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Ranking of Decision-Making Units with Multi-Period Two-Stage Network Structure: Ratio data Envelopment Analysis Based Method

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Abstract

Data envelopment analysis is always considered as a non-parametric method for measuring the efficiency of a set of decision units. The efficiency number obtained from standard models is a criterion for comparing the performance of each decision unit with other units. Despite the many strengths of these models, one of their weaknesses is the lack of distinction between efficient units. Also, these models do not pay attention to the internal structure of the units and have a black box view. To solve these problems, relational data envelopment analysis models are used, which are much more cost-effective in terms of time and cost; But these models are static and do not take time into evaluation. In this paper, a method has been proposed for grading of the decision making units with multi- period two stages network structure using relative data envelopment analysis. Three different perspective are introduced for assessment of efficiency in time periods via relative data envelopment analysis. Proportionate with each perspective, an efficiency number is obtained for any decision making unit. Then three efficiency numbers obtained in the mentioned method is combined with Shannon entropy method and a total efficiency criterion is defined for each unit. Finally, this measure is considered as the main indicator for the units grading. The results of implementation of the mentioned algorithm on the real example and comparison with the similar methods clarify the strength of this algorithm.

Keywords:
ratio data envelopment analysis
network
multi-period
ranking
efficiency

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INTRODUCTION

Nowadays, many organizations use data envelopment analysis (DEA) for evaluating the efficiency of decision making units (DMUs). Standard data envelopment analysis models assess the efficiency of homogeneous units compared to other units by considering multiple inputs and outputs. The first introduced models are Charnes, Cooper and Rhodes (CCR) and Banker, Charnes and Cooper (BCC). Standard models consider units as black boxes and take into account only their inputs and outputs in the evaluation and from this perspective, the units under evaluation are divided into two categories of efficient (units with efficiency number one) and inefficient. However, since we deal with the units that are not exactly available and they are achievable as ratio data in most real-world problems, ratio data envelopment analysis (DEA-R) models can be the best alternative to standard models. Ratio data envelopment analysis models were first introduced by Despic and Paradi (2007). The ratio data envelopment analysis model uses all possible ratios between outputs and inputs to calculate the relative efficiency, which can be considered as outputs in standard models. Converting a set of outputs and inputs to a set of ratios is an advantage where one or more outputs are used to generate one or more output not all of them. Wei et al. (2011a) compared efficiency scores on DEA and DEA-R and showed that the input based DEA-R efficiency score is always greater than or equal to the input based CCR efficiency score and also indicated that the origin of this difference in efficiency score is the assumption of weight limitation. The theory of ratio data envelopment analysis models has been extended in Wei et al (2011b, 2011c) papers in new directions. In these articles, the authors have focused on the relationships between standard DEA and DEA-R models and used the DEA-R models for analyzing the efficiency of 21 medical centers in Taiwan. Mozaffari et al. (2014) calculated cost and revenue efficiency using DEA-R models and compared the results with the DEA model. Ostovan et al. (2020) proposed fuzzy DEA and DEA-R models for evaluating the efficiency of a two-stage network. In the studied example of this paper, the authors considered airlines as two-stage network and when ratio data were available, DEA-R models were used to evaluate efficiency. By access to ratio data in 10 bank branches, using their proposed method, Mozaffari et al. (2020) calculated the super-efficient levels for these branches. Hosseinzadeh Lotfi et al. (2020) used R codes to solve DEA models with definite and fuzzy data. R is a mathematical and subject-oriented programming language designed primarily for statistical calculations and data mining. The R programming language covers a wide range of linear and nonlinear programming, integer and quadratic models as well as statistical tests and time series analysis and has a high graphical capability. Moghaddas et al. (2022) developed combined scale returns of DEA models in the presence of integer input and output data. They corrected the previous topic principles to introduce a minimal set of technical extrapolations and also formulated a pair of correct and incorrect linear programming models to assess the efficiency. Moghaddas et al. (2020) evaluated revenue efficiency according to the piece linear theory in non-competitive situations. In doing so, they introduced a step-by-step pricing function that allows prices to change relative to the output. As a new idea, they proposed a more accurate mathematical modeling for revenue efficiency and defined a dynamic weight function in the maximum revenue optimization model that no longer takes into account the fixed prices. Moghaddas et al. (2021) proposed an assessment method based on the network data proposed envelopment analysis to provide an efficient strategy for each step of a sustainable supply chain network. Their approach offers a robust design with decision making units to avoid imposing additional costs on supply chains due to non-compliance with environmental and social issues. For doing so, they considered the inputs and outputs related to the concept of sustainability in the DEA network to select the most efficient strategy for sustainable supply chain design. The output of their proposed method enables decision making units to select the appropriate strategy for each stage of the sustainable supply chain network and maximize the efficiency of the entire network.

