Inputs and outputs.

We have carefully selected these variables based on their critical importance in the banking sector, supported by both theoretical and empirical research. The chosen inputs and outputs reflect key operational and financial activities essential for evaluating bank branch efficiency. Similar approaches have been employed in prior studies, such as Paradi et al. [25], which further validates our selection. However, we acknowledge that additional inputs and outputs could be explored in future studies to provide a more comprehensive analysis.

Kao et al. [23] introduced a relational model for efficiency decomposition in network data envelopment analysis. It explores the decomposition of efficiency scores into different components, providing a comprehensive understanding of the factors influencing overall efficiency. Consider a basic two-stage production system, as illustrated in Figure 1. The inputs, intermediate products, and outputs are denoted by $x_{ij}(i=1,\dots,m)$, $z_{pj}(p=1,\dots,q)$ and $y_{rj}(r=1,\dots,s)$ respectively. To evaluate two sub-processes together, the model must describe the relationships between the whole process and the two sub-processes. The way to calculate the overall efficiency E_{kk}^{o}, taking the series relationship of the two sub-processes into account, is to incorporate the ratio constraints of the two sub-processes into classical DEA ratio form

$$
\begin{aligned}
E_{kk}^{o} &= max\frac{\sum_{r=1}^{s}u_{rk}y_{rk}}{\sum_{i=1}^{m}v_{ik}x_{ik}}\\
s.t.\ \ &\frac{\sum_{r=1}^{s}u_{rj}y_{rj}}{\sum_{i=1}^{m}v_{ij}x_{ij}} \le 1,\ j=1,\dots,n,\\
&\frac{\sum_{p=1}^{q}w_{pj}z_{pj}}{\sum_{i=1}^{m}v_{ij}x_{ij}} \le 1,\ \ j=1,\dots,n,\\
&\frac{\sum_{r=1}^{s}u_{rj}y_{rj}}{\sum_{p=1}^{q}w_{pj}z_{pj}} \le 1,\ \ j=1,\dots,n,\\
&u_{rj},v_{ij},w_{pj}\ \ge 0,\ \ i=1,\dots,m,\ \ r=1,\dots,s,\ p=1,\dots,q.
\end{aligned}
\tag{1}
$$

Let, u_r^*, v_i^*, w_p^* be optimal solution to the model (1) for DMU_k , the system efficiency and the two process efficiencies are calculated using the following formulae, respectively:

$$E_{kk}^{o} = \frac{\sum_{r=1}^{s}u_{rk}^{*}y_{rk}}{\sum_{i=1}^{m}v_{ik}^{*}x_{ik}}, \tag{2}$$

$$E_{kk}^{1} = \frac{\sum_{p=1}^{q}w_{pk}^{*}z_{pk}}{\sum_{i=1}^{m}v_{ik}^{*}x_{ik}}, \tag{3}$$

and

$$E_{kk}^{2} = \frac{\sum_{r=1}^{s}u_{rk}^{*}y_{rk}}{\sum_{p=1}^{q}w_{pk}^{*}z_{pk}}. \tag{4}$$

Clearly, the system efficiency is the product of the two process efficiencies, $E_{kk}^{o}=E_{kk}^{1}\times E_{kk}^{2}$.

Group evaluation

In most DEA models, the focus is on ranking individual DMUs. Now, if someone decides to compare a group of production units with another group, while the units of each group are trying to improve the situation of the group, a new method must be applied to evaluate the groups and calculate their efficiency score separately. Cook et al. [24] were the first to mention the concept of evaluating group efficiency in the DEA framework. Ang et al. [21] explored this new topic in more detail and presented models for calculating group efficiency. They considered two approaches for evaluating group efficiency: one based on average and another on the weakest performance criteria.

