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The Use of Semi Inherited LU Factorization of Matrices in Interpolation of Data

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Abstract

The polynomial interpolation in one dimensional space R is an important method to approximate the functions. The Lagrange and Newton methods are two well known types of interpolations. In this work, we describe the semi inherited interpolation for approximating the values of a function. In this case, the interpolation matrix has the semi inherited LU factorization.

Keywords: Semi Inherited LU factorization; Interpolation matrix; Semi inherited interpolation

1 Introduction

The LU triangular factorization of matrices is a traditional and applicable method to solve a square linear system. The LU factorization has different types. One of them which is constructed easily is the inherited LU factorization [1]. In this work, we use a special kind of the inherited LU factorization and we constitute the interpolation matrix such that this matrix has this type of factorization. For simplicity, we call this type of LU factorization, semi inherited LU factorization. At first, in section 2, we introduce the semi inherited LU factorization [1,5] and present some preliminaries briefly [7,8]. In section 3, we describe the semi inherited interpolation which depends on the semi inherited LU factorization and we prove that the interpolation matrix in this case has the semi inherited LU factorization. In section 4, we describe the behavior of semi inherited polynomials and matrices when the number of nodes increases and in section 5 we illustrate the mentioned methods in some numerical examples.

2 Preliminaries

Definition 2.1 Let A be an $n\times n$ matrix that $a_{ii} \neq 0$ for $i = 1, \dots, n$ and write $A = B + D + C$, that B is strictly lower triangular, D is diagonal, and C is strictly upper triangular. Then, A has the semi inherited LU factorization if and only if

A = $(I + BD^{-1})(D + C)$. (I is the identity matrix)

Theorem 2.2 Let $A = B + D + C$ be a decomposition of matrix A with invertible diagonal entries where B is strictly lower triangular, D is diagonal, and C is strictly upper triangular. Then, A has the semi inherited LU factorization if and only if $BD^{-1}C = 0$.

Proof. Suppose that the matrix $A_{n\times n}$ has the semi inherited LU factorization. Then,

$$A=B+D+C=(I+BD^{-1})(D+C)=D+C+B+BD^{-1}C \Rightarrow$$

$$BD^{-1}C = 0.$$

Conversely, suppose $BD^{-1}C= 0$. Then,

$$A=B+D+C+BD^{-1}C \Rightarrow A=(I+BD^{-1})(D+C).\square$$

For example:

$$A=\begin{pmatrix} 2 & 0 & 12 & 6 \\ 0 & 6 & 12 & 6 \\ 2 & -6 & 1 & 0 \\ -4 & 12 & -3 & 1 \end{pmatrix}=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ -2 & 2 & -3 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 & 12 & 6 \\ 0 & 6 & 12 & 6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}=$$

$$(I + BD^{-1})(D + C).$$

Remark 2.3 A useful property of matrices which have semi inherited LU factorization is this fact that the calculation of LU factorization of them is very easy, because the matrix U is completely inherited and the matrix L is the product of the strictly lower triangular matrix and the diagonal matrix plus to the identity matrix [1].

Suppose $X=\{x_1, x_2, \dots, x_n\}$ is a set of n distinct points in R, that these are called nodes and X is the nodes set. Also, for each x_i an ordinate $y_i \in R$ is given. The problem of interpolation is to find a suitable function F: $R \to R$ where, $F(x_i) = y_i$ for $i=1,\dots,n$. F is called the interpolation function of $\{(x_i, y_i)\}_{i=1}^n$.

