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Bayesian Prediction for Chris-Jerry Model Using Unified Progressive Hybrid Censored Sample

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Abstract

From a real analysis standpoint, modeling hinges on the choice of the most suitable analysis method, whether it be a frequentist or Bayesian approach, to obtain an updated model. Modeling lifetime data with heavy tail has been a problem among many management researchers. Also, predicting unobserved data based on available data is one of the most important challenges in various sciences such as management and engineering. Bayesian predictive point in Chris-Jerry distribution is considered, in this paper. Te observed data is censored using a unified progressive hybrid censoring scheme. For evidence of the effectiveness of the given methodology, an application is explored.

Keywords:
Bayesian point Prediction
HPD predictive
Chris-Jerry distribution
Unified Progressive Hybrid Censoring

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INTRODUCTION

Prediction problems arise naturally in important fields of statistical research and have been very useful in many areas of application such as management, quality control, life experiments and so on. In experiments where failure information is available only on a part of the data, the sample are said to be censored data. Nowadays, the quality and lifetime of any industrial product have become higher because of modern science and technology. Thus, it has become more challenging to increase product qualities with reliability testing in a short period with low expenditure. In this context, censoring is sensible and important to study the failure data in a reliability experiment. Time (Type-I) and (failure) censoring are the most popular censorship techniques. To reduce the cost and duration associated with experiment, progressively Type-II censoring has been developed, which allows the researcher to eliminate items at periods other than the end time. Also, Gorny & Cramer (2018) introduced unified progressive hybrid censoring scheme, where the experimental time may exceed the predetermined time T and the effective sample sizes m, k are prefixed. Let T_1 and T_2 $(T_1 < T_2)$, be the two time points. This experiment will be stopped at $T^* = max\{min\{X_{k:m:n}, T_2\}, min\{X_{m:m:n}, T_1\}\}$. In recent years, this censoring and its generalization have been introduced and studied quite extensively (Nagy & Alrasheedi, 2022; Dutta & Kayyal, 2024; Nik et al., 2021; Asadi et al., 2024; Elshahhat & Ashour, 2016).

The Chris-Jerry distribution is a heavy-tailed distribution for analyzing real data (Onyekwere & Obulezi, 2022). It has been introduced as a mixture of two distributions called the exponential (with parameter υ) and the gamma (with scale and shape parameters as 3 and υ, respectively) when the mixing proportion is taken as $p = \frac{\upsilon}{\upsilon+2}$. The positive random variable X, is said to be the Chris-Jerry random variable with parameter υ denoted by $CJ(\upsilon)$, if the cumulative distribution function (cdf) is

$$F(x;\upsilon) = 1 - \left[1 + \frac{\upsilon x(\upsilon x + 2)}{\upsilon + 2}\right] e^{-\upsilon x}; \quad x > 0,\ \upsilon > 0 \qquad (1)$$

and the probability density function (pdf) is given as

$$f(x;\upsilon) = \frac{\upsilon^2}{\upsilon + 2}(1 + \upsilon x^2) e^{-\upsilon x} \qquad (2)$$

For more details, see Onyekwere and Obulezi (2022), Innocent et al. (2023), Obulezi et al. (2023), and Chinedu et al. (2023). For various parameter values, the probability density function (pdf) plot can be shown below.

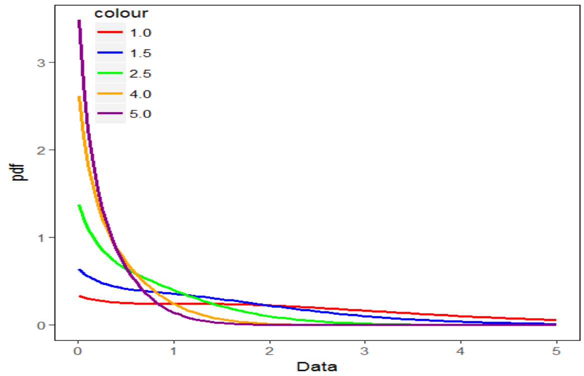

Fig. 1. Pdf plot of Chris-Jerry distribution

Let $X_{1:m:n}, ..., X_{Q:m:n}$ denote a unified progressive hybrid censored sample from a $CJ(\alpha, \beta)$ distribution, with censoring scheme $R = (R_1, R_2, ..., R_m)$. Then, the joint density function of unified progressive hybrid censored order statistics is given by:

$$L(\alpha, \beta \mid data) \propto \prod_{i=1}^{Q} f(x_i; \alpha, \beta) \times (1 - F(x_i; \alpha, \beta))^{R_i} \times (1 - F(T; \alpha, \beta))^{R^*} \qquad (3)$$