However, one of the most challenging issues in the discussion of data envelopment analysis is the issue of ranking decision making units. In this sense that standard data envelopment analysis models do not distinguish between efficient units and units with an efficiency number of one. Among the most well-known ranking methods, AP (Anderson-Peterson) methods, super-efficiency methods and cross-efficiency method can be mentioned. Many researchers have proposed different methods for ranking of the decision making units from different perspectives. Among the new papers in this field it can be referred to the relative entropy method (Si and Ma, 2019), Alder and Volta (2019), Kao and Liu (2020), Izadikhah and Saen(2019), Aparico et al. (2020), Blouri et al. (2020), Soleimani Chamkhorami et al., (2020). In the reviewed articles, the standard models of data envelopment analysis have been examined and developed and no ratio models have been mentioned in any of the reviewed articles. Due to the fact that in real world data, ratio models are considered more and because of their comparative advantage over standard data envelopment analysis models, ratio data envelopment analysis models have been studied in this article. In addition, in the studied ranking articles, two main characteristics (time and network structure) that affect the efficiency of systems in the real world are not considered, so the advantage of this paper is to focus on network structures with ratio data with time element. The main goal is ranking of the units with network structure and in different time periods using ratio data envelopment analysis. The structure of the network system discussed in this article is a system with a parallel structure consisting of several subsystems and each subsystem consists of two stages that are connected in series. The main contribution of this paper is to study this structure in different time periods. In order to evaluate the efficiency of the system with network structure and several periods using three different methods, the efficiency of the whole system is evaluated and finally these three methods are combined with Shannon entropy weighting and ranked based on the obtained combined efficiency number. The paper is organized as follows: the ratio data envelopment analysis is introduced briefly in a two-stage network and the Shannon entropy algorithm. In the third section, the proposed method for evaluating the efficiency of multi-period two-stage systems is introduced. In the fourth section, the proposed method is implemented and analyzed on a real example. The final section of the paper includes conclusion.

PRELIMINARIES

This section briefly introduces the models of ratio data envelopment analysis in the two-step structure and Shannon's entropy algorithm.

Ratio data envelopment analysis in a two-stage network

Suppose $J = \{1,2,3,\dots,n\}$، is a definite set with n decision making units with two steps structure. For the selected decision making unit $j\epsilon J$, the first stage input vector is defined as$\mathrm{X}_j^1 = \{{x_{ij}}^1 \geq 0,$ $1 \leq i \leq m\}$ for producing the final output of the first stage or $Y_j^1 = \{y_{rj}^1 \geq 0, r = 1, \dots, s\}$ and the middle vector $Z_j^1 = \{z_{dj}^1 \geq 0, d = 1, \dots, p\}$. The second stage begins using middle vector $Z_j^1 = \{z_{dj}^1 \geq 0,\ d = 1, \dots, p\}$ and the second stage external input vector $X_j^2 = \{x_{ij}{}^2 \geq 0,\ 1 \leq i \leq l\}$ produces the final output of $Y_j^2 = \{y_{rj}^2 \geq 0, r = 1, \dots, f\}$. Since the two-stage network structure are connected in serial (Fig.1), so the non-negative outputs $Y_j^1 = (y_{rj}^1)$ and $Y_j^2 = (y_{rj}^2)$ are final outputs of the two-stage system.

