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# Centralized Resource Allocation With the Possibility of Downsizing to Evaluate Two-Stage Production System With Shared Inputs

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## Abstract

This paper proposes a centralized network data envelopment analysis model that combines the centralized data envelopment analysis model with possibility of downsizing and two-stage network data envelopment analysis. In the proposed model, this paper also considers the situation in which shared inputs are jointly consumed in each stage. We also assume some outputs can be produced by the first and second stages by using separate inputs. The proposed model is illustrated in an empirical example of twenty sale representatives in two provinces of Golestan and Mazandaran. The results provide valuable information for the centralized decision-maker on how to reallocate resources among the units.

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## INTRODUCTION

Data envelopment analysis (DEA), proposed by Charnes et al. (1978) is a mathematical programming method to evaluate the performance of a group of homogeneous decision-making units (DMUs) using multiple inputs to produce multiple outputs. Nowadays, DEA is regarded as a well-known and powerful mathematical method for determining the relative efficiency of a set of functionally similar DMUs such as banks, brokerage firms, and insurance companies in a wide range of applications (Emrouznejad and Yang, 2018).

Recent developments in the resource allocation problem exhibit an additional planning orientation. In order to integrate target setting and resource allocation in multi-level planning problems. Athanassopoulos (1995) created an interface between goal programming and Data Envelopment Analysis (GoDEA). Beasley (2003) demonstrated that data envelopment analysis (DEA) could be viewed as maximizing the average efficiency of an organization's decision-making units (DMUs). The application of DEA provides an alternative solution to the resource allocation problem, allowing for the consideration of feasible production plans and trade-offs between inputs and outputs (Korhonen and Syrjänen, 2004).

Centralized efficient resource allocation models are used when certain variables are controlled by a central authority rather than individual unit managers. In such a case, the central decision maker's goal is to optimize aggregate resource utilization by all units in an organization rather than to maximize individual output generation and/or minimize resource consumption by each unit individually (Lozano and Villa, 2004; Lozano and Villa, 2005). Asmild et al (2009) reconsidered one of the centralized models proposed by Lozano and Villa (2004) and proposed modifying it only to consider adjustments of previously inefficient units. Lotfi et al. (2010) proposed a Centralized Data Envelopment Analysis (CDEA) model based on the enhanced Russell measure, allowing all DMUs to be easily projected onto the efficient frontier by solving only one model. Fang (2013) attempted to control all decision-making units through a centralized unit by combining technical and attribute efficiency components. They used structural efficiency to further decompose the two components' combined efficiency into aggregate technical efficiency, aggregate allocative efficiency, and re-transferable efficiency. Mar-Molinero et al. (2014) demonstrated that the centralized resource allocation model can be significantly simplified and demonstrated how the model works with real data from Spanish public schools. Wu et al. (2018) proposed a model for determining the maximum income of the evaluated units in the resource reallocation process based on the best income. The efficiency of the performance of some independent units of police organization was evaluated using the reallocating resources method to optimize the total used inputs and total produced outputs of each unit (Khatibi and Rahmani 2018). In the DEA framework, Nemati and Matin (2019) proposed a new approach for resource allocation and efficiency estimation of production units by considering partial impacts among inputs and outputs. At the University of Andalusia in Spain, Contreras and Lozano (2020) have been researching the problem of allocating additional resources in a centralized DEA in order to maximize total revenue.

However, the DEA's research mentioned above studies on resource allocation only use a single-stage structure as a basic unit to analyze and plan allocation plans for their members. Each DMU's operation is treated as a black box, converting external inputs directly into final outputs with no intermediate steps. Kao and Hwang (2008, 2010) proposed two-stage DEA models in which a DMU's overall efficiency can be decomposed into the product of the efficiencies of the two stages. Chen et al. (2006) investigated the sharing of some input resources between two stages and developed an improved two-stage DEA model under the CRS assumption. The authors proposed a relational nonlinear programming model to assess the impact of shared inputs on two stages, as well as information on how to distribute the shared inputs to maximize efficiency. Moghaddas (2019) proposed a model for evaluating a network's efficiency with inputs, outputs, intermediate products, and feedbacks. Nemati et al. (2020) developed a couple of new mathematical programming models in the DEA framework to calculate aggregate, overall, and subunit efficiencies, as well as resource usage by production lines, for a two-stage production system.

There have been a few studies on the resource allocation problem of the two-stage network DEA. One of the studies looks at the allocation of shared input resources used in both the first and second stages based on efficiency. Chen et al. (2010) investigated DMUs with a two-stage network process that shared input resources and was used in both stages of operations, and developed DEA models to assess the performance of two-stage network processes. Zha and Liang (2010) depicted a situation in which shared inputs can be freely allocated between both stages of a two-stage production process. They proposed a product-form cooperative efficiency model to demonstrate the DMU's overall efficiency and the relationship between the stages. Wu et al. (2016) described a method for analyzing the reuse of undesirable intermediate outputs in a two-stage manufacturing process with a shared resource. In this paper, shared resources are input resources used by both the first and second stages and have the property that the proportion used by each stage cannot be conveniently divided and allocated to the operations of the two stages. Toloo et al. (2017) proposed a new relational linear DEA model for calculating the efficiency score of two-stage processes with shared inputs under constant returns-to-scale assumptions. A few studies dealing with the issue of central resource allocation in two-stage production systems could be found. Yu and Chen (2016) proposed a centralized network data envelopment analysis model that took carbon emissions into account and linked them to energy savings. Chen et al. (2018) proposed a network centralized resource allocation model for optimal resource allocation for each shipping line in order to achieve optimal output and undesirable output levels. Yadollahi and Matin (2021) considered and developed the centralized models with downsizing potential proposed by Lozano and Villa (2005) for two-stage production systems.

The main contributions of this paper are the following three aspects. Firstly, the special structure of the production system. Most of the previous studies concentrated on single-stage structures while our organizational system is composed of two-stage structures. Secondly, different from the basic two-stage network structure, there is existence shared inputs that are used in each two stage and existence separate inputs and outputs in each stage. Thirdly, we used special central decision maker for resource allocation in special two-stage production systems for aiming to maximize the total outputs and to minimize the total inputs that can deactivate some of the DMUs for achieving this purpose.

