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DEA models without explicit inputs or outputs: radial and non-radial approaches

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Abstract

any applications and studies are concerns about studying DEA models without explicit inputs or outputs for performance evaluation of Decision Making Units (DMUs). On basis of what provided in DEA literature in this paper it has been verified that in the mentioned approach convexity assumption has not been properly dealt with. Thus, a modified approach has been presented and for clarity, an application of the DEA models without inputs and outputs are presented.

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INTRODUCTION

Data envelopment analysis (DEA) is a non-parametric method for performance evaluation of set of Decision Making Units (DMUs). In this technique those DMUs constructed the efficient frontier are called efficient and the efficiency score of the remaining ones, inefficient DMUs, obtained through a comparison process to this frontier. The first DEA paper was published by Charnes et al. (1978), which called ad CCR model. The standard DEA models have been taken into account the input and output data of DMUs. In many application data sets includes only inputs or only inputs and sometimes the original input output data cannot be distinguished. Such data in performance evaluations of business, human development, health service are very applicable. and others. As mentioned in Liu et al. (2011) by considering x_i and y_r as the input and output of a decision making unit (DMU) the index data have the form $e_{ir} = \frac{y_r}{x_i}$. In this paper considering index data two models for radial and non-radial measurement have been introduced which overcome the shortcoming of the models presented by Liu et al. (2011). Also, for sake of clarity the application used in Liu et al. (2011) has been considered again the results obtained through solving the presented models in this paper are compared to those Liu et al. (2011).

The paper unfolds as follows: In the next section some preliminaries will be briefly discussed, next the modified approach well be presented and also with an application these models will be compared and finally section 5 concludes the paper.

PRELIMINARIES

Consider n DMUs with the input and output vectors as $X = (x_1, ..., x_m)$ and $Y = (y_1, ..., y_s)$. As indicated in Liu et al. (2011) the divided data are defined below is a $m\times$ vector as;

$$Z = \frac{Y}{X} = \left(\frac{y_1}{x_1}, ..., \frac{y_s}{x_1}, \frac{y_1}{x_2}, ..., \frac{y_s}{x_2}, ..., \frac{y_1}{x_m}, ..., \frac{y_s}{x_m}\right)$$

The DEA-WEI model as presented by Liu et al. (2011) is as follows:

$Max\left\{\phi \middle| \phi Y_o \in PPS\right\}$, that is:

$$\begin{aligned} & Max\,\varphi \\ & s.t. \\ & \sum_{j=1}^{n} \lambda_j z_{kj} \le \varphi z_{ko},\ k = 1, ..., l, \\ & \sum_{j=1}^{n} \lambda_j = 1, \\ & \lambda_j \ge 0,\ j = 1, ..., n. \end{aligned} \tag{1}$$

where the input variables have not been explicitly considered and z_{kj} for all k and j, are an index constructed from the input-output data. Furthermore, in many applications radial contraction or expansion is not proper thus one can adopt the Russell measurement, and have the following model:

$$\begin{aligned} & Max\,\frac{1}{l}\sum_{k=1}^{l} \varphi_k \\ & s.t. \\ & \sum_{j=1}^{n} \lambda_j z_{kj} \le \varphi_k z_{ko},\ k = 1, ..., l, \\ & \sum_{j=1}^{n} \lambda_j = 1, \\ & \varphi_k \ge 1, k = 1, ..., l, \\ & \lambda_j \ge 0,\ j = 1, ..., n. \end{aligned} \tag{2}$$

As mentioned in Liu et al. (2011) one can use DEA-WEI models of SBM type to deal with such applications using some variable transformation.

MODIFIED APPROACH

Considering DEA technique convexity axiom is one of the fundamental assumptions in defining efficiency measurement. As exist in literature many researchers have paid attention to this subject, such as Petersen (1990), Bogetoft (1996), and Bogetoft et al. (2000) and Podinovski (2005).

As indicated in Emrouznejad and Amin (2009) when at least one of the input or output variables is ratio the convexity assumption may fail. Emrouznejad and Amin (2009) has indicated that:

Lemma1. The standard DEA models cannot be used directly if at least one of the input/output data is in the form of ratio.

According to what Emrouznejad and Amin (2009) has presented, the correct convexity for the ratio variables should be defined as ratio of convex combination of numerator to the convex combination of denominator rather than a simple convex combination of ratio variable. This means that the convex combination of DMUs should have the kth-output as a ratio data is as follows:

$$\text{The correct convex combination} = \frac{\sum_{j=1}^{n}\lambda_j n_{kj}}{\sum_{j=1}^{n}\lambda_j d_{kj}}$$

where $z_{kj} = \frac{n_{kj}}{d_{kj}} \forall k, j$, and each of n_{kj} and d_{kj} for all k and j, can be input and (or) output.

