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Upgrading Inefficient Decision Making Units (with Negative Data) Towards Common Weights (Using DEA)

Hossein Abbasiyan*

Department of Mathematics, Aliabad Katoul Branch, Islamic Azad University, Aliabad Katoul, Iran

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Abstract

The main purpose of this paper is to upgrade and improve inefficient units by common weights obtained from all units studied. In fact, we consider the common weight vector as the direction in which inefficient units rise. The methodology of this research is to consider the semi-essential radial model and we want to use the duality of this model to find the common weights of inputs and outputs, some of which are negative. For this purpose, we present a multi-objective problem of generating common weights and use ideal programming to solve it, which leads to the production of a nonlinear problem, which for this particular problem, by a linearization method, is called We turn a linear programming problem. Since the necessary and sufficient condition for the boundary of the semi-essential radial model in the nature of input (output) is that there is an input (output) with at least one positive value, so we observe this condition here. Finally, we will explain our method with an example and the remarkable thing about the promotion method in the present study is that negative data is promoted and improved as negative data.

Keywords:
Common weights
Data Envelopment Analysis
Inefficient Units
Negative data
SORM

***Correspondence E-mail**: abbasiyan58@gmail.com

INTRODUCTION

In data envelopment analysis methods, unlike some numerical methods, it is not necessary to know the weights in advance and assign them to inputs and outputs. Also, these methods do not require predefined functional forms (such as statistical regression methods) or explicit forms of the production function (such as some parametric methods). Data envelopment analysis allows the study of units with multiple inputs and multiple outputs. The basis of data envelopment analysis is based on linear algebra and its ability is mostly due to the use of linear programming. Linear programming enables data envelopment analysis to use linear programming problem-solving methods and duality theorems, thus determining the source and amount of inefficiency for each input and output. It also creates many opportunities for collaboration analyst and the decision maker. These collaborations can be in order to select the input and output of the units under evaluation and how to operate and model the efficient border. To deal with negative data, one can either make abnormal changes to the variables or simplify the units to positive data by deleting negative values in some variables, which was done in the year 1989 by Charens et al. Some examples of what has been done to deal with negative data include: Pasteur evaluated 24 bank branches with two outputs that were free. Zhou (1994) as well as Seiford and Zhou (2002) used the variable of profit and taxes as an output for 35 companies, which is negative when there is a lack of finance. In the year 2002, Seiford and Zhou proposed a method in which to classify units efficiently or inefficiently despite negative data, assuming units with negative output data to be efficient, and units with positive output data to be considered inefficient. they got. In 2004, Portella et al.Introduced a new model based on directional distance functions that has relatively good properties in both radial and collective models, and their proposed model is known as the RDM model, which is a performance score by the distance function. Orientation is obtained by comparing the unit under evaluation with the ideal point. Sharp et al.'s method in 2006 is to introduce a modified SBM called MSBM that can control both negative and negative inputs. In this paper, we use a semi-essential radial model to deal with negative data. We use the dual semi-essential radial model (SORM) to construct a multi-objective model whose answer gives common weights, and we use this set of weights as a direction vector to improve inefficient units. In Section 2, the semi-essential radial model and the linear model for finding common weights are presented. A new model for finding common weights when there is negative data is presented in Section 3, and also in Section 4 the common weights vector is presented as a direction vector to improve inefficient units. In Section 5, we provide a numerical example to illustrate our method, and finally in Section 6, the discussion and conclusion are presented.

