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Portfolio Selection using Data Envelopment Analysis with common weights

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Abstract

The stock evaluation process plays an important role in portfolio selection because it is the prerequisite for investment and directly influences on the stock allocation. This paper presents a methodology based on Data Envelopment Analysis for portfolio selection, decision making units which can be stocks or other financial assets. First, DMUs efficiencies are computed based on input/output common weights, and then the generation of a portfolio is carried out by a mathematical model. Finally the methodology is illustrated numerically on the market of Iran stock exchange.

Keywords; DEA, Portfolio Selection, MOLP, Common weights, Efficiency

1. Introduction

Portfolio optimization has been conducted by many researchers for more than 50 years. Most of these researches have been proposed for portfolio selection. For instance, Saaty et al. (1980) proposed Analytic Hierarchy Process (AHP) to deal with the stock portfolio decision problem by evaluating the performance of each company in different level of criteria. Houng (2008) defined a new definition of risk and used genetic algorithm to deal with stock portfolio selection. Some of representative of researches for stock portfolio selection used fuzzy concepts. For example: The model with fuzzy probabilities (Tanaka et al., 2000), the fuzzy goal programming model (Parra et al, 2001), the admissible efficient portfolio selection

model (Zhang & Nie, 2004), A new fuzzy ranking and weighting algorithm to obtain the investment ratio of each stock (Tiryaki & Ahlatcioglu, 2005). The maximizing probability model in the stochastic environment (Williams, 1997) , and minimax models (Cai et al., 2000), (Deng et al., 2005).

Generally, portfolio selection problem considers simultaneously conflicting objectives such as rate of return, liquidity and risk. Therefore, MOLP techniques such as Goal Programming (GP) have been used to choose the portfolio (Ogryczak, 2000), (Ballestero, 2001), (Aouni et al., 2005), (Ben Abdelaziz et al., 2007). The portfolio selection problem can also be seen as a problem of rare resource allocation (funds, labor, etc.) in order to maximize well-being (return, satisfaction, etc.) (Dia, 2009) But, the mean variance methodology for portfolio selection proposed by Markowitz (1952) has been central to research activities in the traditional securities investment field.

Markowitz model is essentially based on two criteria, risks and returns, where the returns are approximated by the mathematical mean return and the risks are measured either by the return dispersion (variance or standard deviation) or by the covariance with the market (beta). However, Markowitz model contains at least three major weaknesses. First, it is a bi-criterion model which is based on the maximization of returns and the minimization of risks. Second, the mean-variance model of Markowitz is by definition a quadratic optimization model which is difficult to solve, particularly for large problems, even when linearization techniques are used (Konno et al., 1993). Finally, the data in this model are, for the most part, inexact, uncertain or even vague in real situations.

In this paper, we present a portfolio selection methodology based on Data Envelopment Analysis (DEA) which allows us to overcome the first two weaknesses of Markowitz model. DEA aims at comparing the inputs and outputs of a set of decision-making units (DMU) by evaluating their relative efficiency. In portfolio selection, DMUs can be stocks, mutual funds, or other assets. Some of the financial applications of DEA methodology in order to evaluation or choose assets are as follows: Murthi et al. (1997), Basso & Funari (2001), Emel et al. (2003), Eilat et al. (2006), Edirisinghe & Zhang (2007), Chen (2008), Ke et al. (2008), Lozano & Gutierrez (2008), Edirisinghe & Zhang (2008) and Amiri et al. (2010). The remainder of the paper is organized as follows. In Section 2, the mathematical formulation of a method for finding common weights is provided. In Section 3, the portfolio selection methodology is presented. Numerical example is presented in section 4 and finally, section 5 draws the conclusive remarks.

2.DEA preliminaries and common weights concept

Data envelopment analysis (DEA) is a method of evaluating relative performance of a group of similar units, called decision-making units (DMU). DMUs essentially perform the same task using similar multiple inputs to produce similar multiple outputs. DEA was first introduced, in 1978, by Charnes, Cooper and

Rhodes as a generalization of technical efficiency proposed by Farrell (Farrell, 1957).