Let U be a vector space of functions with the basis $\{u_1, u_2, ..., u_n\}$, we can consider F in the form of $F=\sum_{j=1}^{n}\lambda_j u_j$. Then

$$F(x_i)=y_i \quad i=1,\dots,n \quad \Rightarrow \quad \sum_{j=1}^{n}\lambda_j u_j(x_i)=y_i \quad \forall i=1,...,n$$

$$\begin{pmatrix} u_1(x_1) & u_2(x_1) & \dots & u_n(x_1) \\ u_1(x_2) & u_2(x_2) & \dots & u_n(x_2) \\ . & . & & . \\ . & . & & . \\ . & . & & . \\ u_1(x_n) & u_2(x_n) & \dots & u_n(x_n) \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ . \\ . \\ . \\ \lambda_n \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ . \\ . \\ . \\ y_n \end{pmatrix}$$

The matrix $A=[u_j(x_i)]_{n\times n}$ is called the interpolation matrix. Therefore, the problem of interpolation is to solve the linear system $A\lambda = y$ where, $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)^t$ and $y = (y_1, y_2, ..., y_n)^t$.

If we consider $u_j(x)=x^{j-1}(j=1,...,n)$ the matrix A has the form of Vandermonde matrix that its determinant is $\Pi_{1\le j<i\le n}(x_i - x_j)$ Hence, the interpolation function has a unique solution if and only if the nodes x_i are distinct. In this case, the interpolation function is a polynomial of degree at most n-1.

Definition 2.4 A vector space U of functions is said a Haar space if the only element of U which has more than n-1 roots is the element of zero [2].

Theorem 2.5 Let U is a vector space with the basis $\{u_1,...,u_n\}$, then the following statements are equivalent:

i) U is a Haar space.

ii) $det[u_j(x_i)]\neq 0$ for any set of distinct points $x_i, i, j=1,2,..., n$.

Proof. ($i\to ii$): Suppose U is a Haar space and $det[u_j(x_i)] = 0$ for a set of distinct points $x_1,..., x_n$ in R. Then, there exists a nonzero vector $c^t=(c_1, c_2, ..., c_n)$ such that $\sum_{j=1}^{n}c_j u_j(x_i)=0$ for $i=1,...,n$. Put $u=\sum_{j=1}^{n}c_j u_j$. Then $u\in U$ and we know that $dim(U)= n$. But $u(x_i)=0 \quad \forall i=1,\dots,n$. Hence, U is not a Haar space.

$(ii\to i)$: Suppose $det[u_j(x_i)]\neq 0$ for any set of distinct points $x_1,..., x_n$ in R, and also suppose U is not a Haar space. Then, there exists $u=\sum_{j=1}^{n}c_j u_j \neq 0$ in U that has at least n roots $x_1,...x_n$ in R. then, $\sum_{j=1}^{n} c_j u_j(x_i)=0$ for $c^t=(c_1, c_2, \ ... \ , c_n)\neq 0$. Hence, $det[u_j(x_i)]=0$ which contradicts the hypothesis. □

3 Introducing of semi inherited interpolation

We suppose that the nodes x_1, ..., x_n are available and U is a vector space of functions with the basis $\{u_1,u_2,\ldots,u_n\}$ and also $A=[u_j(x_i)]_{n\times n}$ is nonsingular. Then, for ordinates $y_1,y_2,\ldots,y_n$ there exists a unique function $u\in U$ that $u(x_i)= \ y_i$ for $i=1,....,n$. Let $y^T=(y_1, y_2, ..., y_n)$,

$$A\lambda = y \qquad , \qquad u=\sum_{j=1}^{n}\lambda_i u_i \ .$$

Finding the function u is various for different bases. One of the schemes which we want to describe is semi inherited interpolation. In this case, we consider the basis $\{h_1, h_2,..., h_n\}$ in the form of,

$$h_{2i-1}(x)=(x-x_2)(x-x_4)...(x-x_{2i})(x-x_{2i+1})...(x-x_n), \qquad (1)$$

$$h_{2i}(x)=(x-x_2)(x-x_4)...(x-x_{2i-2}), \qquad (2)$$

where $1\leq i\leq \left[\frac{n+1}{2}\right]$ and $h_2(x)=1$.