RESEARCH REQUIREMENTS

Suppose we have n units, each with m input and s output. Assume also $I = \{i | x_{ij} \geq 0, \forall j \in \{1, \dots, m\}\}$ and $L = \{l | \exists j \in \{1, \dots, n\}, x_{ij} < 0\}$ so that $I \cup L = \{1, \dots, m\}$ and $R = \{r | y_{rj} \geq 0, j = 1, \dots, n\}$ and $K = \{k | \exists j, y_{kj} < 0\}$ and $R \cup K = \{1, \dots, s\}$. Emrouznezhad and et al introduced new variables x_{lj}^1 and x_{lj}^2 such that $x_{lj} = x_{lj}^1 - x_{lj}^2$ and $x_{lj}^1 = \begin{cases} x_{lj} & x_{lj} \geq 0 \\ 0 & x_{lj} < 0 \end{cases}$ and $x_{lj}^2 = \begin{cases} -x_{lj} & x_{lj} < 0 \\ 0 & x_{lj} \geq 0 \end{cases}$. Similarly, $y_{kj} = y_{kj}^1 - y_{kj}^2, \forall k \in K$, where $y_{kj}^1 = \begin{cases} y_{kj} & y_{kj} \geq 0 \\ 0 & y_{kj} < 0 \end{cases}$ and $y_{kj}^2 = \begin{cases} -y_{kj} & y_{kj} \leq 0 \\ 0 & y_{kj} > 0 \end{cases}$. The dual SORM model in the orient of input is as follows:

$$
\begin{aligned}
\max \quad & u_0 + \sum_{r\in R} y_{ro} u_r + \sum_{k\in K} y_{ko}^1 u_k^1 - \sum_{k\in K} y_{ko}^2 u_k^2 \\
s.t. \quad & u_0 + \sum_{r\in R} y_{rj} u_r + \sum_{k\in K} y_{kj}^1 u_k^1 - \sum_{k\in K} y_{kj}^2 u_k^2 - \sum_{i\in I} x_{ij} v_i - \sum_{l\in L} x_{lj}^1 v_l^1 + \sum_{l\in L} x_{lj}^2 v_l^2 \leq 0, \forall j \\
& \sum_{i\in I} x_{io} v_i + \sum_{l\in L} x_{lo}^1 v_l^1 - \sum_{l\in L} x_{lo}^2 v_l^2 = 1 \\
& v_i \geq 0, \forall i \in I; v_l^1 \geq 0, v_l^2 \geq 0, \forall l \in L \\
& u_r \geq 0, \forall r \in R; u_k^1, u_k^2 \geq 0, \forall k \in K; u_0\ free
\end{aligned}
\tag{1}
$$

And the dual SORM model in the orient of output is as follows:

Min $v_0+\sum_{i\in I}x_{io}v_i+\sum_{l\in L}x_{lo}^1v_l^1-\sum_{l\in L}x_{lo}^2v_l^2$

St:

$$v_0+\sum_i x_{io}v_i+\sum_l x_{lo}^1v_l^1-\sum_l x_{lo}^2v_l^2-\sum_r y_{rj}u_r-\sum_k y_{kj}^1u_k^1+\sum_{k\in} y_{kj}^2u_k^2\geq 0,\forall j$$

$$\sum_{r\in R}y_{rj}u_r+\sum_{k\in K}y_{kj}^1u_k^1-\sum_{k\in K}y_{kj}^2u_k^2=1$$

$$v_i\geq 0,\forall i\in I; v_l^1\geq 0, v_l^2\geq 0,\forall l\in L$$

$$u_r\geq 0,\forall r\in R; u_k^1,u_k^2\geq 0,\forall k\in K; v_0\, free$$

(2)

In fact, in SORM models, negative output values are considered as inputs because in order to find better answers, the absolute value of the negative output value must be reduced. On the other hand, negative input values are considered as output, which in turn increases the absolute value of the negative input value.

The following is a linearization method for a fractional programming without negative data. The following ideal planning model was presented by Hosseinzadeh Lotfi et al.

$$z=\min\sum_{j=1}^n\eta_j$$

$$s.t.\quad \frac{u_0+\sum_{r=1}^s u_ry_{rj}}{\sum_{i=1}^m v_ix_{ij}}+\eta_j=1,\ j=1,\cdots,n(*)$$

$$\frac{u_0+\sum_{r=1}^s u_ry_{rj}}{\sum_{i=1}^m v_ix_{ij}}\leq 1,\ j=1,\cdots,n; u_r>0, r=1,\cdots,s; v_i>0, i=1,\cdots,m; \eta_j\geq 0,\forall j$$

.