The model of obviously most widely used DEA model, CCR with constant returns to scale (CRS) characteristic, is based on the assumption that all input/output parameters are positive (Charnes et al., 1978). This model maximizes the ratio of a linear combination of outputs to a linear combination of inputs and subject to production constraints to determine the (managerial) DEA-efficiency of a given DMU relative to other DMUs. Transforming this fractional mathematical program to a linear program, the relative efficiency is computed as follows for a given DMU_p, relative to remaining $(n-1)$ DMUs.

$$
\begin{aligned}
&(CCR_m) \quad Z_p = Max \sum_{r=1}^{s} u_r y_{rp} \\
&s.t. \qquad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0,\ j = 1,\ldots,n \\
&\qquad \sum_{i=1}^{m} v_i x_{ip} = 1 \\
&\qquad u_r \ge 0, r = 1,\ldots,s \\
&\qquad v_i \ge 0, i = 1,\ldots,m
\end{aligned}
\tag{1}
$$

The above model is called the input-oriented CCR multiplier model that determines the relative efficiency of the DMU_p (where $p = 1,\ldots,n$) using the output nonnegative multipliers u_r and input nonnegative multipliers v_i , the measured level of the output r for DMU_j is y_{rj} and the measured level of the input i for DMU_j is x_{ij}. Z_p is termed the DEA efficiency score of DMU_p. By applying (1) to each DMU independently, an efficiency score Z_j for each DMU_j is computed. Those DMUs with $Z_j = 1$ are termed efficient among the given n DMUs, while those with $Z_j < 1$ are termed DEA-inefficient. The dual problem will also be used afterwards and this is called the input-oriented CCR envelopment model:

$$
\begin{aligned}
&(CCR_e) \quad Min\theta_p \\
&s.t. \qquad \sum_{j=1}^{n} \lambda_j x_{ij} - \theta_p x_{ip} \le 0, i = 1,\cdots,m \\
&\qquad \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rp}, r = 1,\cdots,s \\
&\qquad \lambda_j \ge 0,\ j = 1,\cdots,n
\end{aligned}
\tag{2}
$$

However, it is possible that some of the input parameters for a given firm have non-positive observations, then the traditional CCR model is infeasible, that is,

constraint $\sum_{i=1}^{m} v_i x_{ip} = 1$ cannot be satisfied. While it makes no sense to assign an efficiency score in such a case, for computing an underlying strength index for the firm, Edirisinghe et al (2008) proposed a model, which is a slight generalization of the CCR model. The only difference is that the equality constraint is now replaced with an inequality. Hence, the two models are equivalent if at least one of the input parameters is strictly positive for the evaluated DMU_p, and if not, the efficiency for DMU_p is zero.

However, DEA gives a measure of efficiency, which is essentially defined as a ratio of weighted outputs to weighted inputs. But, the problem of allowing total flexibility of the weights in DEA models, caused to the production of weights which are often in contradiction to prior views or additional available information. This flexibility in selecting the weights, on the other hand, deters the comparison among DMUs on a common base. A possible answer to this difficulty lies in the specification of a common set of weights, which was first introduced by Roll and Golany (1993). Research about the idea of common weights has developed gradually in recent years. Some of the other studies in this field are as follows: Karsak & Ahiska (2005), Jahanshahloo et al (2005), Kao & Hung (2005) and Zohrebandian et al (2010). In this study, we use the proposed approach by Zohrebandian et al (2010) and then, we compute the efficiency scores by using these common weights. In other words, we did as following steps:

Step 1: Computation of the efficiency ratios.

This step consists of computing the efficiency scores $\left(\theta_j^*, j = 1,\ldots,n\right)$ for all DMUs by using model (2).

Step 2: Computation of the projection of DMU_p on the efficient frontier.

This step consists of computing the projection of DMU_p on the efficient frontier by using $\left(\theta_j^*\right)$, as $\left(\hat{x}_j, \hat{y}_j\right) = \left(\theta_j^* x_j, y_j\right), j = 1,\cdots,n$. This projection point is an efficient (virtual) DMU.