We call the basis polynomials in this type, semi inherited polynomials. For example, the semi inherited polynomials for n = 6 are:

$$h_1(x)=(x-x_2)(x-x_3)(x-x_4)(x-x_5)(x-x_6),$$

$$h_3(x)=(x-x_2)(x-x_4)(x-x_5)(x-x_6),$$

$$h_5(x)=(x-x_2)(x-x_4)(x-x_6),$$

$$h_2(x)=1,$$

$$h_4(x)=(x-x_2),$$

$$h_6(x)=(x-x_2)(x-x_4).$$

Next theorem shows that the matrix $A=[h_j(x_i)]$ for any set of distinct points $x_1, x_2,...., x_n$ has the semi inherited LU factorization and also A is a nonsingular matrix.

Theorem 3.1 Let A be the semi inherited interpolation matrix with distinct node points. Then, A has the semi inherited LU factorization.

Proof. Suppose $A=[h_j(x_i)]_{n\times n}$, according to the structure of semi inherited polynomials:

$$\begin{array}{lll} h_i(x_i)\neq 0 & i=1,\dots,n & (a) \\ h_{2j-1}(x_i)=0 & 2j-1<i, & (b) \\ h_j(x_{2i})=0 & 2i<j. & (c) \end{array}$$

We put:

$$A=\begin{pmatrix} h_1(x_1) & h_2(x_1) & \cdots & h_n(x_1) \\ h_1(x_2) & h_2(x_2) & \cdots & h_n(x_2) \\ \vdots & \vdots & \vdots & \\ h_1(x_n) & h_2(x_n) & \cdots & h_n(x_n) \end{pmatrix}=B+D+C\ ,$$

where, B is strictly lower triangular, D is diagonal, and C is strictly upper triangular. By the theorem 2.2, it is sufficient to show that $BD^{-1}C=0$. Under the condition (a), $D=diag(h_1(x_1),\dots,h_n(x_n))$ is a nonsingular matrix and $D^{-1}=diag$ $(h_1(x_1)^{-1},\dots,h_n(x_n)^{-1})$, [3,4,6]. Let $E=BD^{-1}$. Hence, E has the following form:

$$E=\begin{pmatrix} 0 & 0 & \dots & 0 & 0 \\ \frac{h_i(x_2)}{h_1(x_1)} & 0 & \dots & 0 & 0 \\ \cdot & & & \cdot & \cdot \\ \cdot & & & \cdot & \cdot \\ \cdot & & & \cdot & \cdot \\ \frac{h_1(x_n)}{h_1(x_1)} & \frac{h_2(x_n)}{h_2(x_2)} & \dots & \frac{h_{n-1}(x_n)}{h_{n-1}(x_{n-1})} & 0 \end{pmatrix}.$$

Consider the ith row of E and jth column of C $(1<i,j\leq n)$. Then

$$e^i c_j = \left(\tfrac{h_1(x_i)}{h_1(x_1)}\,\tfrac{h_2(x_i)}{h_2(x_2)}\cdots\tfrac{h_{i-1}(x_i)}{h_{i-1}(x_{i-1})}\,0\ \ \ldots\ \ 0\right)\begin{pmatrix} h_i(x_1) \\ h_j(x_2) \\ . \\ . \\ . \\ h_j(x_{j-1}) \\ 0 \\ . \\ . \\ . \\ 0 \end{pmatrix}.$$

Now, it is sufficient to show that $\frac{h_k(x_1)}{h_k(x_k)}h_j(x_k)=0(k<\min(i,j),1<i,j\le n)$. If k is odd, under the condition (*b*) $h_k(x_i)=0$, and if k is even, under the condition (*c*) $h_j(x_k)=0$. Then, A has the semi inherited LU factorization. □

According to the theorem 3.1, $A=[h_j(x_i)]$ for distinct points $x_1,\ldots,x_n$ has the semi inherited LU factorization. Then, $A=(I+BD^{-1})(D+C)$. Therefore, we conclude $det\ (A)\neq 0$ for any set of distinct points $\{x_1,\ldots,x_n\}$ [4]. Therefore, by the theorem 2.5, the basis polynomials $\{h_1,\ldots,h_n\}$ form a Haar space and also these are linearly independent [2]. Then, interpolation is unique, that in this case the interpolation function is a polynomial of degree at most n-1.

