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Evaluation of the Performance in Dynamic Network Data Envelopment Analysis with Undesirable Output

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Abstract

Data Envelopment Analysis (DEA) is a mathematical technique to assess the performance of Decision Making Units (DMUs) with similar inputs and outputs. The traditional DEA models disregard the internal structure of units and have a “black box” view. Thus, to evaluate the structures with more than one stage, the network DEA (NDEA) models expanded. On the other hand, the dynamic optimization models have been presented to eliminate the limitations of static models in optimization. In the article, for the first time, a systematic approach is used to present a dynamic NDEA with constant inputs and undesirable outputs.

First, we used an axiomatic approach in DEA with undesirable output and presented an NDEA model with undesirable output. Then, we extended the proposed approach and presented a dynamic NDEA with undesirable output and a constant input. Afterward, we applied this model to evaluate hospitals’ performance in an experimental study to estimate the efficiency of their dynamic network.

Keywords:
Data envelopment analysis (DEA)
Dynamic network DEA
Undesirable output
Evaluation of hospitals

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INTRODUCTION

Farrell (1957) was the first researcher who introduced non-parametric methods for determining the efficiency when there are multiple inputs with one output(Farrell, 1975). Charnes, Cooper, and Rhodes extended this method for multiple inputs and outputs and introduced it under the topic of the data envelopment analysis (DEA) model(Charnes et al., 1978). In other words, the CCR model extended the efficiency of Farrell's "multiple input-one outputs" model to the general status of multiple input-multiple outputs. Later on, in 1984, the BCC article was published by Banker, Charnes, and Cooper(Banker et al., 1984). Besides CCR and BCC models, several basic models like additive model and SBM (Slacks-Based Measure) were presented(Tone, 2001). All of these models are grouped under classic DEA models. During the last couple of years, many articles have been published about extending various DEA models. Some of them have extended the mathematical models, and some used the applications of these models in real-world problems. For instance, Emrouznejad et al. (2017) reviewed the DEA models after 40 years of their introduction(Emrouznejad & Yang, 2018). Also, Kaffash et al. (2019) reviewed the DEA models in the Insurance sector(Kaffash et al., 2019).

Hosseinzadeh et al. (2020) explained the implementation of DEA models in R software to use in applied examples(Hosseinzadeh Lotfi et al., 2020). Also, some other DEA models have been submitted(Moghaddas et al., 2020; Vaez-Ghasemi et al., 2020).

One of the problems of classic DEA models is that they consider the decision-making units (DMUs) black box and disregard their internal structure and function(Holod & Lewis, 2011; Kao, 2009; Kao, 2014a; Kao, 2014b; Kao, 2019; Kao, 2010; Kao, 2009; Kao, 2009b; Khoveyni et al., 2019). In classic EDA models in which DMU is considered a black box, a DMU may be found efficient while its substructures are not(Kao, 2009).

To fix the problems of classic and independent models, Fare and Grosskopf introduced network DEA (NDEA) models(Fare & Grosskopf, 1989). In these models, the operation process in evaluating the efficiency of DMUs is performed by a multi-stage NDEA model. Based on works done by Fare et al. (1996), Fare and Tiaker (1995), and Sengo Petta (1995), they proposed three models that consider the internal structure and the relationships between the components(Fare & Grosskopf, 1989). Unlike the classic models, the NDEA of Kao (2009) standard format and their models depend on the DMU structure, substructures relationships, and input and output types Kao, C. (2009a). Considering the concept of "variations effect," Khoveyni et al. (2019) studied the topic in two-stage NDEA that if intermediate products increase after increasing the inputs of the first stage, how the output products will change(Khoveyni et al., 2019).

Tajik Yabr et al. (2020) discussed an interval cross efficiency in a general two-stage network(Tajik Yabr et al., 2022).

On the other hand, with even the same structure, it is possible that several different models with various approaches and perspectives have been proposed for a unique structure(Fukuyama & Weber, 2010; Kao, 2009a; Kao, 2019).