Consider the standard DEA the convex combination as defined below:

$$\sum_{j=1}^{n}\lambda_j z_{kj} = \sum_{j=1}^{n}\frac{n_{kj}}{d_{kj}}\lambda_j$$

Therefore, the convexity assumption, while DMU_o is being assessed, should be taken into the model as follows:

$$\frac{\sum_{j=1}^{n}\lambda_j n_{kj}}{\sum_{j=1}^{n}\lambda_j d_{kj}} \geq \frac{n_{kj}}{d_{kj}} = z_{ko}$$

Hence model (1) for DMU_o should be written in the following form:

$$\begin{aligned} & Max\,\varphi \\ & s.t. \\ & \sum_{j=1}^{n}\lambda_j n_{kj} - \varphi z_{ko}\sum_{j=1}^{n}\lambda_j d_{kj} \geq 0, \; k = 1,\dots,l, \\ & \sum_{j=1}^{n}\lambda_j = 1, \quad \lambda_j \geq 0, \; j = 1,\dots,n. \end{aligned} \tag{3}$$

As Russel model without explicit inputs has been introduced in Liu et al. (2011) and it has been used in the application provided in that paper, we also introduce the correct form of this model. Considering the correct convex combination, the proposed Russel model without explicit input is as follows:

$$\begin{aligned} & Max\,\frac{1}{l}\sum_{k=1}^{l}\varphi_k \\ & s.t. \\ & \sum_{j=1}^{n}\lambda_j n_{kj} - z_{ko}\varphi\sum_{j=1}^{n}\lambda_j d_{kj} \geq 0, \; k = 1,\dots,l, \\ & \sum_{j=1}^{n}\lambda_j = 1, \\ & \varphi_k \geq 1, k = 1,\dots,l, \\ & \lambda_j \geq 0, \; j = 1,\dots,n. \end{aligned} \tag{4}$$

Considering the above-mentioned models, considering

$$\lambda_o = 1, \; \lambda_j = 0 \;\; \forall j \neq o, \;\; \varphi = 1$$

Since $y_{ko} = \frac{n_{ko}}{d_{ko}}$ for all k is a feasible solution for model (3). The same idea can be yield a feasible solution for model (4).

Considering model (3) since $\varphi = 1$ is feasible solution so $\varphi^* \geq 1$.

Also, for model (4), we know

$$\varphi_k \geq 1, \forall r \rightarrow \varphi_k^* \geq 1, \; \forall k,$$

therefore; $\frac{\sum_{k=1}^{l}\varphi_k^*}{s} \geq 1$.

One of the signified feature of DEA model is to introduce a benchmark unit for each inefficient unit. Considering faze one and two correspond to model (3) as follows:

Phase 1:

$$\begin{aligned} & Max\,\varphi \\ & s.t. \\ & \sum_{j=1}^{n}\lambda_j n_{kj} - z_{ko}\varphi\sum_{j=1}^{n}\lambda_j d_{kj} - s_k^+ = 0, \; \forall k, \\ & \sum_{j=1}^{n}\lambda_j = 1, \quad \lambda_j \geq 0, \forall j. \end{aligned} \tag{5}$$

Phase 2:

$$Max\,1s^{+}$$

$$s.t.$$

$$\sum_{j=1}^{n}\lambda_j n_{kj} - z_{ko}\varphi^{*}\sum_{j=1}^{n}\lambda_j d_{kj} - s_k^{+} = 0,\ \forall k, \qquad (6)$$

$$\sum_{j=1}^{n}\lambda_j = 1,\ \lambda_j \ge 0, \forall j.$$

The optimal solution of model (5) shows the maximum radial increase in outputs of DMU under evaluation and the optimal solution of model (6) shows the maximum non radial improvements for each output.

Thus considering these two model a benchmark unit can be introduced. Taking into account the discussion about benchmark units, it can be concluded that:

Theorem 1. Projection point for the DMU under evaluation is efficient.

Proof.

It should be showed that $\varphi^{*}y_{ko} + s^{+*}$ is efficient. To this end consider:

$$Max\,\varphi$$

$$s.t.$$

$$\frac{\sum_{j=1}^{n}\lambda_j n_{kj}}{\sum_{j=1}^{n}\lambda_j d_{kj}} \ge \varphi(\varphi^{*}y_{ko} + s^{+*}),\ \forall k, \qquad (7)$$

$$\sum_{j=1}^{n}\lambda_j = 1, \quad \lambda_j \ge 0, \forall j.$$

we suppose that φ' is optimal solution of model (7), $\varphi' \neq 1$ so $\varphi' > 1$ at result $\varphi'\varphi^{*} > \varphi^{*}$ and this is a contradiction. so $\varphi' = 1$ Now consider model faze 2, so:

$$Max\,1s^{+}$$

$$s.t.$$

$$\frac{\sum_{j=1}^{n}\lambda_j n_{kj}}{\sum_{j=1}^{n}\lambda_j d_{kj}} = \varphi(\varphi^{*}y_{ko} + s^{+*}),\ \forall k, \qquad (8)$$

$$\sum_{j=1}^{n}\lambda_j = 1, \quad \lambda_j \ge 0, \forall j.$$

It should be showed that $\varphi' = 1$, thus first we prove that $1s^{+} = 0$ end suppose $1s^{+} > 0$ and $\varphi^{*}y_{ko} + s^{+} = s^{+*} + s^{+}$ to be a new solution thus $s^{+*} + s^{+} > s^{+*}$ and this a contradiction. so $1s^{+} = 0$ and ($\varphi^{*}y_{ko} + s^{+}$) and it can be concluded that the DMU under assessment is efficient. While considering the proposed models here in the section to follow we consider the application which has been considered in Liu et al (2011).