(3)

The model (3) is a nonlinear model. Davoodi and Rezaei presented a method for linearizing this model, which we briefly describe below. Because for each i and j, $x_{ij}>0$ and $v_i>0$, we can set the constraints (*) for each j in $\sum_i v_ix_{ij}$ and the objective function in $\frac{\sum_i v_i}{\sum_i v_i}$ Multiply. After making these changes, the model looks like this:

$$z=\min\quad \sum_{j=1}^n\frac{\eta_j(\sum_{i=1}^m v_i)}{\sum_{i=1}^m v_i}$$

$$s.t:\quad u_0+\sum_{r=1}^s u_ry_{rj}+\eta_j\left(\sum_{i=1}^m v_ix_{ij}\right)=\sum_{i=1}^m v_ix_{ij},\forall j$$

$$u_r,v_i>0,\forall r,i;\ \eta_j\geq 0,\forall j.$$

(4)

We now define: $\frac{\sum_{i=1}^m v_i}{t}=1$ and since $t>0$ we have $\sum_{i=1}^m v_i=t$. By placing this new variable in the objective function and dividing the constraints by t and defining the new variables as and $\beta_0=\frac{u_0}{t}$ and $\pi_i^j=\eta_j\frac{v_i}{t}\geq 0,\beta_r=\frac{u_r}{t},\alpha_i=\frac{v_i}{t}$, the following linear model is obtained:

$$z=\min\sum_{j=1}^n\sum_{i=1}^m\pi_i^j$$

s.t:

$$\beta_0+\sum_{r=1}^s\beta_ry_{rj}+\sum_{i=1}^m\pi_i^jx_{ij}-\sum_{i=1}^m\alpha_ix_{ij}=0,\ j=1,\cdots,n$$

$$\sum_{i=1}^m\alpha_i=1;\beta_r\geq\varepsilon,\alpha_i\geq\varepsilon,\pi_i^j\geq 0,\forall i,\forall j,\forall r$$

(5)

INTRODUCE A NEW LINER MODEL FOR FINDING COMMON WEIGHTS WHEN THERE IS NEGATIVE DATA

Each variable that has positive values in some units and negative values in others is written as the difference of two non-negative variables. We have the following fractional input model for new variables:

$$\max\frac{u_0+\sum_{r\in R}u_ry_{ro}+\sum_{k\in K}u_k^1y_{ko}^1+\sum_{l\in L}v_l^2x_{lo}^2}{\sum_{i\in I}v_ix_{io}+\sum_{l\in L}v_l^1x_{lo}^1+\sum_{k\in K}u_k^2y_{ko}^2}$$

$$\frac{u_0+\sum_{r\in R}u_ry_{rj}+\sum_{k\in K}u_k^1y_{kj}^1+\sum_{l\in L}v_l^2x_{lj}^2}{\sum_{i\in I}v_ix_{ij}+\sum_{l\in L}v_l^1x_{lj}^1+\sum_{k\in K}u_k^2y_{kj}^2}\leq 1,\forall j; v_i,v_l^1,v_l^2,u_r,u_r^1,u_r^2>0,\forall i,l,r,k$$

(6)

Similarly, we write the fractional model in the orient of the output as follows:

$$\min\frac{v_0+\sum_{i\in I}v_ix_{io}+\sum_{l\in L}v_l^1x_{lo}^1+\sum_{k\in K}u_k^2y_{ko}^2}{\sum_{r\in R}u_ry_{ro}+\sum_{k\in K}u_k^1y_{ko}^1+\sum_{l\in L}v_l^2x_{lo}^2}$$

$$\frac{v_0+\sum_{i\in I}v_ix_{ij}+\sum_{l\in L}v_l^1x_{lj}^1+\sum_{k\in K}u_k^2y_{kj}^2}{\sum_{r\in R}u_ry_{rj}+\sum_{k\in K}u_k^1y_{kj}^1+\sum_{l\in L}v_l^2x_{lj}^2}\geq 1,$$

$$\forall j\, ; v_i,v_l^1,v_l^2,u_r,u_r^1,u_r^2>0,\forall i,l,r,k$$

(7)