The remaining portion of the paper is structured as follows. Bayesian predictors and predictive intervals are proposed in Section 2. Section 3 evaluates the estimators' performance via simulations. Finally, in Section 4, we conclude the study with a brief discussion.

BAYESIAN APPROACH

The Bayes estimation (BE) of the Chris-Jerry distribution under the unified progressive hybrid will be covered in this section. It is assumed that the random variable υ follows gamma prior distribution with hyper-parameters $(\hbar_1, \bar{\lambda}_1);\ \hbar_1, \bar{\lambda}_1 > 0$. The prior of υ can be given as:

$$\pi(\upsilon) \propto \upsilon^{\hbar_1 - 1} e^{-\bar{\lambda}_1 \upsilon}; \qquad \upsilon > 0. \tag{4}$$

The corresponding posterior density given the observed data $X = (X_{1:m:n}, ..., X_{Q:m:n})$ is given by:

$$\eta\left(\upsilon \middle| data\right) = \frac{L(\upsilon \mid data)\pi(\upsilon)}{\int_0^\infty L(\upsilon \mid data)\pi(\upsilon) d\upsilon} \tag{5}$$

Thus, under the squared error loss function, the BE of any function on model parameters say $g(\upsilon)$**,** can be obtained as the posterior mean as:

$$\hat{g}_{Bayes_{SE}} \propto \frac{\int_0^\infty g(\upsilon)\eta\left(\upsilon \middle| data\right) d\upsilon}{\int_0^\infty \eta\left(\upsilon \middle| data\right) d\upsilon} \tag{6}$$

Bayesian Prediction

This section deals with the point and interval prediction for $\mathcal{J}$-th order statistic out of $R*$ removed units based on the unified progressive hybrid sample $X = (X_{1:m:n}, ..., X_{Q:m:n})$ with progressive censoring scheme $R = (R_1, R_2, ..., R_m)$. The conditional density function of $X_{\mathcal{J}:R*}; \mathcal{J} = 1, ..., R^*$, can be formulated as:

$$\begin{aligned} f\left(x_{\mathcal{J}} \mid data\right) = & \\ & \frac{(R^*)!}{(\mathcal{J}-1)!(R^*-\mathcal{J})!} \\ & \times \left((1 - F(T;\alpha,\beta)) - (1 - F(x_{\mathcal{J}};\alpha,\beta))\right)^{\mathcal{J}-1} \\ & \times \left(1 - F(x_{\mathcal{J}};\alpha,\beta)\right)^{R^*-\mathcal{J}} \\ & \times \left((1 - F(T;\alpha,\beta))\right)^{-R^*} f(x_{\mathcal{J}}) \end{aligned} \tag{7}$$

Here, $x_{\mathcal{J}} > T$. Considering the cumulative distribution function and density function of the Chris-Jerry model, the conditional density function of $X_{\mathcal{J}:R*}$ as follows:

$$\begin{aligned} f\left(x_{\mathcal{J}} \mid data\right) = & \frac{(R^*)!}{(\mathcal{J}-1)!(R^*-\mathcal{J})!} \\ & \times \left(\left[1 + \frac{\upsilon T(\upsilon T + 2)}{\upsilon + 2}\right] e^{-\upsilon T} - \left[1 + \frac{\upsilon x_{\mathcal{J}}(\upsilon x_{\mathcal{J}} + 2)}{\upsilon + 2}\right] e^{-\upsilon x_{\mathcal{J}}} \right)^{\mathcal{J}-1} \\ & \times \left(\left[1 + \frac{\upsilon x_{\mathcal{J}}(\upsilon x_{\mathcal{J}} + 2)}{\upsilon + 2}\right] e^{-\upsilon x_{\mathcal{J}}} \right)^{R^*-\mathcal{J}} \\ & \times \left(\left[1 + \frac{\upsilon T(\upsilon T + 2)}{\upsilon + 2}\right] e^{-\upsilon T} \right)^{-R^*} \frac{\upsilon^2}{\upsilon + 2}(1 + \upsilon x_{\mathcal{J}}^2) e^{-\upsilon x_{\mathcal{J}}}. \end{aligned} \tag{8}$$

So, the Bayesian predictive density of $X_{\mathcal{J}:R*}$ can be written as:

$$\ddot{f}^*\left(x_{\mathcal{J}} \mid data\right) = \int_0^\infty \ddot{f}^*\left(x_{\mathcal{J}} \mid data\right) \eta\left(\upsilon \mid data\right) d\upsilon \qquad ; x_{\mathcal{J}} > T \tag{9}$$

To avoid the complexity of the integration given in (9), the Monte Carlo Markov chain samples $\upsilon^{(i)}$, for $i = 1, 2, ..., M$ have been used. So, the $\mathcal{J}$-th ordered statistic from the $R*$ censored data is given by:

$$\ddot{X}^*_{\mathcal{J}:R*} = \frac{1}{M} \sum_{i=1}^{M} \int_T^\infty x_{\mathcal{J}} \ddot{f}^*\left(x_{\mathcal{J}} \mid data, \upsilon^{(i)}\right) dx_{\mathcal{J}}.$$

SIMULATION STUDIES

This Section is devoted to the comparative study of the proposed estimates under different unified progressive hybrid censored schemes. The simulation is carried out using *R* software. We considered different values for *n*, *m*, *k* and $T_i;\ i = 1, 2$ to generate 10^4 unified progressive hybrid censored schemes from the Chris-Jerry model. For given (n, m), different censoring schemes are adopted as:

- ***CSI :*** $R_i = 0$ and $R_m = n - m;\ i = 1, ..., m-1.$
- ***CSII :*** $R_1 = n - m$ and $R_i = 0;\ i = 2, ..., m.$
- ***CSIII:*** $R_i = 1,\ R_{n-2m+1} = ... = R_m = 0; i = 1, ..., n-m.$

Table 1: Bayesian point predictors and HPD predictive for $X_{\mathcal{J}:R*}$.

(n, m)	k	*Censoring Schemes*	$\mathcal{J}$	*Point Prediction*	*Interval Prediction*	
(60,30)	10	*CS I*	1	1.67345	1.234	1.875
(60,30)	10		2	1.74324	1.289	1.952
(60,30)	10	*CS II*	1	1.34832	1.098	1.599
(60,30)	10		2	1.37869	1.112	1.543
(60,30)	10	*CS III*	1	1.70546	1.249	1.903
(60,30)	10		2	1.72384	1.265	1.937
(60,30)	15	*CS I*	1	1.79345	1.293	1.917
(60,30)	15		2	1.83552	1.365	2.013
(60,30)	15	*CS II*	1	1.39894	1.212	1.587
(60,30)	15		2	1.43760	1.268	1.621
(60,30)	15	*CS III*	1	1.82543	1.378	2.014
(60,30)	15		2	1.86089	1.500	2.112

CONCLUSION

Modeling heavy-tailed data has been a problem among many researchers. One of the common features among standard heavy-tailed distributions is the parsimonious validity of the models in terms of the number of parameters. The new one-parameter Chris-Jerry distribution furnishes a better fit compared to other heavy tailed models, called Lindley, Exponential and Pareto distributions. The Bayes point and interval prediction for observable unified progressive hybrid censored data from the Chris-Jerry model are derived. The results show that the Bayes point predictors lie between the lower and the upper bounds in each case. The lengths of the prediction intervals are decreasing by increasing the effective sample size (k). It has also been observed that different censoring schemes do not have much effect on the simulation results.

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