APPLICATION

Here we used a pilot study, which is used in Liu et al (2011), on applying DEA methodology for performance assessment of 15 basic research institutes in Chinese Academy of Sciences (CAS), and, as did so, also compare CCR and two DEA-WEI models. Here according to what Liu et al (2011) have done, the most important inputs and outputs have been selected, and some comparisons between the original results and the results by using the DEAWEI models have been carried out. In this study, we have employed the output-oriented CCR and DEA-WEI models to compare analysis. To apply the DEA-WEI models they have translated these indicators into indexes by using outputs/inputs separately, and as a result there exist 8 indexes defined as $Y_j = (y_{1j}, y_{2j}, ..., y_{6j})^T$ where:

$y_{1j} = \text{SCI Pub./staff}$

$y_{2j} = \text{SCI Pub./Res. Expen}$

$y_{3j} = \text{High Pub./staff}$

$y_{4j} = \text{High Pub./Res. Expen}$

$y_{5j} = \text{Exter. Fund./Staff}$

$y_{6j} = \text{Grand.Enroll./Staff}$

$y_{7j} = \text{Exter.Fund./Res.Expen}$

$y_{8j} = \text{Exter.Fund./Res.Expen}$

As Liu et al (2011) have claimed, since the percentage of external funding used for research is unknown, and the expenditure for graduate education are normally very small in China, in their paper they excluded y_{7j} and y_{8j}. Liu et al. (2011) have claimed that the formula to standardize these indexes is $y_{rj} = (y_{rj} / Max_j y_{rj}) \times 100$. As Liu et al. (2011) have stated, their purpose to standardize indexes is to remove measurement differences in these weighted sums. In these models, all the outputs and the six indexes are regarded as non-substitutable and equally important. However, this assumption may not be suitable for the current situations in CAS. Considering Table 1 in which the mentioned data are listed.

Table 1: Data

Units	y_{1j}	y_{2j}	y_{3j}	y_{4j}	y_{5j}	y_{6j}
1	380	59880	201	28	386	35368
2	418	79910	480	196	354	69763
3	68	13150	78	72	57	5747
4	1105	92710	153	45	642	49074
5	478	18920	68	18	165	13801
6	828	134240	167	64	229	73748
7	481	52460	38	13	136	32797
8	493	40840	94	6	115	12743
9	198	23110	43	16	79	15964
10	243	32580	42	11	48	20731
11	553	62100	156	34	105	67927
12	347	49510	64	8	190	31616
13	445	78280	440	162	529	62448
14	260	27530	113	23	137	33952
15	304	59450	94	19	263	70015

According to what has been discussed in this paper about two different approaches, radial and non-radial models without explicit inputs, are listed in Tables 2 and 3. In Table 2 radial efficiency scores of Liu et al. (2011) model and the presented model in this paper, model (3), are gathered. As can be seen the obtained efficiency scores through solving model (3) is different from those obtained by the model provided by Liu et al. (2011).

Table 2: Efficiency scores

DMUs	Model (3)	Model (1)
1	85.45	85.45
2	100.00	100
3	100.00	100
4	48.87	48.87
5	62.34	62.4
6	23.29	42.98
7	23.80	31.85
8	38.32	38.32

DMUs	Model (3)	Model (1)
9	33.58	43.48
10	21.70	14.03
11	41.82	62.78
12	46.07	51.68
13	100.00	100
14	68.33	74.87
15	72.82	100

Moreover, considering model (1), presented by Liu et al, (2011), and the presented model in this paper, model (4), there exist a significant difference between these two sets of efficacy scores. In accordance to the aforementioned models, in which the convexity assumption has been considered in the correct way, it can be seen that the obtained result, in both radial and non-radial models, are differ significantly. As it was seen there exist huge difference between the acquired results of the modified models with the conventional ones.

Table 3: Efficiency scores

DMUs	Model (2)	Model (4)
1	25.47	1.27
2	100	100.00
3	100	100.00
4	11.43	0.31
5	20.71	0.39
6	15.42	0.61
7	6.8	0.45
8	4.29	0.18
9	18.38	0.56
10	10.74	0.57
11	18.71	0.83
12	6.7	0.59
13	100	100.00
14	30.82	0.89
15	100	1.58

CONCLUSION

Data envelopment analysis is a non-parametric technique for efficiency evaluation of a set of decision making units. One of the underlying assumption in DEA is convexity assumption. As stated in literature there exists many cases in which DEA is used while ratio variables are being taken into consideration. This paper demonstrated that using the DEA models of ”without inputs” for performance evaluation may lead into incorrect results. Thus, a modified DEA model is presented taking into account the correct convexity of DMUs when a ratio variable is included in the assessment model. The shortcoming of this method, as exist in the work by Liu et al. (2011) has been removed and a modified model has been introduced.

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