4 Calculation of inherited polynomials and matrices when the number of the points is added

We suppose the semi inherited polynomials are available for the nodes $x_1,\ldots,x_n$ If a distinct point is added to the previous nodes, it is not necessary to calculate the semi inherited polynomials from the beginning and we can use the previous calculations to find the new semi inherited polynomials.

The meaning of $h_i^{(n)}$ is the ith semi inherited polynomial for n nodes. For example, suppose the semi inherited polynomials are available for the nodes x_1, x_2 Then,

$$h_1^{(2)}(x)=(x-x_2) \qquad h_2^{(2)}(x)=1$$

Now, if the point $x_3 \neq x_1, x_2$ is added to the previous nodes then we have,

$$h_1^{(3)}(x) = (x - x_2)(x - x_3) = h_1^{(2)}(x)(x - x_3)$$

$$h_2^{(3)}(x) = 1 = h_1^{(3)}(x) \qquad h_3^{(2)}(x) = (x - x_2) = h_1^{(2)}(x)$$

Hence we used the $h_i^{(2)}(x)(i = 1,2)$ to calculate the $h_j^{(3)}(x)(j = 1,2,3)$.

If the point $x_4 \neq x_1, x_2, x_3$ is added to the nodes x_1, x_2, x_3 then,

$$h_i^{(4)}(x) = (x - x_2)(x - x_3)(x - x_4) = h_1^{(3)}(x)(x - x_4)$$

$$h_2^{(4)}(x) = 1 = h_2^{(3)}(x).$$

$$h_3^{(4)}(x) = (x - x_2)(x - x_4) = h_3^{(3)}(x)(x - x_4)$$

$$h_4^{(4)}(x) = (x - x_2) = h_3^{(3)}$$

Hence, we can conclude that if the semi inherited polynomials are available for *n* nodes and if one point distinct from previous nodes is added to the nodes, the new semi inherited polynomials are calculated in the following form:

If the index of semi inherited polynomial is even and smaller than n + 1 then it remains invariant. If the index of inherited polynomial is odd and smaller than *n* + 1 then $h_{2i-1}^{(n+1)}(x) = h_{2i-1}^{(n)}(x)(x - x_{n+1})$ and the semi inherited polynomial $h_{n+1}^{(n+1)}$ is depended on the index $n+1$ *if n+1* is odd, then $h_{n+1}^{(n+1)} = h_{n-1}^{(n)}$ and if $n+1$ is even then $h_{n+1}^{(n+1)} = h_n^{(n)}$.

Now, we talk about the semi inherited matrices. Let $A^{(n)}$ be the semi inherited interpolation matrix at the distinct points $x_1, x_2, ..., x_n$. Then,

$$A^{(n)} = \begin{pmatrix} h^n(x_1) & h_2^{(n)}(x_1) & \dots & h_n^{(n)}(x_1) \\ h_1^{(n)}(x_2) & h_2^{(n)}(x_2) & \dots & h_n^{(n)}(x_2) \\ . & . & & . \\ . & . & & . \\ . & . & & . \\ h_1^{(n)}(x_n) & h_2^{(n)}(x_n) & \dots & h_n^{(x)}(x_n) \end{pmatrix} =$$

$$\begin{pmatrix} \frac{h_1^{(n)}(x_1)}{h^{(n)}(x_1)} & 0 & \cdots & 0 \\ \frac{h_1^{(n)}(x_2)}{h_1^{(n)}(x_1)} & \frac{h_2^{(n)}(x_2)}{h_2^{(n)}(x_2)} & & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \frac{h_1^{(n)}(x_n)}{h_1^{(n)}(x_1)} & \frac{h_2^{(n)}(x_n)}{h_2^{(n)}(x_2)} & \cdots & \frac{h_n^{(n)}(x_n)}{h_n^{(n)}(x_n)} \end{pmatrix} \begin{pmatrix} h_1^{(n)}(x_1) & h_2^{(n)}(x_1) & \cdots & h_n^{(n)}(x_1) \\ 0 & h_2^{(n)}(x_2) & \cdots & h_n^{(n)}(x_2) \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & h_n^{(n)}(x_n) \end{pmatrix} = L^{(n)}U^{(n)}$$