Group evaluation based on average performance

In this section, we will examine the calculation approach based on the average performance of the group members. A group's performance and potential are determined or influenced by performance of its members. Excellent members exert positive effects on the group's performance, while negative effects are imposed by underperforming members. Suppose, n DMUs are organized into K groups with D_k members for each group $k, (k = 1, \dots, K)$, and each DMU_{d_k}, $d_k = 1, \dots, D_k$ has m inputs $X_{d_k} = (x_{id_k})$ and s outputs $Y_{d_k} = (y_{rd_k})$. For each group t, $(t = 1, \dots, K)$ under evaluation, group efficiency score based on average performance is obtained by solving the following optimization model

$$E_t^A = max \frac{\sum_{r=1}^{s} \sum_{d_t=1}^{D_t} u_{rt}\, y_{rd_t}}{\sum_{i=1}^{m} \sum_{d_t=1}^{D_t} v_{it} x_{id_t}}$$

$$.\frac{\sum_{r=1}^{s} u_{rt} y_{rd_k}}{\sum_{i=1}^{m} v_{it} x_{id_k}} \leq 1, k = 1, \dots, K, \ \ d_k = 1, \dots, D_k,$$

$$\frac{\sum_{r=1}^{s} \sum_{d_t=1}^{D_t} u_{rt}\, y_{rd_t}}{\sum_{i=1}^{m} \sum_{d_t=1}^{D_t} v_{it} x_{id_t}} \leq 1$$

$$v_{it}, u_{rt} \geq 0, \ \ i = 1, \dots, m, \ \ r = 1, \dots, s. \tag{5}$$

Model (5) in linear form is as follows:

$$E_t^A = max \sum_{r=1}^{s} \sum_{d_t=1}^{D_t} u_{rt}\, y_{rd_t}$$

$$s.t \sum_{r=1}^{s} u_{rt} y_{rd_k} - \sum_{i=1}^{m} v_{it} x_{id_k} \ \leq 0, \quad k = 1, \dots, K, \ \ d_k = 1, \dots, D_k,$$

$$\sum_{i=1}^{m} \sum_{d_t=1}^{D_t} v_{it} x_{id_t} = 1,$$

$$v_{it}, u_{rt} \geq 0, \ \ i = 1, \dots, m, \ \ r = 1, \dots, s. \tag{6}$$

Suppose u_{rt}^*, v_{it}^* are optimal solution for model (6), so the optimal solution for model (6) provides the average group efficiency score for group t as follows:

$$E_t^{A*} = \sum_{r=1}^{s} \sum_{d_t=1}^{D_t} u_{rt}^*\, y_{rd_t} \tag{7}$$

When the efficiency of the group reaches the optimal level, the efficiency values of each DMU in group t can be calculated as follows:

$$e_{d_t}^{A*} = \frac{\sum_{r=1}^{s} u_{rt}^* y_{rd_t}}{\sum_{i=1}^{m} v_{it}^* x_{id_t}}, \ \ d_t = 1, \dots, D_t. \tag{8}$$

roup evaluation based on weakest performance

Another approach in evaluating group efficiency is to consider the weakest performance of each group as the efficiency score of that group. According to this idea, group efficiency score based on the weakest performance is determined by the group members with the worst performance in operations. In other words, the overall group performance score is based on the weakest performance of the worst group member. Suppose efficiency of each unit in a group $p, (\, p = 1, .., K)$ be denoted by $e_{d_p}\big(d_p = 1, \dots, D_p\big)$, then efficiency score based on the weakest performance, E_p^W, is as follows:

$$E_p^W = min_{d_p} e_{d_p}. \tag{9}$$

For each group $t, (\, t = 1, .., K)$, group efficiency score based on the weakest performance is obtained by solving the following model:

$$E_t^W = maxmin_{d_t} \frac{\sum_{r=1}^{s} \sum_{d_t=1}^{D_t} u_{rt}\, y_{rd_t}}{\sum_{i=1}^{m} \sum_{d_t=1}^{D_t} v_{it} x_{id_t}}$$

$$s.t. \ \frac{\sum_{r=1}^{s} u_{rt} y_{rd_k}}{\sum_{i=1}^{m} v_{it} x_{id_k}} \leq 1,$$

$$k = 1, \dots, K, \ \ d_k = 1, \dots, D_k,$$

$$v_{it}, u_{rt} \geq 0, \ \ i = 1, \dots, m, \qquad r = 1, \dots, s. \tag{10}$$

The optimal solution of the model (10) and the best group efficiency for group t is:

$$E_t^{W*} = min_{d_t} \frac{\sum_{r=1}^{s} u_{rt}^*\, y_{rd_t}}{\sum_{i=1}^{m} v_{it}^* x_{id_t}} \tag{11}$$