The structure of this paper is as follows: Section 2 presents notions that used the Lozano and Villa's (2005) approach. Section 3 is devoted to introducing a new approach on the possibility of downsizing in the two-stage production system with shared inputs. Section 4 presents the application of the new two-stage centralized network DEA model for evaluating twenty sale representatives in two provinces of Iran. Finally, Section 5 concludes the whole research.

## BASIC PRELIMINARIES

### Notation

Let

Indexes

j, r  indexes for existing DMUs

i  index for inputs (first stage)

c  index for outputs (first stage)

l  index for shared inputs

g index for intermediate products
d index for inputs (second stage)
k index for outputs (second stage)

*Parameters*

n number of existing DMUs
m number of inputs (first stage)
q number of outputs (first stage)
p number of shared inputs
h number of intermediate products
t number of inputs (second stage)
s number of outputs (second stage)

*Decision variables*

$\theta_i$ radial contraction of total amount of *i*th input

$\varphi_k$ radial contraction of total amount of *k*th output

$(\lambda^1_{1r}, \lambda^1_{2r}, \dots, \lambda^1_{nr})$ vector for projecting for first stage

$(\lambda^2_{1r}, \lambda^2_{2r}, \dots, \lambda^2_{nr})$ vector for projecting for first stage

$$\delta_r = \begin{cases} 1 & \text{if } DMU_r \text{ is active} \\ 0 & \text{if } DMU_r \text{ is not active} \end{cases}$$

**Lozano and villa (2005)'s approach**

For resource allocation, Lozano and villa (2005) presented the centralized resource allocation models with possibility of downsizing. They focus on two properties, first, eliminated entirely some existing DMUs may be more efficient for the organization, second they proposed three radial input oriented models for possibility of reducing the number of existing DMUs in separate situations for possibility of downsizing (minimizing total inputs with flexible downsizing, fixed downsizing and with fixed total input reduction). They proposed two phases model for three ideas, in the first phase of the input-oriented model formulated as follows:

(phase I)

$$\begin{aligned} \min \quad & \theta \\ \text{s.t.} \quad & \sum_{r=1}^{n} \sum_{j=1}^{n} \lambda_{jr} x_{ij} \le \theta \sum_{r=1}^{n} x_{ir} \quad \forall i \\ & \sum_{r=1}^{n} \sum_{j=1}^{n} \lambda_{jr} y_{kj} \ge \sum_{r=1}^{n} y_{kr} \quad \forall k \\ & \sum_{j=1}^{n} \lambda_{jr} = \delta_r \quad \forall r \\ & \theta \text{ free}, \lambda_{jr} \ge 0, \quad \delta_r \in \{0,1\} \qquad (1) \end{aligned}$$

After solving model (1), the optimal value $\theta^*$ is obtained.

(phase II)

$$\begin{aligned} \max \quad & \sum_{i=1}^{m} s_i + \sum_{k=1}^{p} t_k \\ \text{s.t.} \quad & \sum_{r=1}^{n} \sum_{j=1}^{n} \lambda_{jr} x_{ij} = \theta^* \sum_{r=1}^{n} x_{ir} - s_i \quad \forall i \\ & \sum_{r=1}^{n} \sum_{j=1}^{n} \lambda_{jr} y_{kj} = \sum_{r=1}^{n} y_{kr} + t_k \quad \forall k \\ & \sum_{j=1}^{n} \lambda_{jr} = \delta_r \quad \forall r \\ & \lambda_{jr} \ge 0, s_i, t_k \ge 0 \quad \delta_r \in \{0,1\} \qquad (2) \end{aligned}$$

After solving model 2, we have the optimal values of the additional amount of total input reduction ($s_i^*$) and total output increase ($t_k^*$), and the total number of operating units to optimize the overall system performance is $\sum_{r=1}^{n} \delta_r^*$. The set of optimal vector $(\lambda^*_{1r}, \lambda^*_{2r}, \dots, \lambda^*_{nr})$ that define each of optimal operating units. The input and output targets corresponding to this operating points can be calculated as follows:

$$\hat{x}_{ir} = \sum_{j=1}^{n} \lambda^*_{jr} x_{ij}$$
$$\hat{y}_{kr} = \sum_{j=1}^{n} \lambda^*_{jr} y_{kj}$$

Yadollahi and Kazemi matin (2021) used this idea and expanded for two-stage production network. According to this idea, in the following section we present the centralized resource allocation with the possibility of downsizing in Two-stage processes with shared inputs.

## EVALUATING TWO-STAGE PROCESSES WHIT SHARED INPUTS WHIT POSSIBILITY OF DOWNSIZING

Fig. 1 shows a two-stage process network where some inputs are associated with both stages and used jointly.

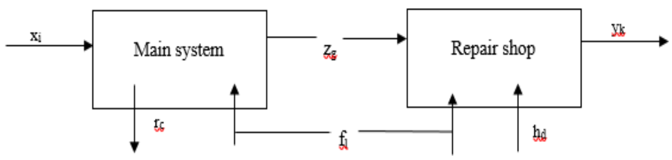


Fig. 1. The production system

In fig. 1, the first stage (Main system) uses input $x_i$ to produce intermediate product $z_g$, that is imperfect output and perfect output $r_p$. The output $z_g$ are incomplete or defective and they need to be repaired. We assume that $f_l$ should be shared among the two stages. The second stage (Repair shop) consume the external input $h_d$. The final product of this stage is $y_k$. The production process is depicted in Fig. 1. for evaluating this two-stage production system, we use the proposed method Kao and Hwang (2008) for two-stage production system with shared inputs.

The main purpose of this study is (re)allocating resources in this special production system with possibility of downsizing. We following two models for resource reallocation, while considering the possibility of downsizing for Two-stage processes with shared inputs.