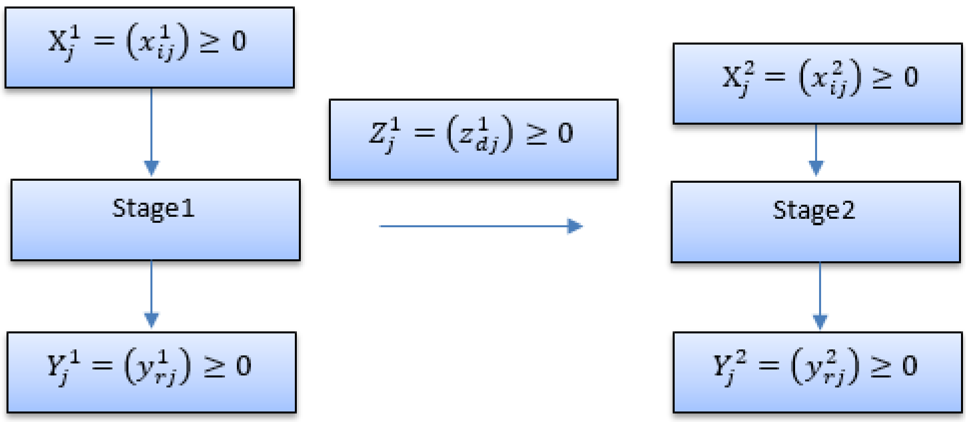

Fig. 1. Two-stage network

To evaluate the efficiency of the two-stage system mentioned in Figure (1), Kamyab et al. (2018) proposed the following model based on the advantages of ratio data envelopment analysis models:

$$\varphi_o = \min \sum_{p=1}^{2} \varphi_p$$

$$s.t. \sum_{j=1}^{n} \lambda_j^1 \left(\frac{x_{ij}^1}{z_{dj}^1}\right) \leq \varphi_1 \left(\frac{x_{io}^1}{z_{do}^1}\right), \quad i = 1, \dots, m; d = 1, \dots, p \tag{1}$$

$$\sum_{j=1}^{n} \lambda_j^1 \left(\frac{x_{ij}^1}{y_{rj}^1}\right) \leq \varphi_1 \left(\frac{x_{io}^1}{y_{ro}^1}\right) \quad i = 1, \dots, m;\ r = 1, \dots, s$$

$$\sum_{j=1}^{n} \lambda_j^2 \left(\frac{z_{dj}^1}{y_{rj}^2}\right) \leq \varphi_2 \left(\frac{z_{dj}^1}{y_{ro}^2}\right) \quad i = 1, \dots, l; d = 1, \dots, p$$

$$\sum_{j=1}^{n} \lambda_j^2 \left(\frac{x_{ij}^2}{y_{rj}^2}\right) \leq \varphi_2 \left(\frac{x_{io}^2}{y_{ro}^2}\right) \quad i = 1, \dots, l; r = 1, \dots, f$$

$$\sum_{j=1}^{n} \lambda_j^k = p_k, 1 \leq k \leq 2$$

$$\sum_{k=1}^{2} p_k = 1$$

$$\lambda_j^k \geq 0 \ \ \forall 1 \leq j \leq n;\ 1 \leq k \leq 2$$

Model (1) is an input-based linear programming problem based on ratio data. As can be seen from the limitations of the model, all data are used proportionally in the model. The first constraint of radial reduction (φ_1) calculates the ratio of the value of the first stage inputs $x_{ij}^1$to the value of the intermediate dimensionsz_{dj}^1. The second constraint calculates the radial reduction of the ratio of the value of the first stage inputs x_{ij}^1 to the final outputsy_{rj}^1. For the second step, the third and fourth constraints are written and the contraction value is displayed withφ_2 in the above constraints. The variables λ_j^1 and λ_j^2 correspond to the first and second stages, respectively. If $\sum_{j=1}^{n} \lambda_j^2 = 0$then only step 1 of the network is considered and the overall efficiency of the network is the same as the efficiency of step 1. Similarly, if $\sum_{j=1}^{n} \lambda_j^1 = 0$then only step 2 is considered and the overall efficiency of the network is the same as the efficiency of step 2. If a$\sum_{j=1}^{n} \lambda_j^1 = p_1$and $\sum_{j=1}^{n} \lambda_j^2 = p_2$ where $p_1 + p_2 = 1$, $p_1, p_2 > 0$ then optimal solution of model (1) defines the overall efficiency of the unit under evaluation. Problem (1) is always possible, because assuming that $\varphi_o{}^*$ is the optimal value of the objective function of model (1), then $\lambda_j^k = 0\,, 1 \le j \le n, j \neq o$ and $\lambda_o^k = 1$ is a possible answer. In addition, it follows that the optimal value does not exceed one and is always greater than zero.