The efficiencies of units in group t are calculated as follows:

$$e_{d_t}^{W*}=\frac{\sum_{r=1}^{s}u_{rt}^{*}y_{rd_t}}{\sum_{i=1}^{m}v_{it}^{*}x_{id_t}},\ \ d_t=1,\dots,D_t. \qquad (12)$$

Model (10) is nonlinear and we must write its linear equivalent. By applying the constraint $\sum_{r=1}^{s}u_{rt}=1$ and without changing the optimal solution of the model, we can obtain an approximation of the optimal solution. So, the model (10) can be rewritten as follows:

$$\begin{aligned}
E_t^W &= maxmin_{d_t}\ \frac{\sum_{r=1}^{s}\sum_{d_t=1}^{D_t}u_{rt}\,y_{rd_t}}{\sum_{i=1}^{m}\sum_{d_t=1}^{D_t}v_{it}x_{id_t}}\\
s.t.\ &\frac{\sum_{r=1}^{s}u_{rt}y_{rd_k}}{\sum_{i=1}^{m}v_{it}x_{id_k}}\le 1,\\
&k=1,\dots,K,\ \ d_k=1,\dots,D_k,\\
&\sum_{r=1}^{s}u_{rt}=1,\\
&v_{it},u_{rt}\ge 0,\ \ i=1,\dots,m,\qquad r=1,\dots,s.
\end{aligned} \qquad (13)$$

By introducing auxiliary variable δ_t for group t we achieve:

$$\delta_t = min_{d_t}\frac{u_tY_{d_t}}{v_tX_{d_t}}, \qquad (14)$$

the model (13) could be expressed as

$$\begin{aligned}
E_t^W &= max\delta_t\\
s.t.\quad &\sum_{r=1}^{s}u_{rt}\,y_{rd_t}-\delta_t\sum_{i=1}^{m}v_{it}\,x_{id_t}\ge 0,\ \ d_t=1,\dots,D_t,\\
&\sum_{r=1}^{s}u_{rt}y_{rd_k}-\sum_{i=1}^{m}v_{it}x_{id_k}\ \le 0,\quad k=1,\dots,K,\ \ d_k=1,\dots,D_k,\\
&\sum_{r=1}^{s}u_{rt}=1,\\
&v_{it},u_{rt}\ge 0,\ \ i=1,\dots,m,\qquad r=1,\dots,s.
\end{aligned} \qquad (15)$$

Model (15) is linear and δ is a parameter located in $0\le\delta_t\le 1$.

Network group efficiency evaluation

Shahbazifar et al. [22]introduced this method for the first time. This method is based on a two-stage network model that assigns a reliable ranking to each group by considering the internal structure of the group members. Suppose a brand competes with its rival brands in the market. Subordinate units do not probably function at the same level. Some units have positive or negative effects on group performance. Now, if a manager takes a closer look at the internal structure of production as an evaluation criterion, it is necessary to define the group's performance evaluation model using the network structure.

Network group efficiency evaluation based on average performance

In this method, the average efficiency score of the members of a group is the comparison and evaluation criterion. We solve the following models to calculate the overall and first and second stages network group efficiency scores.

$$\begin{aligned}
E_t^{OA} &= max\frac{\sum_{r=1}^{s}\sum_{d_t=1}^{D_t}u_{rt}\,y_{rd_t}}{\sum_{i=1}^{m}\sum_{d_t=1}^{D_t}v_{it}x_{id_t}}\\
\mathrm{s.t.}\ &\frac{\sum_{r=1}^{s}u_{rt}y_{rd_k}}{\sum_{i=1}^{m}v_{it}x_{id_k}}\le 1, k=1,\dots,K,\ \ d_k=1,\dots,D_k,\\
&\frac{\sum_{p=1}^{q}w_{pt}z_{pd_k}}{\sum_{i=1}^{m}v_{it}x_{id_k}}\le 1, k=1,\dots,K,\ \ d_k=1,\dots,D_k,\\
&\frac{\sum_{r=1}^{s}u_{rt}y_{rd_k}}{\sum_{p=1}^{q}w_{pt}z_{pd_k}}\le 1, k=1,\dots,K,\ \ d_k=1,\dots,D_k,\\
&v_{it},u_{rt},w_{pt}\ge 0,\ \ i=1,\dots,m,\ \ r=1,\dots,s,\ \ p=1,\dots,q.
\end{aligned} \qquad (16)$$