**Model (I): system efficiency evaluation with flexible downsizing option**

In this model, we proposed a non-radial centralized resource allocation model for two-stage network system with shared inputs, in which maintaining the active status for all existing units is not required. The main purpose of discussing this section is to study the intra-organizational performance of production systems for calculating new operating points, while allowing for some deactivated decision-making units. The possibility of reducing the number of the existing DMUs is included in the model by allowing them to be projected onto a virtual empty operating point that takes no inputs and produces no outputs. Deactivating some decision-making units may be beneficial to the whole organization since the total amount of reduced inputs can be allocated among the remaining active decision-making units, following by an increase in the total produced outputs.

$$\min_{\theta,\varphi,\lambda,\delta} \frac{\frac{1}{m}\sum_{i=1}^{m}\theta_i + \frac{1}{t}\sum_{d=1}^{t}\theta_d}{\frac{1}{q}\sum_{c=1}^{q}\varphi_c + \frac{1}{s}\sum_{k=1}^{s}\varphi_k}$$

s.t

$$\sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^{1}x_{ij} \le \theta_i\sum_{r=1}^{n}x_{ir} \qquad i=1,\dots,m$$

$$\sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^{1}y_{cj} \ge \varphi_c\sum_{r=1}^{n}r_{cr} \qquad c=1,\dots,q$$

$$\sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^{1}z_{gj} \ge \sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^{2}z_{gj} \qquad g=1,\dots,h$$

$$\sum_{j=1}^{n}\lambda_{jr}^{1}(\alpha_j f_{lj}) \le f_{lr}^{1} \qquad l=1,\dots,p,\ r=1,\dots,n$$

$$\sum_{j=1}^{n}\lambda_{jr}^{2}(1-\alpha_j)f_{lj} \le f_{lr}^{2} \qquad l=1,\dots,p, r=1,\dots,n$$

$$f_{lr}^{1}+f_{lr}^{2}=f_{lr} \qquad l=1,\dots,p,\ r=1,\dots,n$$

$$\sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^{2}h_{dj} \le \theta_d\sum_{d=1}^{n}h_{dr} \qquad d=1,\dots,t$$

$$\sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^{2}y_{kj} \ge \varphi_k\sum_{r=1}^{n}y_{kr} \qquad k=1,\dots,s$$

$$\sum_{j=1}^{n}\lambda_{jr}^{1} = \delta_r \qquad r=1,\dots,n$$

$$\sum_{j=1}^{n}\lambda_{jr}^{2} = \delta_r \qquad r=1,\dots,n$$

$$\lambda_{jr}^{1},\lambda_{jr}^{2} \ge 0 \qquad j,r=1,\dots,n$$

$$\theta_i \le 1, \theta_d \le 1 \qquad i=1,\dots,m,\ d=1,\dots,t$$

$$\varphi_k \ge 1, \varphi_c \ge 1 \qquad k=1,\dots,s,\ c=1,\dots,q$$

$$\delta_r \in \{0,1\} \qquad r=1,\dots,n$$

$$0 \le \alpha_j \le 1 \qquad j=1,\dots,n$$

(3)

In model 3, $f_{lr}^{1}$ and $f_{lr}^{2}$ are allocating points for joint inputs in first stage and second stage, respectively, that determine consume of shared inputs in each stage too. Let $\alpha_j$ and $(1-\alpha_j)$ denote the proportions of shared inputs $f_{lj}$ allocated to first and second stages for $DMU_j$, respectively. If the $l^{th}$ shared input is solely used by $DMU_j$ in the first stage, then we have $\alpha_j = 1$. In contrast, if shared input is solely used by $DMU_j$ in second stage, then we have $\alpha_j = 0$. Some portion $0 \le \alpha_j \le 1$ of the shared inputs $f_{lj}$ is allocated to first stage and the remainder $0 \le 1-\alpha_j \le 1$ is allocated to second stage. With this constraint $f_{lr}^{1}+f_{lr}^{2}=f_{lr}$, we suppose that all of the shared inputs divided between two stages. With these constraints for shared inputs, when a unit is deactivating, the shared inputs not allocating between other units. $\theta_i$ denotes radial contraction of the total $i^{th}$ input in the first stage, $\theta_d$ denotes radial contraction of the total $d^{th}$ input in the second stage, $\varphi_c$ represents the radial contraction of the total $c^{th}$ output in the first stage, and $\varphi_k$ represents the radial contraction of the total $k^{th}$ output in the second stage.

The constraints $\theta_i$, $\theta_d \le 1$, $\varphi_k \ge 1, \varphi_c \ge 1$ are requirements for dominance. The objective function can be decomposed into two terms:

the first term $\frac{1}{m}\sum_{i=1}^{m}\theta_i+\frac{1}{t}\sum_{d=1}^{t}\theta_d$, is the Russell-input measure of first stage and second one, the second term $\frac{1}{q}\sum_{c=1}^{q}\varphi_c+\frac{1}{s}\sum_{k=1}^{s}\varphi_k$ is the Russell-outputs measure for first and second one.

In the optimal solution of model (3), in the first stage, the optimal value $\theta_i^*$ is radial reduction of total amount of $i^{th}$ input and the optimal value $\varphi_c^*$ is the radial increase of the total amount of $c^{th}$ output. In the second stage, the optimal value $\theta_d^*$ is radial reduction of total amount of $d^{th}$ input and the optimal value $\varphi_k^*$ is the radial increase of the total amount of $k^{th}$ output. The set of optimal vectors of multipliers for stages 1 and 2 is $(\lambda_{1r}^{1*},\lambda_{2r}^{1*},\dots,\lambda_{mr}^{1*})$ and $(\lambda_{1r}^{2*},\lambda_{2r}^{2*},\dots,\lambda_{mr}^{2*})$, which defines each optimal (benchmark) operating unit. The input, output and intermediate product targets corresponding to these operating points can be calculated as follows:

*First stage*

$$\hat{x}_{ir}=\sum_{j=1}^{n}\lambda_{jr}^{1*}x_{ij}\quad i=1,\dots,m,r=1,\dots,n$$

$$\hat{r}_{cr}=\sum_{j=1}^{n}\lambda_{jr}^{1*}r_{cj}\quad c=1,\dots,q,r=1,\dots,n$$

$$\hat{z}_{gr}^{out}=\sum_{j=1}^{n}\lambda_{jr}^{1*}z_{gj}\quad g=1,\dots,h,r=1,\dots,n$$

*Second stage*

$$\hat{h}_{dr}=\sum_{j=1}^{n}\lambda_{jr}^{2*}h_{dj}\quad d=1,\dots,t,r=1,\dots,n$$

$$\hat{y}_{kr}=\sum_{j=1}^{n}\lambda_{jr}^{2*}y_{kj}\quad k=1,\dots,s,r=1,\dots,n$$

$$\hat{z}_{gr}^{in}=\sum_{j=1}^{n}\lambda_{jr}^{2*}z_{gj}\quad g=1,\dots,h,r=1,\dots,n$$

The proposed model satisfies the following properties:

- Total inputs and outputs in each stages are improved.
- The shared inputs, shared between two stages and with deactivating DMUs reducing the total amount of these inputs.
- Some DMUs may be deactivated and these resources allocating between other units.