Simple models only address the local optimization in a certain period in a specified and independent time period. Thus, the network optimization model is not suitable for the performance evaluation of more complex supply networks with multiple levels. This model disregards the individual or joint relationships in the system's internal structures and cannot assess the efficiency and performance in several successive and interdependent stages. To overcome this problem and consider the efficiency in a long time, the researchers use the dynamic DEA that contains the transfer operation. This model can measure the efficiency of a particular period based on long-term optimization (Tajik Yabr et al., 2022).The functions of the active organizations are like an interdependent chain. Thus, evaluating their performance during multiple periods is necessary and provides better information for the managers. In this regard, Nemoto and Goto(1999; 2003) introduced dynamic DEA (DDEA) models. These models consider the relationship of each unit with itself in successive periods and present the efficiency of each period, as well as the whole efficiency. However, these models consider the structure of units in each period a black box and disregard the internal structure.

As we have discussed, researchers for solving the black box problem of classic models extended the network models for evaluating the efficiency of units in different processes. However, the network models are static, and the proposed DDEA models mainly consider DMU in any period as a single-stage form. Thus, a model is needed that simultaneously considers the unit internal structure and time, providing richer information of units' performances. Kao (2104) comprehensively reviewed the NDEA and DDEA models and proposed three suggestions for future studies(Kao, 2014b). One of them is to extend the dynamic models into network structures, i.e., designing DNDEA models. Besides Kao (2014), other researchers emphasized the extension of these models(Fukuyama & Weber, 2010; Tone & Tsutsui, 2010, 2014).

DMUs may produce undesirable outputs like pollution besides desirable outputs. For the first time, Fare et al. (1984) formulated this concept using the DEA and presented a non-linear model for assessing efficiency(Fare & Grosskopf, 2003, 1984). Using the stability of the Fare et al. model, Sieford and Zhu (2002) proposed a linear model to determine the efficiency of undesirable data(Seiford & Zhu, 2002).

Considering undesirable output as the input, Hailo and Veeman (2000) assessed the paper and wood industry in Canada(Hailu & Veeman, 2001). Fare et al. (2003), in their article, mentioned the violation of taking undesirable outputs as input with the disposability axiom of Shepherd (1970)(Fare & Grosskopf, 2003). They added a downsizing factor for outputs (desirable and undesirable) of all DMUs and presented a new formulation for undesirable data. Kuosmanen (2005), in an article, rejects the adequacy of a constraint factor for all DMUs and, by presenting an example, shows that the set of production possibility is non-convex(Kuosmanen, 2005). Eventually, Kuosmanen and Poidinovski (2009) proved that the production possibility set of Kuosmanen (2005) is the smallest convex set containing undesirable outputs that hold with the Shepherd disposability principle(Kuosmanen & Poidinovski, 2009).

This article aims to take an axiomatic and extends a dynamic NDEA with undesirable output and a constant input. In other words, we first presented a DEA with undesirable output. For this purpose, we use an axiomatic approach and production possibility set presented by Kuosmanen and Poidinovski (2009). Then, we express this NDEA model in a dynamic state and present a new dynamic NDEA with undesirable outputs and constant inputs. Finally, to demonstrate the importance of this topic, we apply the proposed model for assessing hospitals' performance.

The article is organized as follows. In the next section, the prerequisites of the article and the axiomatic approach in relation to the undesirable output are explained. The next section includes the main part of this paper. In this section, an NDEA model with undesirable output is presented, and then a dynamic NDEA with undesirable output and a constant input will be presented. In the fourth section, an application from the real world is presented for the proposed model. The fifth section includes the conclusion and some suggestions.

THE AXIOMATIC APPROACH IN EVALUATIONG DMU$_s$ WITH UNDESIRABLE DATA

Assume that we have K DMUs with N input, M desirable output, and J undesirable output. Assume that $x \in R_+^N$ is the vector of consumed inputs and $v \in R_+^M$, $w \in R_+^J$ are vectors of desirable and undesirable outputs, respectively. In this section, we introduce the weak disposability axiom of Shepherd and the production possibility set of Kuosmanen. Shepherd (1970) introduced the weak disposability axiom as follows(Shephard, 1970):

Definition 1(Shepherd, 1970): T technology holds in weak disposability principle, if for each given x vector, the feasible output vector (v,w) can be downsized by factor θ ($0\leq \theta \leq 1$). In fact, if x vector could produce output vector (v,w), then x vector could produce output vector (θv, θw).