The corresponding multi-objective problem of model (7) to find the set of common weights can be presented as follows:

$$\text{Max:}\begin{Bmatrix}\frac{u_0+\sum_{r\in R}u_ry_{r1}+\sum_{k\in K}u_k^1y_{k1}^1+\sum_{l\in L}v_l^2x_{l1}^2}{\sum_{i\in I}v_ix_{i1}+\sum_{l\in L}v_l^1x_{l1}^1+\sum_{k\in K}u_k^2y_{k1}^2}\\ \vdots\\ \frac{u_0+\sum_{r\in R}u_ry_{rn}+\sum_{k\in K}u_k^1y_{kn}^1+\sum_{l\in L}v_l^2x_{ln}^2}{\sum_{i\in I}v_ix_{in}+\sum_{l\in L}v_l^1x_{l1}^1+\sum_{k\in K}u_k^2y_{kn}^2}\end{Bmatrix}$$

S.t:

$$\frac{u_0+\sum_{r\in R}u_r y_{rj}+\sum_{k\in K}u_k^1 y_{kj}^1+\sum_{l\in L}v_l^2 x_{lj}^2}{\sum_{i\in I}v_i x_{ij}+\sum_{l\in L}v_l^1 x_{lj}^1+\sum_{k\in K}u_k^2 y_{kj}^2}\le 1,$$

$$\forall j; v_i, v_l^1, v_l^2, u_r, u_r^1, u_r^2>0, \forall i,l,r,k$$

(8)

Now we come to the part where we talk about the middle ground. Since the value of 1 is considered as the ideal level for the jth objective function, the ideal planning problem will be as follows:

$$z=\min\sum_{j=1}^{n}\theta_j$$

$$\frac{u_0+\sum_{r\in R}u_r y_{rj}+\sum_{k\in K}u_k^1 y_{kj}^1+\sum_{l\in L}v_l^2 x_{lj}^2}{\sum_{i\in I}v_i x_{ij}+\sum_{l\in L}v_l^1 x_{lj}^1+\sum_{k\in K}u_k^2 y_{kj}^2}+\theta_j=1, \forall j$$

$$v_i, v_l^1, v_l^2, u_r, u_r^1, u_r^2>0, \forall i,l,r,k; \theta_j\ge 0, \forall j$$

(9)

The first we put: $t=\sum_{i\in I}v_i x_{ij}+\sum_{l\in L}v_l^1 x_{lj}^1+\sum_{k\in K}u_k^2 y_{kj}^2$ and define $\pi_i^j=\theta_j\frac{v_i}{t}$ ، $\pi_l^{1j}=\theta_j\frac{v_l^1}{t}$ ، $\pi_k^{2j}=\theta_j\frac{u_k^2}{t}$ ، $\beta_r=\frac{u_r}{t}$ ، $\beta_k^1=\frac{u_k^1}{t}$ ، $\beta_k^2=\frac{u_k^2}{t}$ ، $\alpha_i=\frac{v_i}{t}$ ، $\alpha_l^1=\frac{v_l^1}{t}$ ، $\alpha_l^2=\frac{v_l^2}{t}$ ، $\beta_0=\frac{u_0}{t}$. In this case, the linear model will be as follows:

$$z=\min\sum_{j=1}^{n}\left(\sum_{i\in I}\pi_i^j+\sum_{l\in L}\pi_l^{lj}+\sum_{k\in K}\pi_k^{2j}\right)$$