**Linearization**

We note that Model (3) has a fractional objective function which makes it non-linear and difficult to solve. To transform it into a linear equivalent optimization model, we use Charnes and Cooper (1962) transformation approach. Let use the following variable substitutions:

$\beta=\left(\frac{1}{\frac{1}{q}\sum_{c=1}^{q}\varphi_c+\frac{1}{s}\sum_{k=1}^{s}\varphi_k}\right)$ such that $0<\beta\le 1$ and $\beta\left(\frac{1}{q}\sum_{c=1}^{q}\varphi_c+\frac{1}{s}\sum_{k=1}^{s}\varphi_k\right)=1$.

$\theta_i'=\beta\theta_i(i=1,\dots,m),\theta_d'=\beta\theta_d\ (d=1,\dots,t),\varphi_c'=\beta\varphi_c\ (c=1,\dots,q)$ and $\varphi_k'=\beta\varphi_k\ (k=1,\dots,s)$, $\delta_r'=\beta\delta_r(r=1,\dots,n)$ and $\beta\alpha_j=\acute{\alpha}_j\ (j=1,\dots,n)$.

The intensity weights $\lambda_{jr}^1$ and $\lambda_{jr}^2$ can be partitioned into two components for each weights as follows:

$\lambda_{jr}^{1'}=\beta\lambda_{jr}^1\alpha_j,\quad \acute{v}_{jr}=(1-\beta\alpha_j)\lambda_{jr}^1,\quad \lambda_{jr}^1=\lambda_{jr}^{1'}+\acute{v}_{jr}\ (j,r=1,\dots,n)$ and $\lambda_{jr}^{2'}=\beta\lambda_{jr}^2(1-\alpha_j),\quad \hat{v}_{jr}=\beta\lambda_{jr}^2\alpha_j,\quad \psi_{jr}=(1-\beta)\lambda_{jr}^2,\quad \lambda_{jr}^2=\lambda_{jr}^{2'}+\hat{v}_{jr}+\psi_{jr}\ (j,r=1,\dots,n)$.

By multiplying the objective and constraints of Model (3) by $\beta$, the model could be rewritten as follows:

$$\underset{\theta',\varphi',\lambda',\delta'}{Min}\ \frac{1}{m}\sum_{i=1}^{m}\theta_i'+\frac{1}{t}\sum_{d=1}^{t}\theta_d'$$

$$\begin{aligned}
s.t\quad & \frac{1}{q}\sum_{c=1}^{q}\varphi_c'+\frac{1}{s}\sum_{k=1}^{s}\varphi_k'=1\\
& \sum_{r=1}^{n}\sum_{j=1}^{n}(\lambda_{jr}^{1'}+\acute{v}_{jr})x_{ij}\le\theta_i'\sum_{r=1}^{n}x_{ir}\quad i=1,\dots,m\\
& \sum_{r=1}^{n}\sum_{j=1}^{n}(\lambda_{jr}^{1'}+\acute{v}_{jr})y_{cj}\ge\varphi_c'\sum_{r=1}^{n}r_{cr}\quad c=1,\dots,q\\
& \sum_{r=1}^{n}\sum_{j=1}^{n}(\lambda_{jr}^{1'}+\acute{v}_{jr})z_{gj}\ge\sum_{r=1}^{n}\sum_{j=1}^{n}(\lambda_{jr}^{2'}+\hat{v}_{jr}+\psi_{jr})z_{gj}\quad g=1,\dots,h\\
& \sum_{j=1}^{n}(\lambda_{jr}^{1'}+\acute{v}_{jr})f_{lj}\le f_{lr}^1\quad l=1,\dots,p,\ r=1,\dots,n\\
& \sum_{j=1}^{n}\left(\lambda_{jr}^{2'}+\hat{v}_{jr}+\psi_{jr}\right)f_{lj}\le f_{lr}^2\quad l=1,\dots,p,\ r=1,\dots,n\\
& f_{lr}^1+f_{lr}^2\le f_{lr}\quad l=1,\dots,p,\ r=1,\dots,n\\
& \sum_{r=1}^{n}\sum_{j=1}^{n}(\lambda_{jr}^{2'}+\hat{v}_{jr}+\psi_{jr})h_{dj}\le\theta_d'\sum_{d=1}^{n}h_{dr}\quad d=1,\dots,t\\
& \sum_{r=1}^{n}\sum_{j=1}^{n}(\lambda_{jr}^{2'}+\hat{v}_{jr}+\psi_{jr})y_{kj}\ge\varphi_k'\sum_{r=1}^{n}y_{kr}\quad k=1,\dots,s\\
& \sum_{j=1}^{n}(\lambda_{jr}^{1'}+\acute{v}_{jr})=\delta_r'\quad r=1,\dots,n\\
& \sum_{j=1}^{n}(\lambda_{jr}^{2'}+\hat{v}_{jr}+\psi_{jr})=\delta_r'\quad r=1,\dots,n\\
& \lambda_{jr}^{1'},\acute{v}_{jr},\lambda_{jr}^{2'},\hat{v}_{jr},\psi_{jr}\ge 0\quad j,r=1,\dots,n\\
& \theta_i',\theta_d'\le\beta,\varphi_c',\varphi_k'\ge\beta\quad i=1,\dots,m,\ d=1,\dots,t,c=1,\dots,q,k=1,\dots,s
\end{aligned}$$

$$\delta'_r = (1-t_r)\beta \qquad r = 1,\dots,n$$
$$t_r \in \{0,1\}$$
$$0 \le \acute{\alpha}_j \le \beta \qquad j = 1,\dots,n$$
$$(4)$$

By attention that, model 4 is linear structure in objective function and all the constraints, except the last one; in which the binary variable $t_r$ is exists. This model could be easily solved with Branch-and-Bound (B&B) algorithms and guarantees obtaining the global optimal solutions. A similar linearization approach can be stated for the Model (II).