where, n DMUs are organized into K groups with D_k members for each group $k, (k = 1, \dots, K)$, and each $DMU_{d_k},\ d_k = 1, \dots, D_k$ has m inputs $X_{d_k} = (x_{id_k})$ and q intermediate products $Z_{d_k} = (z_{pd_k})$ and s outputs $Y_{d_k} = (y_{rd_k})$. The optimal solution of model (16) is interpreted as the group's overall efficiency score based on the average performance for group t. The group efficiency scores of the first and second stages are then obtained by solving the following models, respectively.

$$= max \frac{\sum_{p=1}^{q} \sum_{d_t=1}^{D_t} w_{pt}\, z_{pd_t}}{\sum_{i=1}^{m} \sum_{d_t=1}^{D_t} v_{it} x_{id_t}}$$

$$\text{s.t.}\ \frac{\sum_{p=1}^{q} w_{pt} z_{pd_k}}{\sum_{i=1}^{m} v_{it} x_{id_k}} \le 1,\ \ k = 1, \dots, K,\ \ d_k = 1, \dots, D_k, \qquad (17)$$

$$v_{it}, w_{pt} \ge 0,\ \ i = 1, \dots, m,\ \ p = 1, \dots, q.$$

$$E_t^{2A} = max \frac{\sum_{r=1}^{s} \sum_{d_t=1}^{D_t} u_{rt}\, y_{rd_t}}{\sum_{p=1}^{q} \sum_{d_t=1}^{D_t} w_{pt} z_{pd_t}}$$

$$\text{s.t.}\ \frac{\sum_{r=1}^{s} u_{rt} y_{rd_k}}{\sum_{p=1}^{q} w_{pt} z_{pd_k}} \le 1,\ \ k = 1, \dots, K,\ \ d_k = 1, \dots, D_k, \qquad (18)$$

$$w_{pt}, u_{rt} \ge 0,\ \ p = 1, \dots, q,\ \ r = 1, \dots, s.$$

Note that the efficiencies of the overall process and the two sub-processes are calculated independently. Consider u^*_{rt}, v^*_{it} and w^*_{pt} as the optimal solutions of the above models. In conclusion,

$$E_t^{OA} = max \frac{\sum_{r=1}^{s} \sum_{d_t=1}^{D_t} u^*_{rt}\, y_{rd_t}}{\sum_{i=1}^{m} \sum_{d_t=1}^{D_t} v^*_{it} x_{id_t}}$$

$$E_t^{1A} = max \frac{\sum_{p=1}^{q} \sum_{d_t=1}^{D_t} w^*_{pt} z_{pd_t}}{\sum_{i=1}^{m} \sum_{d_t=1}^{D_t} v^*_{it} x_{id_t}}$$

$$E_t^{2A} = max \frac{\sum_{r=1}^{s} \sum_{d_t=1}^{D_t} u^*_{rt} y_{rd_t}}{\sum_{p=1}^{q} \sum_{d_t=1}^{D_t} w^*_{pt} z_{pd_t}}. \qquad (19)$$