$$s.t.$$

$$\beta_0+\sum_{r\in R}\beta_r y_{rj}+\sum_{k\in K}\beta_k^1 y_{kj}^1+\sum_{l\in L}\alpha_l^2 x_{lj}^2+\sum_{i\in I}\pi_i^j x_{ij}+\sum_{l\in L}\pi_l^{1j}x_{lj}^1$$
$$+\sum_{k\in K}\pi_k^{2j}y_{kj}^2=\sum_{i\in I}\alpha_i x_{ij}+\sum_{l\in L}\alpha_l^1 x_{lj}^1+\sum_{k\in K}\beta_k^2 y_{kj}^2$$
$$\sum_{i\in I}\alpha_i+\sum_{l\in L}\alpha_l^1+\sum_{k\in K}\beta_k^2=1, \pi_i^j, \pi_l^{1j}, \pi_k^{2j}\ge 0,\ i\in I, l\in L, k\in K, \forall j.$$

(10)

The above model is a linear programming problem in the nature of input to find the common weights of inputs and outputs with negative data. Similarly, for the nature of the output, we have the following multi-objective problem:

$$\min\left\{\frac{v_0+\sum_{i\in I}v_i x_{i1}+\sum_{l\in L}v_l^1 x_{l1}^1+\sum_{k\in K}u_k^2 y_{k1}^2}{\sum_{r\in R}u_r y_{r1}+\sum_{k\in K}u_k^1 y_{k1}^1+\sum_{l\in L}v_l^2 x_{l1}^2},\cdots,\frac{v_0+\sum_{i\in I}v_i x_{in}+\sum_{l\in L}v_l^1 x_{ln}^1+\sum_{k\in K}u_k^2 y_{kn}^2}{\sum_{r\in R}u_r y_{rn}+\sum_{k\in K}u_k^1 y_{kn}^1+\sum_{l\in L}v_l^2 x_{ln}^2}\right\}$$

$$s.t.\ \frac{v_0+\sum_{i\in I}v_i x_{ij}+\sum_{l\in L}v_l^1 x_{lj}^1+\sum_{k\in K}u_k^2 y_{kj}^2}{\sum_{r\in R}u_r y_{rj}+\sum_{k\in K}u_k^1 y_{kj}^1+\sum_{l\in L}v_l^2 x_{lj}^2}\ge 1, \forall j; v_i, v_l^1, v_l^2, u_r, u_r^1, u_r^2>0, \forall i,l,r,k$$

(11)

Therefore, the linear model will be as follows:

$$z=\min\sum_{j=1}^{n}\left(\sum_{r\in R}f_r^j+\sum_{k\in K}f_k^{1j}+\sum_{l\in L}f_l^{2j}\right)$$
$$\alpha_0+\sum_{i\in I}\alpha_i x_{ij}+\sum_{l\in L}\alpha_l^1 x_{lj}^1+\sum_{k\in K}\beta_k^2 y_{kj}^2$$
$$\forall j\ -\sum_{r\in R}f_r^j y_{rj}-\sum_{k\in K}f_k^{1j}y_{kj}^1$$
$$-\sum_{l\in L}f_l^{2j}x_{lj}^2=\sum_{r\in R}\beta_r y_{rj}$$
$$+\sum_{k\in K}\beta_k^1 y_{kj}^1+\sum_{l\in L}\alpha_l^2 x_{lj}^2$$
$$\sum_{r\in R}\beta_r+\sum_{k\in K}\beta_k^1+\sum_{l\in L}\alpha_l^2=1; f_r^j, f_k^{1j},$$
$$f_l^{2j}\ge 0, r\in R, k\in K, l\in L, \forall j$$

(12)

Where $f_r^j=\rho_j\frac{u_r}{t}$ ، $f_k^{1j}=\rho_j\frac{u_k^1}{t}$ ، $f_l^{2j}=\rho_j\frac{v_l^2}{t}$ ، $\alpha_0=\frac{v_0}{t}$ ، $\alpha_i=\frac{v_i}{t}$ ، $\alpha_l^1=\frac{v_l^1}{t}$ ، $\alpha_l^2=\frac{v_l^2}{t}$ و $\beta_r=\frac{u_r}{t}$ ، $\beta_k^1=\frac{u_k^1}{t}$ ، $\beta_k^2=\frac{u_k^2}{t}$.