**Model (II): system efficiency evaluation with fixed downsizing value**

In this section, with limited number of active units are considered to maximize the overall efficiency by reducing the amounts of total inputs and increasing the amounts of total outputs, and with reallocating attempts to improve the shared input. In this model the number of active units determined by the decision maker, i.e., the decision maker deactivates some decision units to improve the state of the production system. Let $N^{target} < n$ represent the maximum desired number of surviving DMUs. Note also that, it is reasonable to expect that there must be a lower limit on the number of surviving units such that if $N^{target}$ is below it, no feasible solution exists for this model.

$$\operatorname*{Min}_{\theta,\varphi,\lambda,\delta} \frac{\frac{1}{m}\sum_{i=1}^{m}\theta_i + \frac{1}{t}\sum_{d=1}^{t}\theta_d}{\frac{1}{q}\sum_{c=1}^{q}\varphi_c + \frac{1}{s}\sum_{k=1}^{s}\varphi_k}$$

s.t

$$\sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^{1}x_{ij} \le \theta_i \sum_{r=1}^{n} x_{ir} \qquad i = 1,\dots,m$$
$$\sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^{1}y_{cj} \ge \varphi_c \sum_{r=1}^{n} r_{cr} \qquad c = 1,\dots,q$$
$$\sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^{1}z_{gj} \ge \sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^{2}z_{gj} \qquad g = 1,\dots,h$$
$$\sum_{j=1}^{n}\lambda_{jr}^{1}(\alpha_j f_{lj}) \le f_{lr}^{1} \qquad l = 1,\dots,p, r = 1,\dots,n$$
$$\sum_{j=1}^{n}\lambda_{jr}^{2}(1-\alpha_j) f_{lj} \le f_{lr}^{2} \qquad l = 1,\dots,p, r = 1,\dots,n$$
$$f_{lr}^{1} + f_{lr}^{2} \le f_{lr} \qquad l = 1,\dots,p, r = 1,\dots,n$$
$$\sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^{2}y_{kj} \ge \varphi_k \sum_{r=1}^{n} y_{kr} \qquad k = 1,\dots,s$$
$$\sum_{j=1}^{n}\lambda_{jr}^{1} = \delta_r \qquad r = 1,\dots,n$$
$$\sum_{j=1}^{n}\lambda_{jr}^{2} = \delta_r \qquad r = 1,\dots,n$$
$$\sum_{r=1}^{n}\delta_r \le N^{target}$$
$$\lambda_{jr}^{1}, \lambda_{jr}^{2} \ge 0 \qquad j, r = 1,\dots,n$$
$$\theta_i \le 1, \theta_d \le 1 \qquad i = 1,\dots,m, \quad d = 1,\dots,t$$
$$\varphi_k \ge 1, \varphi_c \ge 1 \qquad k = 1,\dots,s, \quad c = 1,\dots,q$$
$$\delta_r \in \{0,1\} \qquad r = 1,\dots,n$$
$$0 \le \alpha_j \le 1 \qquad j = 1,\dots,n$$
$$(5)$$

## APPLICATION

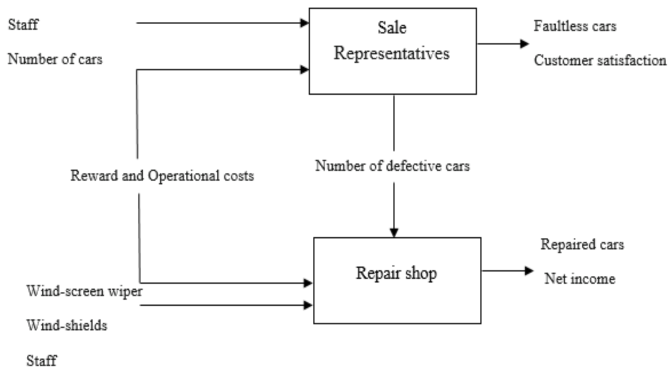


Fig. 2. The two-stage production system

In this section, we will try to improve the overall performance of twenty sales representatives in two counties, Golestan and Mazandaran, using a new technique of allocating resources to allow the potential deactivation of some decision-making units. The finished cars are patched to the sales representatives and then sold to the customers. On average, 11% of all manufactured cars are defective and in need of repair. The majority of defects are caused by the breaking of the windshield and the need to service the car engine. The defective vehicles are returned to the representatives and must be repaired in a repair shop. Each sales representative owns a licensed repair shop, and the defective vehicles will be repaired there. We attempted to improve customer service by (re)allocating resources in this production system.

Amirteimouri's article contains data and a two-stage production system (Amirteimouri 2013). Figure 2 depicts the manufacturing process. The network production system is divided into two stages: Sale Representatives, Staff, and Number of Cars, and the second is Faultless Cars and

Customer Satisfaction. The number of defective cars is an intermediate product; some manufactured cars are defective and must be repaired. The majority of defects are caused by the breaking of the windshield and the need to service the car engine. The second stage is the repair shop, where the inputs are windshield wipers, windshields, and staff, and the outputs are repaired cars and net income. Reward and operational costs are shared inputs for each stage's consumption.

Table 1: The data for the application

| DMU | Staff | Cars | Faultless cars | Customer satisfaction | Defective cars | Wind-screen wiper | Wind-shields | Staff | Repaired cars | Net income | Reward & operational cost |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 11 | 190 | 171 | 93 | 19 | 24 | 1 | 31 | 18 | 473 | 235 |
| **2** | 13 | 206 | 185 | 64 | 21 | 26 | 2 | 30 | 20 | 635 | 310 |
| **3** | 10 | 176 | 158 | 71 | 18 | 23 | 1 | 22 | 17 | 412 | 206 |
| **4** | 9 | 149 | 133 | 76 | 16 | 20 | 1 | 21 | 16 | 410 | 208 |
| **5** | 14 | 191 | 171 | 89 | 20 | 27 | 1 | 28 | 18 | 629 | 316 |
| **6** | 10 | 163 | 146 | 74 | 17 | 21 | 1 | 22 | 17 | 411 | 201 |
| **7** | 8 | 151 | 137 | 91 | 14 | 17 | 1 | 19 | 14 | 401 | 198 |
| **8** | 12 | 169 | 151 | 96 | 18 | 23 | 2 | 33 | 17 | 399 | 204 |
| **9** | 15 | 193 | 172 | 87 | 21 | 29 | 3 | 38 | 20 | 670 | 331 |
| **10** | 14 | 188 | 168 | 89 | 20 | 28 | 2 | 35 | 20 | 650 | 328 |
| **11** | 16 | 199 | 176 | 91 | 23 | 31 | 3 | 41 | 21 | 780 | 349 |
| **12** | 11 | 161 | 142 | 93 | 19 | 23 | 1 | 29 | 19 | 601 | 299 |
| **13** | 10 | 158 | 140 | 98 | 18 | 19 | 1 | 24 | 18 | 430 | 211 |
| **14** | 13 | 171 | 154 | 86 | 17 | 23 | 2 | 28 | 17 | 620 | 312 |
| **15** | 14 | 173 | 154 | 89 | 19 | 24 | 1 | 31 | 18 | 640 | 328 |
| **16** | 15 | 185 | 164 | 91 | 21 | 26 | 3 | 32 | 20 | 703 | 342 |
| **17** | 12 | 159 | 142 | 95 | 17 | 27 | 1 | 30 | 17 | 513 | 261 |
| **18** | 19 | 207 | 182 | 88 | 25 | 33 | 3 | 45 | 22 | 841 | 419 |
| **19** | 12 | 197 | 174 | 83 | 23 | 26 | 2 | 31 | 22 | 591 | 283 |
| **20** | 17 | 201 | 179 | 79 | 22 | 30 | 3 | 41 | 21 | 841 | 408 |
| **Total** | 255 | 3587 | 3199 | 1720 | 388 | 500 | 35 | 611 | 372 | 11650 | 5749 |