Step 3: Generation of a common set of weights.

This step consists of generating a common set of weights by solving below model, which produces by compromise solution approach, where *p* represents the distance parameter.

$$Min \quad D_p(u,v)=\left[\sum_{j=1}^{n}\left(-\sum_{r=1}^{s}u_r\hat{y}_{rj}+\sum_{i=1}^{m}v_i\hat{x}_{ij}\right)^p\right]^{\frac{1}{p}},\ p\geq 1$$

$$\begin{aligned} s.t. \quad & \sum_{r=1}^{s}u_r\hat{y}_{rj}-\sum_{i=1}^{m}v_i\hat{x}_{ij}\leq 0, \qquad j=1,\cdots,n \\ & \sum_{r=1}^{s}u_r+\sum_{i=1}^{m}v_i=1 \\ & v_i,u_r\geq 0, \qquad i=1,\cdots,m, \qquad r=1,\cdots,s \end{aligned} \tag{3}$$

Model (3) is a linearly constrained nonlinear program, where it is completely linear for $p=1$ and ∞.

Step 4: Computation of the efficiency ratios by using common weights.

This step consists of computing the efficiency ratios for DMU_j, $j=1,...,n$ which can be obtained by using the produced common weights (u^*,v^*) from step (3) as follows:

$$\theta_j=\left(\sum_{r=1}^{s}u_r^*y_{rj}\right)\Big/\left(\sum_{i=1}^{m}v_i^*x_{ij}\right),\ j=1,\cdots,n \tag{4}$$

3.Portfolio selection methodology

In this section we present the methodology for generating portfolio. First, the relevant inputs and outputs to evaluate DMUs are specified, and then the DMUs efficiency ratios are calculated based on presented approach with common weights. The generation of the desired portfolio is achieved using the following linear programming proposed by Dia (2009) which maximizes the aggregated efficiencies of the selected DMUs in the portfolio. Moreover, the DM's preferences must be established. In other words, the DM specifies bounds of inputs and outputs in order to determine, for example, the maximum level of risk (or input) which can be taken, or the minimal level of return (or output) expected.

$$\begin{aligned} & MaxP=\sum_{j=1}^{n}\theta_j z_j \\ & s.t.\sum_{r=1}^{s}u_r y_{rj}-\sum_{i=1}^{m}v_i x_{ij}\leq 0,\ j=1,\cdots,n \\ & \sum_{r=1}^{s}u_r y_{rj}\geq \theta_j,\ j=1,\cdots,n \\ & \sum_{j=1}^{n}z_j x_{ij}\leq \alpha_i, i=1,\cdots,m \\ & \sum_{j=1}^{n}z_j y_{rj}\geq \beta_r, r=1,\cdots,s \\ & \sum_{j=1}^{n}z_j=1 \\ & v_i,u_r\geq 0 \end{aligned} \tag{5}$$

In the above model, $(\theta_j, j = 1,..., n)$ is the efficiency ratio of DMU_j. $z_j, j = 1,..., n$ is the proportion (DMUs are stocks) of DMU_j invested in the portfolio. $\alpha_i (i = 1,..., m)$ is the maximal amount of input i to be considered in the portfolio, and $\beta_r (r = 1,..., s)$ is the minimal amount of output r to be considered in the portfolio.

4.Illustrative Example

To illustrate the application of the presented methodology in the portfolio selection of stocks, a problem of portfolio selection from the market of Iran stock exchange is utilized. For this illustrative example, we considered 21 stocks as DMUs. These stocks are accepted in Tehran stock exchange. In this study the risk coefficient β is defined as the only input factor, and rate of return, EPS and turnover are defined as output factors. We obtained basic data from Rahavarde novin Software in the Tehran stock exchange library during the fiscal year 20/03/2007 - 19/03/2008. The data of the study are presented in table 3.