UPGRADING INEFFICIENT UNITS TOWAEDS COMMON WEIGHTS

In this section, we intend to improve inefficient units and for this, we consider the common weights vector of inputs and outputs as direction vector. Suppose we have n units with input levels x_{ij}, i = 1,…, m and output levels y_{rj}, r = 1,…, s corresponding to unit j, and we want to evaluate unit o. Consider the following model for a variable scale return mode:

$$\begin{aligned}&\max\quad \beta_0\\ &s.t:\quad \sum_{j=1}^{n}\lambda_j y_{rj}\ge y_{ro}+\beta_o g_{y_r}\qquad r=1,\dots,s\\ &\qquad\quad \sum_{j=1}^{n}\lambda_j x_{ij}\le x_{io}-\beta_o g_{x_i}\qquad i=1,\dots,m\\ &\qquad\quad \sum_{j=1}^{n}\lambda_j=1;\ \lambda_j,\beta_o,g_{x_i},g_{y_r}\ge 0\end{aligned}$$

(13)

In the above model, no nature is considered and it deals with both input contraction and output expansion at the same time. But if $g_{y_r}=0$, then we have a model in the nature of the input, and if we put $g_{x_i}=0$, then we get a model in the nature of the output. The directional vectors (g_{x_i}, g_{y_r}) are usually selected by the observed input and output surfaces. But if some of the data is negative, this choice can no longer be made because the directional vectors must be negative. We intend to use the common weights of inputs and outputs for this purpose. Input and output weights show their degree of importance, so if we use these weights as directional vectors to represent units on the performance frontier, we actually use the importance of each input and output to show how inefficient units improve. We have. Now if these weights are common to all units, they will certainly have the same conditions for improvement, although they may not be on the edge of efficiency, but they will definitely improve because the common weights obtained to increase the efficiency of all units to Is synchronous and we call it the Common Weight Oriented Model (CSWDM).

NUMERICAL EXAMPLE

Consider the table below, which contains 10 units with one input and two outputs. Y output is positive in some units and negative in others. This example has the conditions to use our method because there is at least one positive value for both outputs and we want to use the nature of the output. By our method, we find the corresponding goals of inefficient units and compare them with other methods. We write the output variable Y as $Y = Y^1 - Y^2$ where $Y^1 = \begin{cases} Y & Y \geq 0 \\ 0 & Y < 0 \end{cases}$ and $Y^2 = \begin{cases} 0 & Y \geq 0 \\ -Y & Y < 0 \end{cases}$.

Table 1: input and output values of units

units	Z	Y	X	units	Z	Y	X
F	27	-8	50	A	11	15	12
G	27	-18	35	B	6	18	35
H	22	-10	40	C	13	20	25
I	19	-7	25	D	20	12	22
J	8	26	16	E	25	-10	40

The linear programming model in the nature of output is as follows:

$$
\begin{aligned}
\min \quad & \left(f_z^1 + f_y^{11}\right) + \cdots + \left(f_z^{10} + f_y^{110}\right) \\
s.t. \quad & \alpha_0 + 12\alpha_x - 11 f_z^1 - 15 f_y^{11} = 11\beta_z + 15\beta_y^1 \\
& \vdots \\
& \alpha_0 + 16\alpha_x - 8 f_z^{10} - 26 f_y^{110} = 8\beta_z + 26\beta_y^1 \\
& \beta_z + \beta_y^1 = 1
\end{aligned}
\tag{14}
$$

By solving the above model, a set of common weights are obtained, which is shown in the table below:

Table 2: common weights

v_0^*	v_x^*	u_y^{*1}	u_y^{*2}	u_z^*
16.30	0.00	0.46	0.00	0.54

According to the above table, the common weights related to the main inputs and outputs are: $w_X = 0.00$, $w_Y = u_Y^{1*} - u_Y^{2*} = 0.46$, $w_Z = 0.54$. Because we want to use the nature of the output, we only use the output weights. The optimal values of the proposed model for all units are given in the first row of the table below. In other lines of this table, the optimal values of the mentioned models are given.