Table 2 presents the results for model (I). As can be seen, in the first stage, the amounts of 42.21 and 15.18 units have been reduced from the initial total value of Staff ($\hat{x}_1$) and Cars ($\hat{x}_1$), respectively. These reductions are obtained while maintaining the Faultless ($\hat{r}_1$) cars and increasing 175.63 unit in Customer satisfaction ($\hat{r}_2$). The Defective cars ($\hat{z}$) is intermediate product which is also considered as the product of the Sale Representatives, have reached the values of 372.8. Given that this product of this stage is used as input for the Repair shop, all of this product used in this stage. In the second stage, the amounts of 48.56, 15 and 45.25 units have been reduced from the initial total value of Wind-screen wiper ($\hat{h}_1$), Wind-shields ($\hat{h}_2$) and Staff ($\hat{h}_3$), respectively. These reductions are obtained while increasing .87 and 84.84 units in Repaired cars ($\hat{y}_1$) and Net income ($\hat{y}_1$), respectively. Reward & operational cost ($\hat{f}$) as shared input, the total amounts of 2493.84 is used in first stage and the total amounts of 3255.15 is used in second stage, the all shared inputs divided between two stages. In this case, none of operating units have been deactivated.

Table 3 presents the results of Model (II). Decision maker decide to allocate resources by deactivating one DMUs, the results of this model are achieved with $N^{target} = 19$. As can be seen, in the first stage, the amounts of 33.2 and 11.8 units have been reduced from the initial total value of Staff ($\hat{x}_1$) and Cars ($\hat{x}_2$), respectively. These reductions are obtained while maintaining the Faultless cars ($\hat{r}_1$) and Customer satisfaction ($\hat{r}_2$). The Defective cars ($\hat{z}$) is intermediate product which is also considered as the product of the Sale Representatives, have reached the values of 378.95, 374.93 of this total value has been used as input for the Repair shop. In the second stage, the amounts of 46.64, 10.9 and 41.14 units have been reduced from the initial total value of Wind-screen wiper ($\hat{h}_1$), Wind-shields ($\hat{h}_2$) and Staff ($\hat{h}_3$), respectively. The total amount of Repaired cars ($\hat{y}_1$) and Net income ($\hat{y}_2$) are constant. Reward & operational cost ($\hat{f}$) as shared input, the total amounts of 4119.81 is used in first stage and the total amounts of 1210.19 is used in second stage, the all shared inputs divided between two stages. Due to the deactivation of the 18th unit, this input reduces 419 units from initial total value.

Table 2: Results from model (I)

| DMU | Staff ($\hat{x}_1$) | Cars ($\hat{x}_2$) | Faultless cars ($\hat{r}_1$) | Customer satisfaction ($\hat{r}_2$) | Defective cars ($\hat{z}^{out}$) | Defective cars ($\hat{z}^{in}$) | Wind-screen wiper ($\hat{h}_1$) | Wind-shields ($\hat{h}_2$) | Staff ($\hat{h}_3$) | Repaired cars ($\hat{y}_1$) | Net income ($\hat{y}_2$) | Reward & operational cost ($f^1$) | Reward & operational cost ($f^2$) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 10.06 | 160.17 | 142.1 | 97.66 | 18.1 | 19 | 23 | 1 | 29 | 19 | 601 | 234.56 | .43 |
| **2** | 11 | 190 | 171 | 93 | 19 | 19 | 23 | 1 | 29 | 19 | 601 | 0 | 310 |
| **3** | 11 | 190 | 171 | 93 | 19 | 19 | 23 | 1 | 29 | 19 | 601 | 101.51 | 104.49 |
| **4** | 11 | 190 | 171 | 93 | 19 | 19 | 23 | 1 | 29 | 19 | 601 | 0 | 208 |
| **5** | 11 | 190 | 171 | 93 | 19 | 19 | 23 | 1 | 29 | 19 | 601 | 316 | 0 |
| **6** | 10.06 | 160.17 | 142.1 | 97.66 | 18.1 | 14.7 | 17.85 | 1 | 20.42 | 14.71 | 429.32 | 0 | 201 |
| **7** | 10.06 | 160.17 | 142.1 | 97.66 | 18.1 | 16.1 | 19.59 | 1 | 23.33 | 16.16 | 487.52 | 0 | 198 |
| **8** | 11 | 190 | 171 | 93 | 19 | 19 | 23 | 1 | 29 | 19 | 601 | 0 | 204 |
| **9** | 10.95 | 188.59 | 169.7 | 93.22 | 18.96 | 19 | 23 | 1 | 29 | 19 | 601 | 0 | 331 |
| **10** | 10.91 | 187.22 | 168.31 | 93.43 | 18.91 | 19 | 23 | 1 | 29 | 19 | 601 | 0 | 328 |
| **11** | 10.06 | 160.17 | 142.1 | 97.66 | 18.06 | 19 | 23 | 1 | 29 | 19 | 601 | 347.72 | 1.28 |
| **12** | 10.11 | 161.76 | 143.65 | 97.41 | 18.11 | 19 | 23 | 1 | 29 | 19 | 601 | 285.92 | 13.08 |
| **13** | 11 | 190 | 171 | 93 | 19 | 19 | 23 | 1 | 29 | 19 | 601 | 210.21 | .79 |
| **14** | 10.96 | 188.79 | 169.82 | 93.19 | 18.96 | 19 | 23 | 1 | 29 | 19 | 601 | 25.39 | 286.61 |
| **15** | 10.96 | 188.75 | 169.79 | 93.19 | 18.96 | 19 | 23 | 1 | 29 | 19 | 601 | 0 | 328 |
| **16** | 10.91 | 187.28 | 168.37 | 93.42 | 18.91 | 19 | 23 | 1 | 29 | 19 | 601 | 10.79 | 331.21 |
| **17** | 10.67 | 179.47 | 160.79 | 94.65 | 18.67 | 19 | 23 | 1 | 29 | 19 | 601 | 261 | 0 |
| **18** | 10.06 | 160.17 | 142.1 | 97.66 | 18.1 | 19 | 23 | 1 | 29 | 19 | 601 | 417.74 | 1.26 |
| **19** | 10.96 | 188.94 | 169.97 | 93.16 | 18.96 | 19 | 23 | 1 | 29 | 19 | 601 | 283 | 0 |
| **20** | 10.06 | 160.17 | 142.1 | 97.66 | 18.1 | 19 | 23 | 1 | 29 | 19 | 601 | 0 | 408 |
| **Total** | 212.79 | 3571.82 | 3199 | 1895.63 | 372.8 | 372.8 | 451.44 | 20 | 565.75 | 372.87 | 11734.84 | 2493.84 | 3255.15 |
| **Optimal** | .834 | .995 | 1 | 1.102 | - | - | .903 | .571 | .925 | 1.002 | 1.007 | - | - |