Obviously, the overall group efficiency is the product of the group efficiencies of its stages:

$$E_t^{OA} = E_t^{1A} * E_t^{2A}\ . \qquad (20)$$

Model (16) is nonlinear and can be transformed into the following linear program:

$$E_t^{OA} = max \sum_{r=1}^{s} \sum_{d_t=1}^{D_t} u_{rt}\, y_{rd_t}$$

$$\text{s.t.}\ \sum_{p=1}^{q} w_{pt} z_{pd_k} - \sum_{i=1}^{m} v_{it} x_{id_k} \le 0,\ k = 1, \dots, K,\ \ d_k = 1, \dots, D_k,$$

$$\sum_{r=1}^{s} u_{rt} y_{rd_k} - \sum_{p=1}^{q} w_{pt} z_{pd_k} \le 0,\ k = 1, \dots, K,\ \ d_k = 1, \dots, D_k,$$

$$\sum_{i=1}^{m} \sum_{d_t=1}^{D_t} v_{it} x_{id_t} = 1,$$

$$v_{it}, u_{rt}, w_{pt} \ge 0,\ \ i = 1, \dots, m,\ \ r = 1, \dots, s,\ \ p = 1, \dots, q. \qquad (21)$$

Also, by converting models (17) and (18) into linear form, we have:

$$E_t^{1A} = max \sum_{p=1}^{q} \sum_{d_t=1}^{D_t} w_{pt}\, z_{pd_t}$$

$$\text{s.t.}\ \sum_{p=1}^{q} w_{pt} z_{pd_k} - \sum_{i=1}^{m} v_{it} x_{id_k} \le 0,\ k = 1, \dots, K,\ \ d_k = 1, \dots, D_k,$$

$$\sum_{i=1}^{m} \sum_{d_t=1}^{D_t} v_{it} x_{id_t} = 1,$$

$$v_{it}, w_{pt} \ge 0,\ \ i = 1, \dots, m,\ \ p = 1, \dots, q, \quad (22)$$

$$E_t^{2A} = max \sum_{r=1}^{s} \sum_{d_t=1}^{D_t} u_{rt}\, y_{rd_t}$$

$$\text{s.t.}\ \sum_{r=1}^{s} u_{rt} y_{rd_k} - \sum_{p=1}^{q} w_{pt} z_{pd_k} \le 0,\ k = 1, \dots, K,\ \ d_k = 1, \dots, D_k,$$

$$\sum_{p=1}^{q} \sum_{d_t=1}^{D_t} w_{pt} z_{pd_t} = 1,$$

$$u_{rt}, w_{pt} \ge 0,\ \ r = 1, \dots, s, \qquad p = 1, \dots, q. \qquad (23)$$

Network group efficiency evaluation based on weakest performance

In the previous part, we introduced the network group efficiency evaluation method based on average performance and obtained the network group efficiency score for ranking the groups. In this section, instead of the average performance of the group members, we set the performance of the weakest member of the groups as a criterion and based on the weakest performance for each group, we obtained the group efficiency score. For the group under evaluation $t, (t = 1, \dots, K)$, consider the following models to obtain group efficiency score of the first stage based on the weakest performance:

$$
\begin{aligned}
E_t^{1W} &= max\delta_t \\
s.t. \quad & \textstyle\sum_{p=1}^{q} w_{pt}\, z_{pd_t} - \delta_t \sum_{i=1}^{m} v_{it}\, x_{id_t} \geq 0,\ d_t = 1, \dots, D_t, \\
& \sum_{p=1}^{q} w_{pt} z_{pd_k} - \sum_{i=1}^{m} v_{it} x_{id_k} \leq 0,\quad k = 1, \dots, K,\ d_k = 1, \dots, D_k, \qquad (24) \\
& \sum_{p=1}^{q} w_{pt} = 1, \\
& v_{it}, w_{pt} \geq 0,\ i = 1, \dots, m, \qquad p = 1, \dots, q,
\end{aligned}
$$

While $\delta_t = min_{d_t} \frac{u_t Y_{d_t}}{v_t X_{d_t}}$ and δ_t are auxiliary variables for group t.