Kuosmanen (2005), with regard to the weak disposability axiom of Shepherd, presented the following production possibility set that holds under the observations, input disposability principle, and convexity(Kuosmanen, 2005).

$$T=\left\{ (x,v,w) \middle| \sum_{k=1}^{K}\theta_k\lambda_k v_{mk} \geq v_m \quad m = 1,\dots,M \ , \sum_{k=1}^{K}\theta_k\lambda_k w_{kj} = w_j \quad j = 1,\dots,J \quad \sum_{k=1}^{K}\lambda_k x_{nk} \leq x_n \quad n = 1,\dots,N\ , \sum_{k=1}^{K}\lambda_k = 1\ ,\ \lambda_k \geq 0 \quad k = 1,\dots,K \quad 0 \leq \theta_k \leq 1 \quad k=1,\dots,K \right\} \tag{1}$$

Kuosmanen and Poidinovski (2009) showed that the smallest convex set holds in the related principles. One of the problems of this formulation is its non-linear form that will be changed to linear after changing the following variable [22]:

$$\mu^k = (1-\theta^k)\lambda^k$$
$$\eta^k = \theta^k\lambda^k$$
$$\theta^k = \frac{\eta^k}{\eta^k+\mu^k} \tag{2}$$

In fact, after changing this variable, the linear disposable production set is produced:

$$T=\left\{ (x,v,w) \middle| \sum_{k=1}^{K}\eta_k v_{mk} \geq v_m \quad m = 1,\dots,M \ ,\ \sum_{k=1}^{K}\eta_k w_{kj} = w_j \quad j = 1,\dots,J\ ,\ \sum_{k=1}^{K}(\eta_k+\mu_k)x_{nk} \leq x_n \quad n = 1,\dots,N\ ,\ \sum_{k=1}^{K}(\eta_k+\mu_k) = 1\ ,\ \eta_k \geq 0\ ,\mu_k \geq 0 \ k = 1,\dots,K \right\} \tag{3}$$

Kuosmanen and Poidinovski (2009) showed that the above set is the smallest convex skin that holds in Shepherd's weak disposability axiom(Kuosmanen & Poidinovski, 2009).

THE PRESENTED APPROACH

In this section, a new approach is presented for evaluating the efficiency of the dynamic network with undesirable output and a constant input. In the first stage, we extend a simple two-stage network efficiency evaluating model with undesirable output based on production possibility set by Kuosmanen and Poidinovski (2009). The following section uses the presented static model, a dynamic network model with undesirable output and a constant input.

The undesirable NDEA model

Assume that we have a simple two-stage network. In this subsection, we introduce a two-stage network for assessing the operations of DMUs. Assume the following two-stage model:

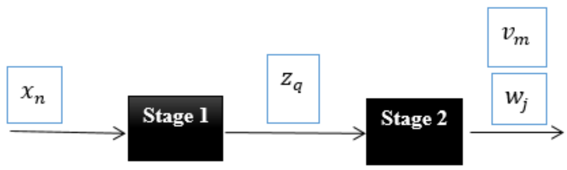

Fig. 1. The process of a simple two-stage network with undesirable output

The production technology for the above two-stage process can be expressed as follows:

$$T^N=\left\{ (x,z,v,w) \middle| \sum_{k=1}^{K}(\eta_k^1+\mu_k^1)x_{nk} \leq x_n \quad n = 1,\dots,N\ ,\ \sum_{k=1}^{K}\eta_k^1 z_{kq} \geq z_q\ ,\ \sum_{k=1}^{K}(\eta_k^2+\mu_k^2)z_{kq} \leq z_q \quad q = 1,\dots,H,\ \sum_{k=1}^{K}\eta_k^2 v_{mk} \geq v_m \quad m = 1,\dots,M\ ,\ \sum_{k=1}^{K}\eta_k^2 w_{kj} = w_j \quad j = 1,\dots,J\ ,\ \sum_{k=1}^{K}(\eta_k^1+\mu_k^1)x_{nk} \leq x_n \quad n = 1,\dots,N\ ,\ \sum_{k=1}^{K}(\eta_k^1+\mu_k^1) = 1\ ,\ \sum_{k=1}^{K}(\eta_k^2+\mu_k^2) = 1\ ,\ \eta_k^1,\eta_k^2 \geq 0\ ,\mu_k^1,\ \mu_k^2 \geq 0 \quad k = 1,\dots,K \right\} \tag{4}$$