SHANON ENTROPY ALGORITHM

Entropy as the meaning of disorder, was first coined in 1865 by Rudolph Colossius in the field of thermodynamics and in 1948 by Claude Shannon in the field of information and communication. The Shannon algorithm is defined in six steps:

Step 1: Form the Shannon entropy matrix. First, Shannon's entropy matrix (this matrix is actually the decision matrix in the multi-criteria decision method) is formed, which in the rows of those units and in its columns are the efficiencies obtained from three methods θ_k^1 (efficiency in each period independently), θ_k^2 (efficiency using hybrid DMUs) and θ_k^3 (efficiency using comprehensive production capability set). Therefore, the above matrix will be $A = [e_{kl}]$, which is a matrix $n * 3$, and $e_{kl} = \theta_k^l, l = 1,2,3\,, k = 1, \dots, n$ ($k = 1, \dots, n$) is the number of DMUs.

Step 2: Normalizing the matrix.

Normalizes the above matrix by dividing the value of each column by the sum of the values in that column, and each normalized value is denoted by P_{kl} in other words:

$$P_{kl} = \frac{e_{kl}}{\sum_{k=1}^{n} e_{kl}}, \forall k, l$$

Step 3: Calculating the entropy value. In this step, using the following formula, the entropy of $E_l$is calculated by the values ofP_{kl}, which according to its definition has a value between zero and one.

$$E_l = -\frac{1}{Ln3} \sum_{k} P_{kl}\, Ln\, P_{kl} \ \ , l = 1,2,3$$

Step 4: Calculating d_l (deviation value). The values of $d_l$are calculated using the following formula, which states how much useful information each indicator provides for decision making. Therefore, the role of that indicator in decision making should be reduced equally.

$d_l = l - E_l, l = 1,2,3$

Step 5: Calculating the weight$\boldsymbol{w_l}$. In this step, the weight of $\boldsymbol{w_l}$is calculated using the following formula:

$$w_l = \frac{d_l}{\sum_{l=1}^{3} d_l} \ \ , l = 1,2,3$$

Step 6: Obtaining the combined efficiency ($\boldsymbol{\tau_k}$). In this step, using the weights obtained for the first, second and third methods in the previous step, the "combined efficiency" for eachDMU_k is obtained as follows, which can be used to rank decision units based on their efficiency in periods.

$$\tau_k = \sum_{l=1}^{3} w_l \theta_k^l \ \ , k = 1,2,3, \dots, n$$

THE PROPOSED METHOD

Efficiency evaluation

In this section, using ratio data envelopment analysis, the efficiency of multi-period two-stage systems is evaluated. In other words, the goal is to involve the time element in evaluating efficiency in a two-stage network structure. Suppose again that J = {1, 2, 3, n}, the set index *n* of the decision unit and each unit has a two-stage network structure at times$(t = 1, \dots, T)$. For the unit under evaluation $k \epsilon J$, in the first step the non-negative input uses $X_j^{1t} = \{x_{ij}{}^{1t},\ 1 \le i \le m_1\}$at time t (1≤t≤T) to produce the negative intermediate product $Z_j{}^{1t} = \{z_{dj}{}^{1t},\ 1 \le d \le p\}$and non-negative final output of the first stage or $Y_j^{1t} = \{y_{rj}^{1t}, r = 1, \dots, s\}$In the second stage, using the middle vector$Z_j^{1t} = \{z_{dj}^{1t}, d = 1, \dots, p\}$, which is also one of the outputs of the first stage, and non-negative input vector $X_j{}^{2t} = \{x_{ij}{}^{2t},\ 1 \le i \le m_2\}$produces a non-negative final output $Y_j^{2t} = \{y_{rj}^{2t},, r = 1, \dots f\}$. Fig. 2 shows the structure of the two stages in independent time.