Table 3: Results of the models

β_0	J	I	H	G	F	E	D	C	B	A
CSWDM	0	0.16	0.23	0.62	0	0	0	0.01	0.32	0
SORM	1	1.36	1.38	1.65	1	1	1	1.01	1.55	1
RDM	1	1.10	1.14	3.94	1	1	1	1.01	1.41	1
MSBM	0	4.86	6.68	33.76	0	0	0	0.01	9.48	0

The following table lists the output levels of the targets obtained:

Table 4: Upgraded output levels for inefficient units

	Z					Y				
	CSW	**SORM**	**MSBM**	**RDM**	**Obs.**	**CSW**	**SORM**	**MSBM**	**RDM**	**Obs.**
B	11.11	8.45	8	12.7	6	22.38	25.4	26	20.5	18
C	13.08	13	13	13.1	13	20.07	20.1	20.17	20	20
G	24.18	23.6	20	20.14	6	-2.42	-3.8	12	11.6	-18
H	25.60	25	24.5	23.64	22	-6.92	-10	0	1.6	-10
I	21.62	20.9	20.75	20.85	19	-4.76	8.3	12.3	8.26	-7

Now we will compare our method with other methods, which we will do as follows. If the observed output and the target output are either positive or both negative, we calculate the ratio of the observed output to the target output. If the target output is positive but the observed output is negative, we do not calculate any ratio. We calculate the average of the resulting ratios for the two outputs and present the results as a percentage in the table below.

Table 5: Average percentage efficiency of inefficient units

Inefficient Units	CSWDM	SORM	MSBM	RDM
B	67.22	70.81	72.12	67.52
C	99.52	99.75	99.58	99.62
G	19.13	23.27	30.00	29.79
H	77.57	94.00	89.80	93.06
I	77.94	70.81	72.12	67.52

DISCUSSION AND CONCLUSION

The main purpose of this paper is to describe a method for improving inefficient decision units in data envelopment analysis with negative data. By introducing a model, we can obtain common weights. In this paper, due to the negativity of some data, we used a multi-objective model, which is the result of a semi-essential radial model. The methodology of this research is that we have created a multi-objective model using the multiplicative SORM model and turned it into a linear programming model through a linearization method. Of course, this linear model is used for inputs and outputs with negative data, which will definitely change the negative data. Using common weights can increase the efficiency of all units equally because in finding weights, increasing the efficiency of all units is considered. We have used the weights obtained from this linear model to build a directional model (CSWDM), which we have used to upgrade inefficient units. The use of common weights in this oriented model has the advantage that all inefficient units recover under equal conditions. Of course, in this article, we have used the output direction, which can be used for other tasks in other directions, such as the input direction or the combined direction. The results obtained in the example above show that the upgraded units of this method presented in this paper are more reasonable than the other methods, and we have even obtained negative upgraded output data for the negative output data. One of the limitations of this research is that the necessary and sufficient condition for limiting the semi-essential radial model to the nature of input (output) is that there is an input (output) with at least one positive value. Another limitation in this research is that there is negative data that if this limitation is not present, a linear model can be obtained without changing the variables, which is considered as a vector for upgrading the common weights and a suitable path for upgrading the units (with Data without negative data).

It is suggested that other methods of multi-objective linear programming can be used for negative data. Other methods such as the optimal weights of each unit can also be used to find the right direction vector. Even as mentioned before, it is possible to use the input direction or the combination direction, and each of these paths will definitely give different upgraded units, which should be selected according to the conditions of the respective company. In this paper, using this model, the number of inefficient units is less and it improves all inefficient units under the same conditions, and we are sure that each inefficient unit will improve. The disadvantage of this method is that the inefficient unit may not improve on the performance frontier. It is suggested that methods such as super-efficiency be used to find more efficient units. He even used multi-criteria decision making methods (such as BWM and SWARA) to select the appropriate model for improving inefficient units, which requires sufficient information from inefficient companies (units).

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