This technology is the extension of the mentioned technology in (3) for the network process displayed in Fig. 1(Kuosmanen, 2005). The DMUs efficiencies can be evaluated with regard to this technology. The two-stage model for evaluating efficiency is as follows:

$$
\begin{array}{ll}
\min \theta & \\
\sum_{k=1}^{K}(\eta_k^1+\mu_k^1)x_{kn} \le \theta x_{on} & n=1,\dots,N \\
\sum_{k=1}^{K}\eta_k^1 z_{kq} \ge \sum_{k=1}^{K}(\eta_k^2+\mu_k^2)z_{kq} & q=1,\dots,H \\
\sum_{k=1}^{K}\eta_k^2 v_{km} \ge v_{om} & m=1,\dots,M \\
\sum_{k=1}^{K}\eta_k^2 w_{kj} = w_{oj} & j=1,\dots,J \\
\sum_{k=1}^{K}(\eta_k^1+\mu_k^1)=1 & \\
\sum_{k=1}^{K}(\eta_k^2+\mu_k^2)=1 & \\
\eta_k^1,\eta_k^2 \ge 0\ ,\ \mu_k^1,\mu_k^2 \ge 0 & k=1,\dots,K
\end{array}
\tag{5}
$$

In the above model, the structural variables of η_k^1,μ_k^1 for the first stage, and the structural variables of η_k^2,μ_k^2 for the second stage are used. The above model is input-oriented, and the condition that the output of the first stage must not be lower than the input of the second stage has been applied in the second group of constraints. The amount of optimal function of this problem ranges between 0 and 1. If the optimal function value is 1 for DMU_0, it is efficient; otherwise, it is inefficient.

The dynamic undesirable NDEA

In this subsection, a two-stage dynamic programming model is presented to evaluate the performance of a set of DMUs with a two-stage structure. In this system, the input indexes consist of variable and constant inputs, as well as some desirable and undesirable output indexes that were used as the input indexes for the next stage. For this purpose, assume n DMUs in T time period with input and output indexes (Fig. 2).

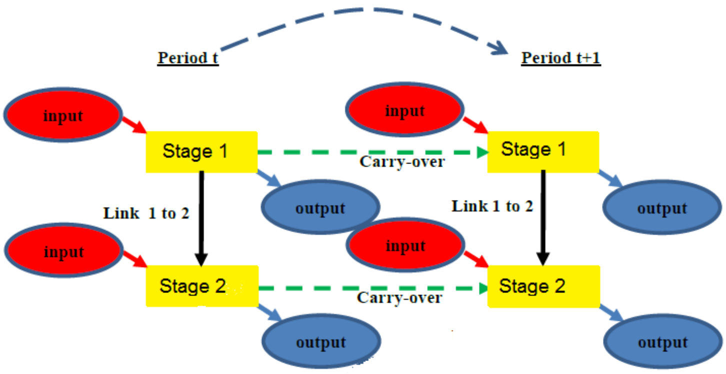

Fig. 2. A dynamic two-stage network model with undesirable output

Fig. 2 is a two-stage network. The constraints of the problem are explained in the following. It is noteworthy that these constraints are based on the perspective and production technology presented by Kuosmanen (2005).