Table 3: Results from model (II); $N^{target} = 19$

| DMU | Staff ($\hat{x}_1$) | Cars ($\hat{x}_2$) | Faultless cars ($\hat{r}_1$) | Customer satisfaction ($\hat{r}_2$) | Defective cars ($\hat{z}^{out}$) | Defective cars ($\hat{z}^{in}$) | Wind-screen wiper ($\hat{h}_1$) | Wind-shields ($\hat{h}_2$) | Staff ($\hat{h}_3$) | Repaired cars ($\hat{y}_1$) | Net income ($\hat{y}_2$) | Reward & operational cost ($f^1$) | Reward & operational cost ($f^2$) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 11.5 | 182.85 | 163.28 | 80.53 | 19.56 | 19 | 23 | 1 | 29 | 19 | 601 | 235 | 0 |
| **2** | 11.1 | 190.42 | 171.34 | 92.4 | 19.1 | 20.24 | 25.62 | 1.76 | 33.38 | 19.84 | 686.48 | 308.72 | 1.28 |
| **3** | 12.2 | 190.2 | 171.14 | 92.7 | 19.02 | 19 | 23 | 1 | 29 | 19 | 601 | 204.54 | 1.46 |
| **4** | 11.1 | 190.5 | 171.42 | 92.13 | 19.1 | 19 | 23 | 1 | 29 | 19 | 601 | 206.56 | 1.44 |
| **5** | 15.4 | 199.5 | 176.6 | 89.3 | 25.6 | 19.21 | 23.48 | 1.14 | 29.83 | 19.14 | 617.73 | 314.72 | 1.28 |
| **6** | 11.4 | 176.22 | 155.5 | 88.77 | 20.69 | 21.27 | 24.94 | 1.63 | 30.66 | 20.69 | 607.78 | 201 | 0 |
| **7** | 12 | 197 | 174 | 83 | 23 | 19.08 | 23.06 | 1.02 | 29.04 | 19.06 | 600.78 | 0 | 198 |
| **8** | 11. | 163.55 | 144.54 | 92.27 | 19.03 | 19 | 23 | 1 | 29 | 19 | 601 | 202.57 | 1.43 |
| **9** | 11.1 | 190.5 | 171.42 | 92.13 | 19.1 | 19 | 23 | 1 | 29 | 19 | 601 | 0 | 331 |
| **10** | 15.4 | 199.06 | 176.23 | 90.01 | 22.83 | 19.92 | 24.72 | 1.55 | 30.51 | 20.47 | 608.47 | 326.44 | 1.56 |
| **11** | 11.2 | 190.52 | 171.28 | 92.88 | 19.23 | 19.23 | 23.17 | 1.1 | 29.11 | 19.17 | 600.45 | 348.91 | .09 |
| **12** | 11.1 | 190.5 | 171.42 | 92.13 | 19.1 | 20.9 | 24.83 | 1.58 | 30.84 | 20.41 | 617.25 | 299 | 0 |
| **13** | 11.4 | 193.39 | 173.96 | 86.97 | 19.41 | 21.66 | 25.4 | 1.77 | 31.22 | 20.97 | 615.35 | 211 | 0 |
| **14** | 11.1 | 190.5 | 171.42 | 92.13 | 19.1 | 19 | 23 | 1 | 29 | 19 | 601 | 310.67 | 1.33 |
| **15** | 10.4 | 168.97 | 150.29 | 92.69 | 18.68 | 20.92 | 24.83 | 1.58 | 30.83 | 20.42 | 616.7 | 0 | 328 |
| **16** | 11.1 | 190.41 | 171.26 | 92.73 | 19.15 | 20.86 | 25.46 | 1.74 | 32.3 | 20.34 | 652.3 | 0 | 342 |
| **17** | 11.1 | 190.5 | 171.42 | 92.13 | 19.1 | 19.64 | 23.85 | 1.25 | 30.14 | 19.49 | 618.71 | 261 | 0 |
| **18** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **19** | 11.1 | 190.5 | 171.42 | 92.13 | 19.1 | 19 | 23 | 1 | 29 | 19 | 601 | 283 | 0 |
| **20** | 11.1 | 190.11 | 171.06 | 92.97 | 19.05 | 19 | 23 | 1 | 29 | 19 | 601 | 406.68 | 1.32 |
| **Total** | 221.8 | 3575.2 | 3199 | 1720 | 378.95 | 374.93 | 453.36 | 24.1 | 569.86 | 372 | 11650 | 4119.81 | 1210.19 |
| **Optimal** | .869 | .996 | 1 | 1 | - | - | .906 | .688 | .932 | 1 | 1 | - | - |

## CONCLUSION

Conventional DEA models cannot be applied to centralized resource allocation in systems with network structure. In specific, this study aimed to evaluate the possibility of closing some DMUs in the interest of non-radial efficiency. This paper, through proposing a model, aims to maximize the reduction in the total inputs and the increase in the total outputs, allowing for any existing DMU to be closed even when selected by the user. This paper provided a brief review of some basic network systems with shared inputs. In real-world applications, some of the network production systems are two-stage with shared inputs structures. In this study, a new two-stage network DEA model with shared inputs was proposed that can be used for centralized resource allocation of the two-stage network structures with possibility of downsizing. Finally, real-world examples were used to illustrate the approach.