Table 1 contains the common set of weights obtained by Zohrehbandian et al's method (for p=1) and the results obtained by our methodology and those by Dia (2009) are summarized in Table 2.

Table 1: Common set of weights

V_1^*	U_1^*	U_2^*	U_3^*
0.9909362	0.0000004704	0.000472263	0.008591074

Table 2:summary of results

	Dia (2009)		Our methodology	
Stock #	Efficiency ratio with CCR_e	Portfolio (Inv. Prop.)	Efficiency ratio with common weights	Portfolio (Inv. Prop.)
1	1	0.0000	0.999996789	0.5368362
2	0.11758227	0.0000	0.112659558	0.0000
3	0.24648551	0.0000	0.232984608	0.0000
4	0.26475204	0.0000	0.242449369	0.0000
5	0.84585324	0.0000	0.55755283	0.0000
6	0.46619564	0.0000	0.397320345	0.0000
7	0.81775684	0.0000	0.599881387	0.0000
8	0.72188369	0.0000	0.684560104	0.0000
9	0.15025888	0.0000	0.13290491	0.0000
10	0.24150518	0.0000	0.236043132	0.0000
11	0.32537266	0.0000	0.284477259	0.0000
12	1	0.4470285	0.789463331	0.0000
13	0.29255049	0.0000	0.286886214	0.0000
14	1	0.2111888	0.999987534	0.3580310

15	0.70925684	0.0000	0.382939472	0.0000
16	0.44725825	0.0000	0.40863437	0.0000
17	0.1743986	0.0000	0.154752451	0.0000
18	0.12841818	0.0000	0.126536377	0.0000
19	1	0.3417827	0.999992913	0.1051328
20	0.56940176	0.0000	0.466239425	0.0000
21	0.89536514	0.0000	0.550765093	0.0000

Note that, we considered DM's preferences for input and output bounds as follows: The risk coefficient must be less than 2, the rate of return must be greater than 3.5, the EPS must be greater than 1100, and the turnover must be greater than 0.0004.These bounds can be transformed by the DM and the process can be repeated until the preferred portfolio is found.

The results in column 3 show the portfolio is generated by Dia (2009) with stocks #12, #14, and #19 that they are efficient stocks resulted in column 2. We can see the results of our methodology in column 5 that the portfolio is generated with stocks #1, #14, and #19. The efficiency ratio for these stocks presented in column 4.

Table 3: inputs and outputs data of 21 stocks

Stock #	Input (risk coefficient β)	Output1 (rate of return)	Output 2 (EPS)	Output 3 (turnover)
1	0.68	2.23	614	0.0045411
2	1.82	1.95	443	0.000638
3	1.45	2.84	445	0.001364112
4	2.32	6.97	1671	0.00076152
5	0.62	1.39	3352	0.000743297
6	1.55	8.57	1785	0.000407519
7	1	7.05	3314	0.000267692
8	0.62	4.32	855	0.00121493
9	1.57	2.35	797	0.000310709
10	0.57	1.31	421	0.000342282
11	0.46	1.75	130	0.000238776
12	0.12	0.46	767	0.000238
13	1.42	1.26	382	0.00275
14	0.5	4.4	1834	0.001367205
15	0.18	0.22	816	0.000105386
16	1.24	5.88	705	0.001256
17	3.22	4.11	2575	0.001139
18	1.1	1.15	389	0.000474
19	0.54	6.92	1082	0.00087
20	0.78	3.1	2156	0.000652
21	0.32	0.13	1755	0.000632923

5.Conclusion

The asset evaluation process plays an important role in portfolio selection because it is the prerequisite for investment and directly influences on the asset allocation. In DEA for calculating the efficiency of different DMUs, different set of weights is obtained, which seems to be unacceptable in reality. In this paper, by using a mathematical model proposed by Dia (2009), a portfolio selection methodology based on data envelopment analysis with common weights is presented. In this paper we consider positive data for input and output factors, whereas, some of the input or output parameters for a given firm may have non-positive observations. Then, future investigations would extend the proposed methodology to situations on negative data and in uncertainty environments.

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