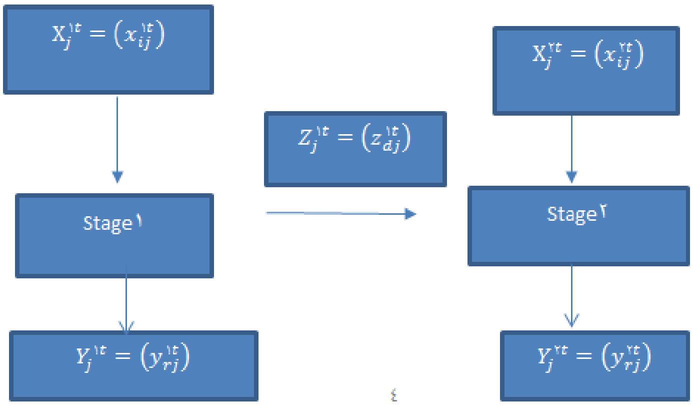

Fig. 2. Two-stage network structure at time $(\mathbf{1} \le \boldsymbol{t} \le \boldsymbol{T})$

Without disturbing the whole argument, suppose that for the decision unit j that $j = 1, \dots, n$ is available with data $\left(\frac{x_{ij}{}^{1t}}{z_{dj}{}^{1t}}, 1 \le i \le m_1, 1 \le d \le p\right)$ ، $\left(\frac{z_{dj}{}^{1t}}{y_{rj}{}^{2t}}, 1 \le d \le p, 1 \le r \le f\right)$ ،$\left(\frac{x_{ij}{}^{2t}}{y_{rj}{}^{2t}}, 1 \le i \le l, 1 \le r \le f\right)$ $\frac{x_{ij}{}^{1t}}{y_{rj}{}^{1t}}, 1 \le i \le m_2, 1 \le r \le s$ at time $t = 1, \dots, T$. Due to the availability of ratio data, using the models of ratio data envelopment analysis mentioned in the previous section, the efficiency of the two-stage systems presented in Figure (2) and in separate time periods $t = 1, \dots, T$ calculated by three separate methods:

Method 1: Calculating efficiency in each period independently

For units under evaluation that have a structure similar to Figure (2) and in time periods $t = 1, \dots, T$, ratio data are available. First, the efficiency of the unit under evaluation $DMU_k\,(k \in J)$ in each time period $t = 1, \dots, T$ is calculated independently using model (2).

$$\theta_k^t = Min\,\{\beta_1 + \beta_2\}$$

$$s.t.$$

$$\sum_{j=1}^{n} \lambda_j^{1t}\left(\frac{x_{ij}{}^{1t}}{z_{dj}{}^{1t}}\right) \le \beta_1\left(\frac{x_{ik}{}^{1t}}{Z_{dk}{}^{1t}}\right),\quad 1 \le i \le m_1,\quad 1 \le d \le p \tag{2}$$

$$\sum_{j=1}^{n} \lambda_j^{1t}\left(\frac{x_{ij}{}^{1t}}{y_{rj}{}^{1t}}\right) \le \beta_1\left(\frac{x_{ik}{}^{1t}}{y_{rk}{}^{1t}}\right),\quad 1 \le i \le m_1, 1 \le r \le s$$

$$\sum_{j=1}^{n} \lambda_j^{2t} \left(\frac{{x_{ij}}^{2t}}{{y_{rj}}^{2t}} \right) \leq \beta_2 \left(\frac{{x_{ik}}^{2t}}{{y_{rk}}^{2t}}\right), 1 \leq i \leq m_2, 1 \leq r \leq$$