and $A^{(n+1)}$ has following from:

$$A^{(n+1)} = \begin{pmatrix} h_1^{(n+1)}(x_1) & h_2^{(n+1)}(x_1) & \cdots & h_{n+1}^{(n+1)}(x_1) \\ h_1^{(n+1)}(x_2) & h_2^{(n+1)}(x_2) & \cdots & h_{n+1}^{(n+1)}(x_2) \\ \vdots & \vdots & \vdots & \vdots \\ h_1^{(n+1)}(x_{n+1}) & h_2^{(n+1)}(x_{n+1}) & \cdots & h_{n+1}^{(n+1)}(x_{n+1}) \end{pmatrix} = L^{(n+1)}U^{(n+1)}, \qquad (3)$$

where,

$$L^{(n+1)} = \begin{pmatrix} & & & 0 \\ & L^{(n)} & & \vdots \\ \frac{h_1^{(n+1)}(x_{n+1})}{h_1^{(n+1)}(x_1)} & \cdots & \frac{h_n^{(n+1)}(x_{n+1})}{h_n^{(n+1)}(x_n)} & \frac{h_{n+1}^{(n+1)}(x_{n+1})}{h_{n+1}^{(n+1)}(x_{n+1})} \end{pmatrix}$$

and if n+1 is odd then:

$$U^{(n+1)} = \begin{pmatrix} h_1^{(n)}(x_1)(x_1 - x_{n+1}) & h_2^{(n)}(x_1) & h_3^{(n)}(x_1)(x_1 - x_{n+1}) & \cdots & h_{n-1}^{(n)}(x_1) \\ 0 & h_2^{(n)}(x_2) & h_3^{(n)}(x_2)(x_2 - x_{n+1}) & \cdots & h_{n-1}^{(n)}(x_2) \\ 0 & 0 & h_3^{(n)}(x_3)(x_3 - x_{n+1}) & \cdots & h_{n-1}^{(n)}(x_3) \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & h_{n-1}^{(n)}(x_{n+1}) \end{pmatrix}$$

and if n+1 is even then:

$$U^{(n+1)}=\begin{pmatrix} h_1^{(n)}(x_1)(x_1-x_{n+1}) & h_2^{(n)}(x_1) & h_3^{(n)}(x_1)(x_1-x_{n+1}) & \cdots & h_n^{(n)}(x_1) \\ 0 & h_2^{(n)}(x_2) & h_3^{(n)}(x_2)(x_2-x_{n+1}) & \cdots & h_n^{(n)}(x_2) \\ 0 & 0 & h_3^{(n)}(x_3)(x_3-x_{n+1}) & \cdots & h_n^{(n)}(x_3) \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & h_n^{(n)}(x_{n+1}) \end{pmatrix}$$ We note that the matrices $L^{(n+1)},U^{(n+1)}$ are $(n+1)\times(n+1)$.

5 Numerical examples

Example 5.1. Find the semi inherited interpolation polynomial for nodes X = {1, 2, -3,7,4} and ordinates Y={3, -4, -1,3,2}.

By using (1) and (2),

$$h_1(x)=(x-x_2)(x-x_3)(x-x_4)(x-x_5) \quad , \quad \mathrm{h}_2(x)=1$$

$$h_3(x)=(x-x_2)(x-x_4)(x-x_5) \quad , \quad \mathrm{h}_4(x)=(x-x_2) \quad , \quad \mathrm{h}_5(x)=(x-x_2)(x-x_4)$$

$$A=\begin{pmatrix} -72 & 1 & -18 & -1 & 6 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -350 & -5 & 50 \\ 0 & 1 & 0 & 5 & 0 \\ 0 & 1 & 0 & 2 & -6 \end{pmatrix}=\left(I+BD^{-1}\right)(D+C)=$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & \frac{2}{5} & 1 \end{pmatrix}\begin{pmatrix} -72 & 1 & -18 & -1 & 6 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -350 & -5 & 50 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & -6 \end{pmatrix}$$