In the same way, by solving the following model, the group efficiency score of the second stage is obtained:

$$
\begin{aligned}
E_t^{2W} &= max\delta_t \\
s.t. \quad & \textstyle\sum_{r=1}^{s} u_{rt}\, y_{rd_t} - \delta_t \sum_{p=1}^{q} w_{pt}\, z_{pd_t} \geq 0,\ d_t = 1, \dots, D_t, \\
& \sum_{r=1}^{s} u_{rt} y_{rd_k} - \sum_{p=1}^{q} w_{pt} z_{pd_k} \leq 0,\quad k = 1, \dots, K,\ d_k = 1, \dots, D_k, \qquad (25) \\
& \sum_{r=1}^{s} u_{rt} = 1, \\
& w_{pt}, u_{rt} \geq 0,\ p = 1, \dots, q, \qquad r = 1, \dots, s.
\end{aligned}
$$

Finally, the overall group efficiency score is obtained by solving the following model.

$$
\begin{aligned}
E_t^{oW} &= max\delta_t \\
s.t. \quad & \textstyle\sum_{r=1}^{s} u_{rt}\, y_{rd_t} - \delta_t \sum_{i=1}^{m} v_{it}\, x_{id_t} \geq 0,\ d_t = 1, \dots, D_t, \\
& \sum_{r=1}^{s} u_{rt} y_{rd_k} - \sum_{p=1}^{q} w_{pt} z_{pd_k} \leq 0,\quad k = 1, \dots, K,\ d_k = 1, \dots, D_k, \\
& \sum_{p=1}^{q} w_{pt} z_{pd_k} - \sum_{i=1}^{m} v_{it} x_{id_k} \leq 0,\quad k = 1, \dots, K,\ d_k = 1, \dots, D_k, \qquad (26) \\
& \sum_{r=1}^{s} u_{rt} = 1, \\
& v_{it}, w_{pt}, u_{rt} \geq 0,\ i = 1, \dots, m, p = 1, \dots, q,\ r = 1, \dots, s.
\end{aligned}
$$

In this approach, based on the method proposed by Shahbazifar et al. [22], we have presented a network group efficiency evaluation model. In the case of two-stage production, we have achieved the group efficiency score of the first and second stages, and finally the overall group efficiency score.

DATA ANALYSIS AND RESEARCH FINDINGS

In this section, we aim to compare three banking groups in Gilan province. The total number of branches is 72, with Group A consisting of 17 branches, Group B having 23 branches, and

Group C comprising 32 branches. The statistical description of the branches can be found in Table 1. Our objective is to assess and compare the performance of these banking groups. The graph illustrates the financial processes conducted in each branch, as depicted in Figure 1. We consider the two scenarios mentioned in the text, the average performance scenario and the weakest performance scenario for comparing bank groups. We have used two methods: 1) Group Evaluation and 2) Two-Stage Group Evaluation. The results are as follows:

Group efficiency evaluation

Average performance Scenario:

From this perspective, as illustrated in Table 2, the Banking Group A has a score of 0.92, and it is the best group among all the groups. On the other hand, Banking Group B with a score of 0.82 is the worst group among all the groups. In fact, the results emphasize the exceptional performance of Banking Group A and its ability to maintain a harmonious balance among its subsidiary branches. This balance is reflected in the similar evaluation scores of individual branch performances, indicating a consistent level of efficiency and effectiveness across the entire banking group. Management of Banking Group A has demonstrated exceptional leadership and coordination in ensuring that all branches operate with comparable levels of performance. This uniformity in performance suggests that the banking group has established standardized strategies and implemented guidelines that are uniformly followed by all branches. The advantage of having such a balanced performance is that it allows the banking group to align its objectives, strategies, and decision-making processes more effectively. The consistent evaluation scores of the individual branches indicate that the banking group's strategies and guidelines have been successfully communicated and implemented across all branches. By achieving a balanced and equitable performance across its branches, Banking Group A is better positioned to execute its overall strategy and deliver consistent service quality to its customers. This demonstrates the banking group's strong management capabilities and its ability to foster a cohesive and synchronized working environment among its branches.