$$\sum_{j=1}^{n} \lambda_j^{2t} \left(\frac{{z_{dj}}^{1t}}{{y_{rj}}^{2t}} \right) \leq \beta_2 \left(\frac{{z_{dk}}^{1t}}{{y_{rk}}^{2t}}\right), 1 \leq r \leq f, 1 \leq d \leq p$$

$$\sum_{j=1}^{n} \lambda_j^{1t} = K_1, \quad \sum_{j=1}^{n} \lambda_j^{2t} = K_2$$

$$K_1 + K_2 = 1, K_1 \geq 0, K_2 \geq 0$$

$$\lambda_j^{1t} \geq 0, \lambda_j^{2t} \geq 0 \ , \ j = 1, \dots, n$$

Based on the logic of model (1), β1 and β2 are the same values of the functions of steps 1 and 2, respectively. Model (2) is possible because assuming that $\theta_k^{t*}$is the optimal value of the objective function of model (2), then$\lambda_j^{kt} = 0, 1 \leq j \leq n, j \neq k$ and $\lambda_k^{kt} = 1$is possible answer. In addition, it follows that the optimal value does not exceed one and is always greater than zero. In this step, the efficiency defined for eachDMU_k‘$(k = 1, \dots, n)$ after T time period is equal to:

$$\theta_k^1 = \frac{1}{T} \sum_{t=1}^{T} \theta_k^t \tag{3}$$

In other words, first the efficiency of each unit in each time period t is calculated and then the average of the efficiencies is considered as the efficiency of the whole decision-making unit in a time period with length T.

$$\theta_k^t = Min \{\beta_1 + \beta_2\} \tag{2}$$

s.t.

$$\sum_{j=1}^{n} \lambda_j^{1t} \left(\frac{{x_{ij}}^{1t}}{{z_{dj}}^{1t}} \right) \leq \beta_1 \left(\frac{{x_{ik}}^{1t}}{{Z_{dk}}^{1t}}\right), 1 \leq i \leq m_1, 1 \leq d \leq p$$

$$\sum_{j=1}^{n} \lambda_j^{1t} \left(\frac{{x_{ij}}^{1t}}{{y_{rj}}^{1t}} \right) \leq \beta_1 \left(\frac{{x_{ik}}^{1t}}{{y_{rk}}^{1t}}\right), 1 \leq i \leq m_1, 1 \leq r \leq s$$

$$\sum_{j=1}^{n} \lambda_j^{2t} \left(\frac{{x_{ij}}^{2t}}{{y_{rj}}^{2t}} \right) \leq \beta_2 \left(\frac{{x_{ik}}^{2t}}{{y_{rk}}^{2t}}\right), 1 \leq i \leq m_2, 1 \leq r \leq$$

$$\sum_{j=1}^{n} \lambda_j^{2t} \left(\frac{{z_{dj}}^{1t}}{{y_{rj}}^{2t}} \right) \leq \beta_2 \left(\frac{{z_{dk}}^{1t}}{{y_{rk}}^{2t}}\right), 1 \leq r \leq f, 1 \leq d \leq p$$

$$\sum_{j=1}^{n} \lambda_j^{1t} = K_1, \quad \sum_{j=1}^{n} \lambda_j^{2t} = K_2$$

$$K_1 + K_2 = 1, K_1 \geq 0, K_2 \geq 0$$

$$\lambda_j^{1t} \geq 0, \lambda_j^{2t} \geq 0 \ , \ j = 1, \dots, n$$

Based on the logic of model (1), β1 and β2 are the same values of the functions of steps 1 and 2, respectively. Model (2) is possible because assuming that $\theta_k^{t*}$is the optimal value of the objective function of model (2), then$\lambda_j^{kt} = 0, 1 \leq j \leq n, j \neq k$ and $\lambda_k^{kt} = 1$is possible answer. In addition, it follows that the optimal value does not exceed one and is always greater than zero. In this step, the efficiency defined for eachDMU_k‘$(k = 1, \dots, n)$ after T time period is equal to:

$$\theta_k^1 = \frac{1}{T} \sum_{t=1}^{T} \theta_k^t \tag{3}$$

In other words, first the efficiency of each unit in each time period t is calculated and then the average of the efficiencies is considered as the efficiency of the whole decision-making unit in a time period with length T.