By solving the system $A\lambda = y$:

$$\lambda_1=\frac{-17}{126},\lambda_2=-4,\ \lambda_3=\frac{-11}{105},\ \lambda_4=\frac{7}{5},\ \lambda_5=\frac{-8}{15}.$$

$$H(x)=\sum_{j=1}^{5}\lambda_j h_j(x)=\frac{-17}{126}x^4+\frac{56}{45}x^3-\frac{59}{90}x^2-\frac{3692}{315}x+\frac{214}{15}$$

We can see;

$$H(1)=3, H(2)=-4,\ H(\text{-}3)=-1,\ H(7)=3,\ H(4)=2.$$

Example 5.2 Find the semi inherited interpolation polynomial for nodes X ={12, -3, 4, 5, 6, -1$_1$8, 9} and ordinates Y={2,5,-6,12,10,-3,7,2}.

Similar to the previous example,

$$A=\begin{pmatrix} 786240 & 1 & 98280 & 15 & 16380 & 105 & 4095 & 1365 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1400 & 7 & -700 & -7 & 175 & -35 \\ 0 & 1 & 0 & 8 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 9 & 378 & 9 & -189 & 63 \\ 0 & 1 & 0 & 2 & 0 & -12 & 0 & 0 \\ 0 & 1 & 0 & 11 & 0 & 33 & -297 & 297 \\ 0 & 1 & 0 & 12 & 0 & 48 & 0 & 480 \end{pmatrix} = \left(I+BD^{-1}\right)\left(D+C\right)=$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{9}{8} & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{4} & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{11}{8} & 0 & \frac{-11}{4} & 1 & 0 \\ 0 & 1 & 0 & \frac{3}{2} & 0 & -4 & 0 & 1 \end{pmatrix}\begin{pmatrix} 786240 & 1 & 98280 & 15 & 16380 & 105 & 4095 & 1365 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1400 & 7 & -700 & -7 & 175 & -35 \\ 0 & 1 & 0 & 8 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 9 & 378 & 9 & -189 & 63 \\ 0 & 1 & 0 & 2 & 0 & -12 & 0 & 0 \\ 0 & 1 & 0 & 11 & 0 & 33 & -297 & 297 \\ 0 & 1 & 0 & 12 & 0 & 48 & 0 & 480 \end{pmatrix}$$

By solving the system $A\lambda = y$,

$$\lambda_1=\frac{252349}{129729600}\ ,\ \lambda_2=5\ ,\ \lambda_3=\frac{-48043}{3326400}\ ,\ \lambda_4=\frac{7}{8}$$

$$\lambda_5=\frac{-241}{44352}\ ,\ \lambda_6=\frac{13}{16}\ ,\ \lambda_7\frac{125}{19008}\ ,\ \lambda_8=\frac{-7}{64}$$

$$H(x)=\sum_{j=1}^{8}\lambda_j h_j(x)=$$

$\frac{252349}{129729600}x^7-\frac{8939449}{129729600}x^6+\frac{37970549}{43243200}x^5-\frac{112690327}{25945920}x^4+\frac{754973}{2494800}x^3+\frac{18820649}{327600}x^2-\frac{19216187}{225225}x-\frac{701577}{5005}$ We can see:

$H(12)=2,\ H(-3)=5,\ H(4)=-6,\ H(5)=12,\ H(6)=10,\ H(-1)=-3,\ H(8)=7,\ H(9)=2.$

Example 5.3 If X={1, 2,3,4,5}, X' ={1, 2,3,4,5,6}, and *X''*={1,2,3,4,5,6,7} then from (3), the semi inherited interpolation matrices $A^{(5)}, A^{(6)}, A^{(7)}$ have the following forms:

$$A^{(5)}=\begin{pmatrix}24&1&-12&-1&3\\0&1&0&0&0\\0&1&2&1&-1\\0&1&0&2&0\\0&1&0&3&3\end{pmatrix}=L^{(5)}U^{(5)}=$$