Weakest performance scenario:

If we consider the scenario of evaluating the weakest individual performance among the banking groups, based on the provided group performance evaluation results in Table 3, Banking Group A is ranked as the worst-performing group with a group performance score of 0.12 among all the groups. Interestingly, in the scenario of evaluating based on the group average performance, this banking group had the best performance. On the other hand, Banking Group C achieved the highest rank among all the groups with a performance score of 0.78. It is worth noting that all the branches under Banking Group

C have acceptable individual evaluation scores. This indicates that the banking group maintains a rigorous monitoring system over the performance of its subsidiary branches, ensuring their alignment with the set objectives. In conclusion, the evaluation of banking groups using the weakest individual performance scenario revealed the underperformance of Banking Group A due to the poor performance of one of its branches. However, in terms of overall performance, Banking Group C emerged as the top-performing group. The meticulous oversight and coordination of all subsidiary branches by Banking Group C reflect its commitment to achieving the predetermined goals.

Two-stages group efficiency evaluation

In this section, we aim to have a more precise evaluation of the banking groups' status, considering the intermediary financial processes that banks undertake in their branches to incur costs and generate revenue. As depicted in Figure 1, each branch incurs expenses in the first stage to collect deposits from customers, and in the second stage, it is required to generate income from these deposits. The two-stage group performance evaluation model provides a more detailed assessment of the banking groups, determining their strengths and weaknesses in each stage. The results obtained from this type of evaluation provide accurate and reliable information to the banking group managers for making future decisions regarding strategies and objectives.

Average performance Scenario:

As shown in Table 2, Group A is the best performing banking group among all the groups, with a score of 0.9. This group maintains a favorable status in both stages. The group's first-stage group performance score is 0.99, indicating excellent performance in converting expenses into deposits. Additionally, in the second stage, it has achieved a satisfactory score of 0.91 in converting deposits into income. In contrast, Group C has the lowest group performance score of 0.82 among the groups. Looking at the table, we can see that the weakness of this group lies in the second stage, namely, the conversion of deposits into income. Improvement is needed in this process to enhance the group's performance. However, this group demonstrates a favorable status in the first stage, with a first-stage group performance score of 0.97.

Weakest performance Scenario:

From this perspective, as seen in Table 3, Group B is the best performing group among all the groups, with an overall performance score of 0.7. This group has a better status in both stages compared to the other groups. On the contrary, Group A is the worst performing group, with a total performance score of 0.14. The reason for this group's placement at the bottom is that it has a member with a very low individual performance score. To improve the situation of this banking group, structural reforms should be implemented in the branch with poor performance.

Comparison of the two presented methods

Now let's compare the results of group evaluation (GE) and network (two-stage) group evaluation (NGE) methods. In the group evaluation method, each banking group is ranked based on its group efficiency score. In this method, only the overall group score is considered, and the evaluation is done in a general manner for all stages and processes within the group. The advantage of this method is its simplicity and ease of implementation, as it provides a single performance indicator for the entire group. On the other hand, the two-stage group evaluation method provides a more detailed analysis of the group's performance. It considers the financial intermediation processes performed in the bank branches, including both cost incurred in acquiring deposits in the first stage and income generated from these deposits in the second stage. This method allows for a more comprehensive evaluation of the group's performance by assessing its efficiency in each stage separately. By incorporating the two-stage evaluation, the strengths and weaknesses of each banking group can be identified more accurately. This provides valuable insights for managers in making informed decisions regarding strategies and objectives. The two-stage evaluation method offers a more nuanced understanding of the performance of banking groups, enabling targeted improvements and optimizations in specific stages where performance is lacking. In conclusion, while the group evaluation method offers a simplified overview of group performance, the two-stage group evaluation method provides a more detailed and comprehensive analysis, allowing for a more precise assessment of the strengths and weaknesses of banking groups. In Figures 2 and 3, we observe comparative graphs between the two methods, depicting the performance comparison of banking groups based on average performance and weakest performance efficiency scores.

