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Lump Solutions of Biharmonic Equation

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Abstract

In this article, through symbolic computation With Maple, we get the solution of the (1 + 1)-dimensional Biharmonic-equation. These solutions, which we call lump solution, obtained using square functions, are rationally localized in all directions in the space. It should be noted that not all nonlinear partial differential equations have lump solution. Finally, by selecting the appropriate parameter, the lump solutions are shown in the figures.

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INTRODUCTION

In mathematics, partial differential equations have received widespread attention because of their important role in describing physical phenomena. Therefore, it is important to obtain the solutions to these equations, especially their exact solutions, which have received much attention in recent years. So, it can be said that studying the exact solutions of the nonlinear equation is one of the hot topics in nonlinear science. Many of the exact solutions of partial differential equations are obtained by various effective methods, such as the homotopy analysis method [S. J. Liao.)2004), A. Aziz, F. Khani et al.(2010)], and the exp-function method[M. T. Darvishi et al.(2011), E. M. E. Zayed et al.(2015), Jian-Guo Liu et al.(2018), Elsayed M. E. Zayed et al.(2016), W. X. Ma et al.(2010), Yakup Yıldırım et al.(2017), J. H. He et al.(2007), F. Khani et al.(2009), B-C. Shin et al.(2009), Yakup Yıldırım et al.(2017), Yakup Yıldırım et al.(2017), X. H. Wu et al.(2008), Zhang.(2008), M. T. Darvishi et al.(2011)], the tanh-method[A. M. Wazwaz (2005), Zayed, E. M. E et al.(2010)] and multiple exp-function method[W. X. Ma et al.(2010), W. X. Ma et al.(2012), A.M. Wazwaz(2017), Jian-Guo Liu et al.(2013)],and so on. Rational solutions are a broad set of exact solutions to nonlinear differential equations. The purpose of this article is to find lump solution. Lump solution is localized in all directions in the space, which is a special kind of rational solutions. Lump functions are, in fact, analytical rational functions of spatial and temporal variables, which are localized in all directions in space. It has been studied by many scholars in recent years and has attracted much attention in the mathematical physics community [Hong-Qian Sun et al.(2017), Sun, H. Q et al.(2017), Yuan Zhoua et al.(2019), Solomon Manukure et al.(2018), Yong Zhang et al.(2017), Shou-Ting et al.(2018), Harun-Or-Roshid et al.(2018),J.Y.Yang et al.(2017), Bang-Qing Li et al.(2018), Jian-bing Zhang et al.(2017), Wen-XiuMa(2015), Wen-Xiu Ma et al.(2018), Li-Li Huang et al.(2017) ,Xiang-min Meng et al.(2019)] The Biharmonic equation is a fourth-order partial differential equation which arises in areas of continuum mechanics and it is important in applied mechanics. It can also be mentioned for its specific application in the modeling of thin structures that respond elastically to external forces. The first time the Biharmonic equation is used is not exactly clear because any harmonic function that applies to the Laplace equation is also a Biharmonic function. In this paper, based on the study of the Biharmonic equation, we obtain the lump solutions of this equation.

LUMP SOLUTION

In this section, our goal is to obtain the solution of the Biharmonic equation. The main idea of this method can be expressed as follows:

Consider the Biharonic equation as follows

$$uu_{yyyy}+2uu_{xxyy}+u_{xxxx}=0 \quad (1)$$

And suppose the solution to Equation (1) is as follows

$$u(x,y)=2\ \ln(f(x,y))_x \quad (2)$$

Where f(x,y) is unknown real function. Biharmonic equation is transformed into the proper bilinear form

$$G(D_x,D_y;f.f)=0, \quad (3)$$

Where D_x and D_y are the bilinear derivative operators and D- operator is defined by

$$D_x^m\ D_y^n\ a(x,y).b(x,y)=(\partial y-\partial y')^n\ (\partial x-\partial x')^m\ a(x,y)b(x',y'),$$

Where m and n are positive integers, a(x,y) is the function of x and y , and b(x,y) is a function of the formal variables x and y.

We search for the quadratic function solutions to find the lump solutions of Biharmonic equation, so define

$$f(x,y)=g^2\ (x,y)+h^2\ (x,y)+a_3 \quad (4)$$

$$g(x,y)=x+a_1\ y+a_2,\quad h(x,y)=b_0\ x+b_1\ y+b_2 \quad (5)$$

Where a_(i),i=1,2,b_i,j=0,1,2 are arbitrary constants. Substituting (4) and (5) into (3), and with the help of Maple software, it gives us a system of algebraic equations of all variables.