$$\begin{pmatrix}1&0&0&0&0\\0&1&0&0&0\\0&1&1&0&0\\0&1&0&1&0\\0&1&0&\frac{3}{2}&1\end{pmatrix}\begin{pmatrix}24&1&-12&-1&3\\0&1&0&0&0\\0&0&2&1&-1\\0&0&0&2&0\\0&0&0&0&3\end{pmatrix}$$

$$A^{(6)}=\begin{pmatrix}-120&1&60&-1&-15&3\\0&1&0&0&0&0\\0&1&-6&1&3&-1\\0&1&0&2&0&0\\0&1&0&3&-3&3\\0&1&0&4&0&8\end{pmatrix}=L^{(6)}U^{(6)}=$$

$$\begin{pmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&1&1&0&0&0\\0&1&0&1&0&0\\0&1&0&\frac{3}{2}&1&0\\0&1&0&2&0&1\end{pmatrix}\begin{pmatrix}-120&1&60&-1&-15&3\\0&1&0&0&0&0\\0&0&-6&1&3&-1\\0&0&0&2&0&0\\0&0&0&0&-3&3\\0&0&0&0&0&8\end{pmatrix}$$

$$A^{(7)}=\begin{pmatrix}720&1&-360&-1&90&3&-15\\0&1&0&0&0&0&0\\0&1&24&1&-12&-1&3\\0&1&0&2&0&0&0\\0&1&0&3&6&3&-3\\0&1&0&4&0&8&0\\0&1&0&5&0&15&15\end{pmatrix}=L^{(7)}U^{(7)}=$$

$$\begin{pmatrix}1&0&0&0&0&0&0\\0&1&0&0&0&0&0\\0&1&1&0&0&0&0\\0&1&0&1&0&0&0\\0&1&0&\frac{3}{2}&1&0&0\\0&1&0&2&0&1&0\\0&1&0&\frac{5}{2}&0&\frac{15}{8}&1\end{pmatrix}\begin{pmatrix}720&1&-360&-1&90&3&-15\\0&1&0&0&0&0&0\\0&0&24&1&-12&-1&3\\0&0&0&2&0&0&0\\0&0&0&0&6&3&-3\\0&0&0&0&0&8&0\\0&0&0&0&0&0&15\end{pmatrix}$$

We can see that the leading submatrix $(n-1)\times(n-1)$ of $L^{(n)}$ is coincided to $L^{(n-1)}$ and the leading submatrix $(n-1)\times(n-1)$ of $U^{(n)}$ is obtained from $U^{(n-1)}$ according to the explanation in section 4. Also, we can see the behaviors of the last column of the $U^{(n)}$, when n is odd or even.

6 Conclusion

In this work, we have suggested a new method to find the interpolation polynomial. We applied the semi inherited interpolation and introduced semi inherited LU factorization of a matrix. In this case, the interpolation matrix is constructed which has the semi inherited LU factorization and help us to find the coefficients in the interpolation polynomial easily. We used this special kind of decomposition of the square matrix A to introduce a new way for interpolating. In this manner, we introduced the new bases, new matrix and new interpolation. The method is simple and the cost of operation in the new method is less than the Lagrange polynomial and almost the same as the Newton's divided differences method. There are some superiors in comparison with the Lagrange interpolation. We can calculate the semi inherited polynomials very easy and fast. Also, we can add a new point in the table and compute the interpolation polynomial very easy by using the previous calculations and matrices as it has been discussed in the article. Since the interpolation polynomial is unique, each new method for finding the interpolation polynomial has the same error like common techniques such as Newton's and Lagrange interpolations. If we use the Lagrange, Newton or semi inherited interpolation methods the results are the same. In order to improve this work, one can introduce rational semi inherited interpolation. It means that we can use the semi inherited interpolation to find the functions which are not polynomial to interpolate a table of data. Also, the new technique can be developed for bivariate interpolation. Consequently, one can use this special factorization to interpolate a table of data easily.

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