In all stages and periods, there are two main inputs whose constraints can be expressed as follows:

$$
\begin{array}{l}
\sum_{k=1}^{K}(\eta_k^{dt}+\mu_k^{dt})x_{nkd}^{t} \le x_{nod}^{t} \quad n=1,\dots,N_1,\ \ d=1,2,\ \ t=1,\dots,T \\
\sum_{k=1}^{K}(\eta_k^{dt}+\mu_k^{dt})x_{nkd}^{fix\,t} = x_{nod}^{fix\,t} \quad n=1,\dots,N_2,\ \ d=1,2,\ \ t=1,\dots,T
\end{array}
\tag{6}
$$

In which, x_{nkd}^{t} is the n[th] variable input of DMU_k for the d[th] stage in the t[th] period. Also, $x_{nkd}^{fix\,t}$ is the constant input of DMU_k for the d[th] stage in the t[th] period. The output constraints (desirable and undesirable) of stages and periods are as follows:

$$
\begin{array}{l}
\sum_{k=1}^{K}\eta_k^{dt}v_{km}^{dt} \ge v_{om}^{dt} \quad m=1,\dots,M\ ,\quad d=1,2,\ \ t=1,\dots,T \\
\sum_{k=1}^{K}\eta_k^{dt}w_{kj}^{dt} = w_{oj}^{dt} \quad j=1,\dots,J\ ,\quad d=1,2,\ \ t==1,\dots,T
\end{array}
\tag{7}
$$

v_{km}^{dt} is the m[th] desirable output of DMU_k for the d[th] stage in the t[th] period. w_{kj}^{dt} is the J[th] desirable output of DMU_k for d^{th} stage in t^{th} period. In the following, the links constraints are presented. In fact, there are intermediate indexes in a two-stage network that contain desirable and undesirable indexes:

$$\sum_{k=1}^{K} \eta_k^{1t} z_{kq}^{g1t} \geq z_{oq}^{g1t} \quad q = 1, \dots, H_1, \ \ t = 1, \dots. T$$
$$\sum_{k=1}^{K} \eta_k^{1t} z_{kq}^{b1t} = z_{oq}^{b1t} \quad q = 1, \dots, H_2, \ \ t = 1, \dots. T$$
$$\sum_{k=1}^{K} (\eta_k^{2t} + \mu_k^{2t}) z_{kq}^{gdt} \leq z_{oq}^{g2t} \quad q = 1, \dots, H_1, \ t = 1, \dots, T$$
$$\sum_{k=1}^{K} (\eta_k^{2t} + \mu_k^{2t}) z_{kq}^{bdt} \leq z_{oq}^{b2t} \quad q = 1, \dots, H_2, \ \ t = 1, \dots. T$$

(8)

In the following, the carry-over constraints are discussed. It is to be noted that these indexes can also be desirable and undesirable:

$$\sum_{k=1}^{K} \eta_k^{dt} z_{kq}^{gd(t,t+1)} \geq z_{oq}^{gd(t,t+1)} \quad q = 1, \dots, H_3, \ \ t = 1, \dots. T-1$$
$$\sum_{k=1}^{K} \eta_k^{dt} z_{kq}^{bdt} = z_{oq}^{bd(t,t+1)} \quad q = 1, \dots, H_4, \ \ t = 1, \dots, T-1$$
$$\sum_{k=1}^{K} \left(\eta_k^{d(t+1)} + \mu_k^{d(t+1)}\right) z_{kq}^{bd(t,t+1)} \leq z_{oq}^{b2(t,t+1)} \quad q = 1, \dots, H_3, \ \ t = 1, \dots, T-1$$
$$\sum_{k=1}^{K} \left(\eta_k^{d(t+1)} + \mu_k^{d(t+1)}\right) z_{kq}^{bd(t,t+1)} \leq z_{oq}^{b2(t,t+1)} \quad q = 1, \dots, H_4, \ \ t = 1, \dots. T-1$$

(9)