## REFRENCES


Amirteimoori, A. (2013). A DEA two-stage decision processes with shared resources. *Central European Journal of Operations Research, 21*(1), pp. 141-151.

Asmild, M., Paradi, J. C., & Pastor, J. T. (2009). Centralized resource allocation BCC models. *Omega, 37*(1), pp. 40-49.

Athanassopoulos, A. D. (1995). Goal programming & data envelopment analysis (GoDEA) for target-based multi-level planning: allocating central grants to the Greek local authorities. *European Journal of Operational Research, 87*(3), pp. 535-550.

Beasley, J. E. (2003). Allocating fixed costs and resources via data envelopment analysis. *European Journal of Operational Research, 147*(1), pp. 198-216.

Charnes, A., & Cooper, W. W. (1962). Programming with linear fractional functionals. *Naval Research logistics quarterly, 9*(3- 4), pp. 181-186.

Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research, 2*(6), pp. 429-444.

Chen, M.-C., Yu, M.-M., & Ho, Y.-T. (2018). Using network centralized data envelopment analysis for shipping line resource allocation. *International journal of environmental science and technology, 15*(8), pp. 1777-1792.

Chen, Y., Du, J., Sherman, H. D., & Zhu, J. (2010). DEA model with shared resources and efficiency decomposition. *European Journal of Operational Research, 207*(1), pp. 339-349.

Chen, Y., Liang, L., Yang, F., & Zhu, J. (2006). Evaluation of information technology investment: a data envelopment analysis approach. *Computers & Operations Research, 33*(5), pp. 1368-1379.

Contreras, I., & Lozano, S. (2020). Allocating additional resources to public universities. A DEA bargaining approach. *Socio-economic planning sciences, 71*, p 100752.

Emrouznejad, A., & Yang, G.-l. (2018). A survey and analysis of the first 40 years of scholarly literature in DEA: 1978–2016. *Socio-economic planning sciences, 61*, pp. 4-8.

Fang, L. (2013). A generalized DEA model for centralized resource allocation. *European Journal of Operational Research, 228*(2), pp. 405-412.

Kao, C., & Hwang, S.-N. (2008). Efficiency decomposition in two-stage data envelopment analysis: An application to non-life insurance companies in Taiwan. *European Journal of Operational Research, 185*(1), pp. 418-429.

Kao, C., & Hwang, S.-N. (2010). Efficiency measurement for network systems: IT impact on firm performance. *Decision Support Systems, 48*(3), pp. 437-446.

Khatibi, M., & Rahmani Parchikolaei, B. (2018). Reallocation of the Organization Forces by the Super Effective Model Despite the Limitations on Number and the Way of Transferring the Forces of Some Units. *Iranian Journal of Optimization, 10*(1), pp. 59-65.

Korhonen, P., & Syrjänen, M. (2004). Resource allocation based on efficiency analysis. *Management Science, 50*(8), pp. 1134-1144.

Lotfi, F. H., Noora, A. A., Jahanshahloo, G. R., Gerami, J., & Mozaffari, M. R. (2010). Centralized resource allocation for enhanced Russell models. *Journal of Computational and Applied Mathematics, 235*(1), pp. 1-10.

Lozano, S., & Villa, G. (2004). Centralized resource allocation using data envelopment analysis. *Journal of Productivity Analysis, 22*(1), pp. 143-161.

Lozano, S., & Villa, G. (2005). Centralized DEA models with the possibility of downsizing. *Journal of the Operational Research Society, 56*(4), pp. 357-364.

Mar-Molinero, C., Prior, D., Segovia, M.-M., & Portillo, F. (2014). On centralized resource utilization and its reallocation by using DEA. *Annals of operations Research, 221*(1), pp. 273-283.

Moghaddas, Z. (2019). Financial efficiency evaluation in a network with feedbacks. *Iranian Journal of Optimization, 11*(2), pp. 115-124.

Nemati, M., Kazemi Matin, R., & Toloo, M. (2020). A two-stage DEA model with partial impacts between inputs and outputs: application in refinery industries. *Annals of operations Research, 295*, pp. 285-312.

Nemati, M., & Matin, R. K. (2019). A data envelopment analysis approach for resource allocation with undesirable outputs: an application to home appliance production companies. *Sādhanā, 44*(1), pp. 1-13.

Toloo, M., Emrouznejad, A., & Moreno, P. (2017). A linear relational DEA model to evaluate two-stage processes with shared inputs. *Computational and Applied Mathematics, 36*(1), pp. 45-61.

Wu, J., Chu, J., An, Q., Sun, J., & Yin, P. (2018). Resource reallocation and target setting for improving environmental performance of DMUs: An application to regional highway transportation systems in China. *Transportation Research Part D: Transport and Environment, 61*, pp. 204-216.

Wu, J., Zhu, Q., Ji, X., Chu, J., & Liang, L. (2016). Two-stage network processes with shared resources and resources recovered from undesirable outputs. *European Journal of Operational Research, 251*(1), pp. 182-197.

Yadolladi, A. H., & Matin, R. K. (2021). Centralized resource allocation with the possibility of downsizing in two-stage network production systems. *RAIRO-Operations Research, 55*(4), pp. 2583-2598.

Yu, M.-M., & Chen, L.-H. (2016). Centralized resource allocation with emission resistance in a two-stage production system: Evidence from a Taiwan's container shipping company. *Transportation Research Part A: Policy and Practice, 94*, pp. 650-671.

Zha, Y., & Liang, L. (2010). Two-stage cooperation model with input freely distributed among the stages. *European Journal of Operational Research, 205*(2), pp. 332-338.