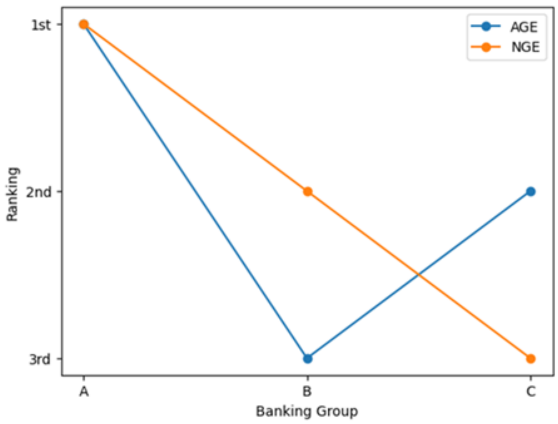

Fig.2. Comparison of AGE and NGE (Average Scenario)

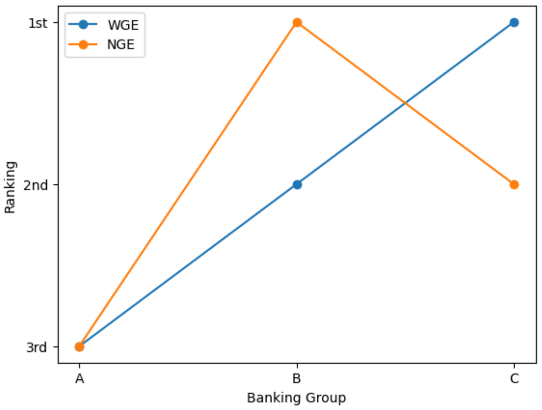

Fig.3.Comparison of WGE and NGE (Weakest Scenario)

CONCLUSION AND FUTURE RESEARCHES

In this article, we have compared the performance of three banking groups in Gilan province using group performance evaluation methods. The two-stage group efficiency evaluation method has provided accurate and reliable results, offering valuable insights to bank managers regarding the performance of branches and groups. These insights enable managers to identify strengths and weaknesses, establish strategies for improvement, and set goals for achieving better outcomes.

Our findings reveal that the two-stage evaluation method is effective in identifying performance disparities across different bank groups, which can guide targeted interventions and policy decisions. However, the constantly evolving nature of the banking industry calls for further advancements in performance evaluation techniques.

For future research, we recommend exploring the following areas:

a) Advancement in Evaluation Methods: Investigate new and innovative methods for evaluating bank group performance, including the integration of qualitative data, artificial intelligence, and advanced modeling techniques.

b) Enhancement of Accuracy and Reliability: Develop and test methods that enhance the precision and consistency of performance evaluations, possibly through comparative analysis and hybrid approaches.

By focusing on these research areas, further progress can be achieved in the field of group efficiency evaluation, leading to more effective management practices and improved outcomes for bank groups.

Table 1: Describe Data of 72 branches of Gilan province banks.

	X_1	X_2	Z	Y_1	Y_2
count	7.20E+01	7.20E+01	7.20E+01	7.20E+01	7.20E+01
mean	1.06E+08	2.75E+07	8.32E+08	1.04E+10	3.68E+09
std	8.44E+07	2.20E+07	6.65E+08	8.48E+09	2.99E+09
min	5.53E+06	1.54E+06	4.42E+07	5.99E+08	1.97E+08
25%	2.12E+07	5.54E+06	1.68E+08	2.04E+09	7.36E+08
50%	9.61E+07	2.46E+07	7.49E+08	9.04E+09	3.13E+09
75%	1.83E+08	4.71E+07	1.47E+09	1.80E+10	6.61E+09
max	2.72E+08	6.97E+07	2.17E+09	2.77E+10	9.98E+09

Table 2: Group network-efficiency evaluation results for Gilan province banking groups. (Average scenario)

GROUP	AGE	$E_t^{A(1)*}$	$E_t^{A(2)*}$	$E_t^{A(o)*}$
Bank A	0.98(1)	0.99	0.91	0.9(1)
Bank B	0.87(3)	0.98	0.87	0.84(2)
Bank C	0.93(2)	0.97	0.84	0.82(3)

Table 3: Group network-efficiency evaluation results for Gilan province banking groups. (Weakest scenario)

GROUP	WGE	$E_t^{W(1)*}$	$E_t^{W(2)*}$	$E_t^{W(o)*}$
Bank A	0.12(3)	0.16	0.12	0.14(3)
Bank B	0.77(2)	0.82	0.78	0.7(1)
Bank C	0.78(1)	0.82	0.77	0.68(2)

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