With regard to the mentioned constraints, various evaluation models can be explained here. As a radial input-oriented model, the following model can be presented:

$$\theta^* = \min \frac{1}{N_1} \sum_{n=1}^{N_1} \theta_n$$

$S.t$

$$\sum_{k=1}^{K} (\eta_k^{dt} + \mu_k^{dt}) x_{nkd}^{t} \leq \theta x_{nod}^{t} \quad n = 1, \dots, N_1, \ d = 1,2, \ t = 1, \dots, T$$
$$\sum_{k=1}^{K} (\eta_k^{dt} + \mu_k^{dt}) x_{nkd}^{fix\,t} = x_{nod}^{fix\,t} \quad n = 1, \dots, N_2, \ d = 1,2, \ t = 1, \dots, T$$
$$\sum_{k=1}^{K} \eta_k^{dt} v_{km}^{dt} \geq v_{om}^{dt} \quad m = 1, \dots, M, \ d = 1,2, \ t = 1, \dots, T$$
$$\sum_{k=1}^{K} \eta_k^{dt} w_{kj}^{dt} = w_{oj}^{dt} \quad j = 1, \dots, J, \ d = 1,2, \ t = 1, \dots, T$$
$$\sum_{k=1}^{K} \eta_k^{1t} z_{kq}^{g1t} \geq z_{oq}^{g1t} \quad q = 1, \dots, H_1, \ t = 1, \dots, T$$
$$\sum_{k=1}^{K} \eta_k^{1t} z_{kq}^{b1t} = z_{oq}^{b1t} \quad q = 1, \dots, H_2, \ t = 1, \dots, T$$
$$\sum_{k=1}^{K} (\eta_k^{2t} + \mu_k^{2t}) z_{kq}^{bdt} \leq z_{oq}^{b2t} \quad q = 1, \dots, H_1, \ t = 1, \dots, T$$
$$\sum_{k=1}^{K} (\eta_k^{2t} + \mu_k^{2t}) z_{kq}^{bdt} \leq z_{oq}^{b2t} \quad q = 1, \dots, H_2, \ t = 1, \dots, T$$
$$\sum_{k=1}^{K} \eta_k^{dt} z_{kq}^{gd(t,t+1)} \geq z_{oq}^{gd(t,t+1)} \quad q = 1, \dots, H_3, \ t = 1, \dots, T-1$$
$$\sum_{k=1}^{K} \eta_k^{dt} z_{kq}^{bdt} = z_{oq}^{bd(t,t+1)} \quad q = 1, \dots, H_4, \ t = 1, \dots, T-1$$
$$\sum_{k=1}^{K} \left(\eta_k^{d(t+1)} + \mu_k^{d(t+1)}\right) z_{kq}^{bd(t,t+1)} \leq z_{oq}^{b2(t,t+1)} \quad q = 1, \dots, H_3, \ t = 1, \dots, T-1$$
$$\sum_{k=1}^{K} \left(\eta_k^{d(t+1)} + \mu_k^{d(t+1)}\right) z_{kq}^{bd(t,t+1)} \leq z_{oq}^{b2(t,t+1)} \quad q = 1, \dots, H_4, \ t = 1, \dots, T-1$$
$$\sum_{k=1}^{K} (\eta_k^{dt} + \mu_k^{dt}) = 1 \quad d = 1,2, \ t = 1, \dots, T$$
$$\eta_d^{kt}, \mu_d^{kt} \geq 0 \quad d = 1,2,3, \ t = 1, \dots, T \quad k = 1, \dots, K$$

(10)

This input-oriented and always feasible model ranges between 0 and 1 with the optiml value of the objective function. If the optimal value of this model gets 1, DMU_0 is efficient; otherwise, it is inefficient. In the objective function, we are looking for the maximum downsizing of the input variables of the DMU understudy. If the optimal value of the objective function model becomes 1, then (n=, 1,…, N1) $\theta_n = 1$, indicating that none of the input variables of the understudy *DMU* be reduced. In other words, the understudy DMU is efficient. In this section, a dynamic model is presented to evaluate and rank the DMUs. In the next section, we explain how to implement this model to rank the DMus in an applied example.

AN APPLIED EXAMPLE

In this section, we use the presented model in the previous section and evaluate the efficiency in an applied example. This applied example includes the three years' successive data (2014 to 2017) of 13 hospitals in Semnan Province, Iran. The indexes of efficiency evaluations are as follows:

Input indexes:

1- Number of beds (constant input)
2- Number of general physicians
3- Number of specialist physicians
4- Number of nurses

Intermediate indexes

1- Income
2- Debt

Output indexes

1- Number of outpatients
2- Number of deaths (undesirable output)
3- The average number of hospitalizations per day

The number of hospital beds does not change during the study period and is always constant. So it is considered a constant input for the hospitals. Also, in the outputs, the number of deaths is a negative outcome for the hospitals and is regarded as an undesirable output.