We solve this system to determinea_i

$,i=1,2,b_i,j=0,1,2$.

$$\{a1=-b_0,a_3=0,b_1=1\}$$

$$\{a_1=b_0, a_3=0, b_1=-1\} \tag{6}$$

Using transformation (2), the quadratic function solution of the Biharmonic equation (1) is given by

$$u=\frac{4g+4b_0h}{g^2+h^2} \tag{7}$$

It is observed that

$$\lim_{y^2\to\infty} u(x,y)=0, \qquad \forall(x,y)\in R^2 \tag{8}$$

Two special pairs of positive quadratic function solutions and lump solutions with choosing specific parameters are given in the following.

First choice

$\{a_1=-b_0,\ a_3=0, a_2=1, b_0=1, b_1=1, b_2=-2\}$

leads to

$$u=\frac{2(4x-2)}{(x-y+1)^2+(x+y\_2)^2} \tag{9}$$

The plots are illustrated in Fig .1.

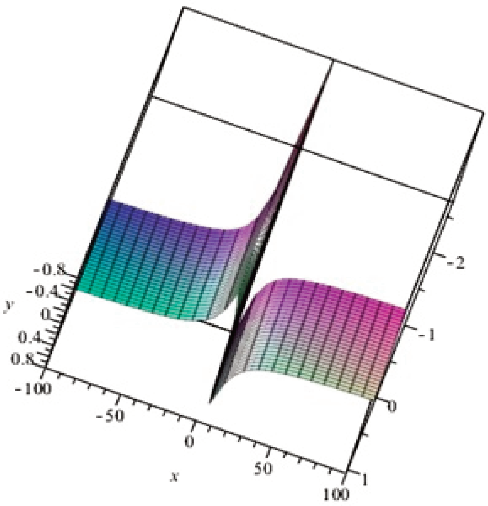

Fig1: The lump solution of Biharmonic equation for $-100\le x\le 100, -0.8\le y\le 0.8,\ a_2=1, b_0=1, b_2=-2$

Second,another selection of the parameters :
$\{a_1=b_0,\ a_3=0, a_2=1, b_0=1, b_1=-1, b_2=-2\}$
The plots are results in Fig 2.

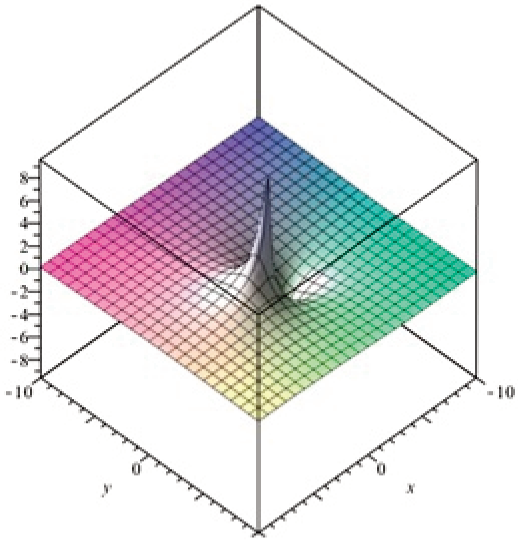

Fig2: The lump solution of Biharmonic equation for $-10\le x\le 10, -10\le y\le 10, a_2=1, b_0=1, b_2=-2$

We draw two plots with particular choice of the involved parameters for lump solutions.(Figure 1,2)

Obviously, for all of lump solutions above, one can see

$$\lim_{y^2\to\infty} u(x,y)=0, \qquad \lim_{y^2\to\infty} u(x,y)=\infty, \forall(x,y)\in R^2$$

The lump solution derived in this paper satisfy this criterion,and they are rationally localized in all direction in the space.

CONCLUSION

In recent years efforts much has been done to obtain the lump solutions to the equations. Trying to find the lampsolutions is very interesting. In this paper, by positive quadratic function solutions, we have getted the lump solutions of the Biharmonic equation ,Our computations are based the symbolic computation software Maple. Finally, with particular choices of the involved parameters which have been made to show the lump solutions, we get different plots. we also hoped that our results will provide some valuable information in the field of nonlinear science.

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