The network process of hospitals is displayed in Fig. 3.

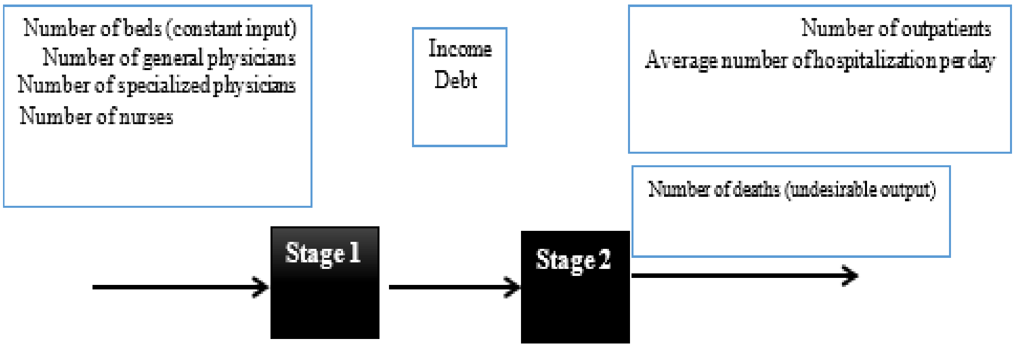

Fig. 3. Network process of the hospitals

Table 1: Data of inputs for efficiency evaluation indexes of hospitals

Hospital	Number of beds			Number of physicians			Number of specialists			Number of nurses		
	x_1^{fix}	x_2^{fix}	x_3^{fix}	x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}	x_{31}	x_{32}	x_{33}
1	42	40	41	8	7	11	9	8	10	12	16	13
2	85	85	99	10	10	10	61	51	51	130	135	140
3	18	18	18	11	14	13	10	8	7	11	14	13
4	72	79	84	8	7	11	51	62	66	109	122	131
5	180	202	221	8	8	11	50	63	72	201	208	210
6	203	231	250	7	9	13	70	80	83	233	234	242
7	130	130	130	8	8	9	44	44	51	107	110	118
8	82	76	76	3	5	8	22	23	27	119	113	115
9	70	78	78	10	8	14	53	67	74	105	116	129
10	75	77	94	9	8	12	52	68	69	107	118	128
11	78	80	90	7	7	13	54	64	73	110	140	142
12	74	76	79	11	11	10	49	55	66	113	108	113
13	75	76	78	15	15	14	53	58	59	120	121	124

Table 1: Data of intermediates for efficiency evaluation indexes of hospitals (continued A)

Hospital	Income			Debt		
	z_1^g	z_2^g	z_3^g	z_1^b	z_2^b	z_3^b
1	772.30	778.44	71.47	811.20	811.20	528.30
2	754.107	113.165	003.200	741.64	741.64	48.142
3	8267	13420	9011	543	543	1551
4	13.69	51.103	14.107	448.21	448.21	596.83
5	51.272	555.384	378.416	102.122	102.122	095.230
6	008.339	554.552	718.642	652.136	652.136	812.437
7	929.143	82.256	083.287	114.22	114.22	140
8	045.61	285.100	429.130	145.11	145.11	774.46
9	2.73	8.90	3.109	17.24	17.24	601.85

10	4.70	3.108	8.111	2.24	2.24	17.85
11	83	447.110	39.109	83.25	83.25	42.82
12	125.71	208.108	381.109	518.24	518.24	911.85
13	108.89	123.105	729.107	23.25	23.25	331.81

Table 1: Data of outputs for efficiency evaluation indexes of hospitals (continued B)

Hospital	Number of outpatients			Number of deaths			Average hospitalization number per day		
	v_{11}	v_{12}	v_{13}	w_1	w_2	w_3	v_{21}	v_{22}	v_{23}
1	57160	57868	61206	46	45	51	23	26	27
2	123549	144983	126927	113	162	125	19	20	23
3	34220	22544	24290	1	2	1	2	2	1
4	101312	132411	133011	101	117	121	13	17	18
5	40712	47312	53412	301	362	511	34	42	49
6	49718	57599	63612	368	417	610	45	58	65
7	70400	82690	93791	199	201	201	97	113	110
8	49712	51374	57602	42	37	42	718	727	
9	10480	12016	13714	108	118	120	14	15	719
10	10382	13507	13620	104	120	123	15	18	19
11	10421	13612	13571	103	119	125	14	17	19
12	10021	1138	12717	107	118	113	14	16	18
13	10721	11332	13173	109	125	138	16	16	17

In the following, we evaluate the efficiency of 13 hospitals using the presented model. This model is a dynamic NDEA (10). The linear programming (10) will be solved by the Lingo application (11). Table 2 presents the efficiency evaluation results.

Table 2: Results of efficiency evaluation of the hospitals using Model 6

Hospital	$\boldsymbol{\theta}^*$	Hospital	$\boldsymbol{\theta}^*$
1	1	8	1
2	0.718	9	0.738
3	1	10	0.772
4	0.809	11	0.728
5	0.720	12	0.734
6	0.741	13	0.616
7	0.860	-	-

As seen, the efficiency dynamic network values of 13 hospitals are presented in columns two and four. Of these 13 hospitals, three hospitals were efficient and others inefficient. Also, the efficiency values of 13 hospitals are shown in Diagram 1.

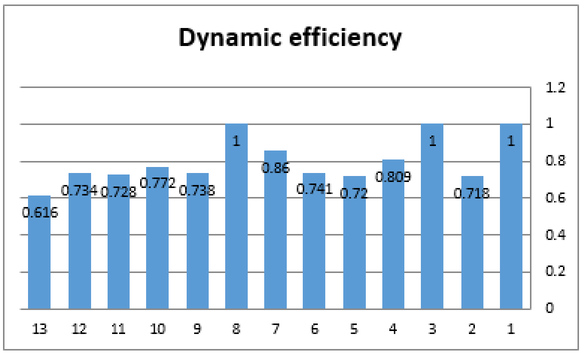

Fig. 3. Dynamic efficiency of hospitals

Fig. 3 displays the efficiency scores of the hospitals. Hospitals 1, 3, and 8 have the highest height, and hospital 13 has the lowest compared to other hospitals, indicating its low performance.

To compare the results, consider hospitals 1 and 13. The inputs of Hospital 1 over 3 years were better (lower) than Hospital 13. Also, the outputs (desirable and undesirable) of Hospital 1 over these 3 years were better than Hospital 13. Because the desirable and undesirable outputs of Hospital 1 during 3 years were respectively more and less than those of Hospital 13, Hospital 1 should have better dynamic performance than Hospital 13. According to efficiency results seen in Table 2 and Figure 3, Hospital 1 is efficient and has the highest performance, and Hospital 13 is inefficient and has the lowest performance, indicating the correct performance of the assessment model. Similar analyses could be carried out to compare other hospitals.

CONCLUSION

In many problems, the DMUs conditions are such a way that some data and information are transferred from one stage to another. Thus, it is impossible to consider the production technology a black box in which an input process turns into an output process. In this investigation, the studied DMUs had some subsections with desirable indexes and some undesirable ones. Then, some of the desirable and undesirable indexes were used as inputs for consumption in the next stages. Also, some input indexes were constant and could not be increased or decreased. Considering the structure of DMU, a dynamic model was presented for the evaluation of their performances, and then the efficiency of each DMU was obtained. Unlike the other two-stage models, the special feature of the proposed model enables it to be used for any number of time periods.

In the end, the presented model was used to assess the network dynamic efficiency of hospitals. The results of implanting the model show that it can assess the hospitals' performances.

The presented model in this article was used for deterministic data. However, the model can be used and extended for stochastic conditions, like probabilistic and fuzzy data. It is also possible to use a directional distance function[2] instead of the used model for the assessment. Also, we can change the network structure model, for example, use integer data besides constant input indexes and undesirable outputs. Each one of these suggestions can be a topic for future investigation.

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