Method 2: Calculating efficiency in a single time period

In this method, the sum of input and output for each DMU in different time periods $t = 1, \dots, T$ is considered as input and output. In other words, suppose our arbitrary decision unit j, or $t = 1, \dots, T$ in the first step is the non-negative inputs $\sum_{t=1}^{T} {x_{ij}}^{1t}$ at times 1 to T consumes to produce the nonnegative first output of the first $\sum_{t=1}^{T} {z_{dj}}^{1t}$ and the non-negative output of the first stage$\sum_{t=1}^{T} {y_{rj}}^{1t}$, also in the second step using the middle vector$\sum_{t=1}^{T} {z_{dj}}^{1t}$ and non-negative input vector $\sum_{t=1}^{T} {x_{ij}}^{2t}$ produces the final negative output$\sum_{t=1}^{T} {y_{rj}}^{2t}$.

Model (4) is used to evaluate the efficiency of the input-based nature:

$$
\begin{aligned}
&\theta_k^2 = Min\,\{\beta_1 + \beta_2\} \\
&s.t. \\
&\sum_{j=1}^{n} \lambda_j^1 \left(\sum_{t=1}^{T} \frac{{x_{ij}}^{1t}}{{z_{dj}}^{1t}} \right) \le \beta_1 \left(\sum_{t=1}^{T} \frac{{x_{ik}}^{1t}}{{Z_{dk}}^{1t}} \right), \\
&\qquad 1 \le i \le m, \quad 1 \le d \le p \\
&\sum_{j=1}^{n} \lambda_j^1 \left(\sum_{t=1}^{T} \frac{{x_{ij}}^{1t}}{{y_{rj}}^{1t}} \right) \le \beta_1 \left(\sum_{t=1}^{T} \frac{{x_{ik}}^{1t}}{{y_{rk}}^{1t}} \right), \\
&1 \le i \le m, \ 1 \le r \le s \\
&\sum_{j=1}^{n} \lambda_j^1 \left(\sum_{t=1}^{T} \frac{{x_{ij}}^{2t}}{{y_{rj}}^{2t}} \right) \le \beta_2 \left(\sum_{t=1}^{T} \frac{{x_{ik}}^{2t}}{{y_{rk}}^{2t}} \right), \\
&1 \le i \le l, \ 1 \le r \le f \\
&\sum_{j=1}^{n} \lambda_j^2 \left(\sum_{t=1}^{T} \frac{{z_{dj}}^{1t}}{{y_{rj}}^{2t}} \right) \le \\
&\beta_2 \left(\sum_{t=1}^{T} \frac{{z_{dk}}^{1t}}{{y_{rk}}^{2t}} \right), 1 \le r \le f, 1 \le d \le p \\
&\sum_{j=1}^{n} \lambda_j^1 = K_1, \quad \sum_{j=1}^{n} \lambda_j^2 = K_2 \\
&K_1 + K_2 = 1, K_1 \ge 0, K_2 \ge 0 \\
&\lambda_j^1 \ge 0, \lambda_j^2 \ge 0 \ , j = 1, \dots, n
\end{aligned} \tag{4}
$$

Similar to model (3), β1 and β2 are the values of the efficiencies of steps 1 and 2, respectively. On the other hand, model (4) is always possible because assuming that θ_k^{2*} is the optimal value of the objective function of model (4), then$\lambda_j^k = 0\,, 1 \le j \le n, j \ne o$ and$\lambda_o^k = 1$ is a possible answer. In addition, it follows that the optimal value does not exceed one and is always greater than zero. In model (4), in fact, the decision unit in different time periods is considered as a single time period.

Method 3: Comprehensive efficiency calculation

In this method, each decision unit is considered as an independent decision unit at any time: so in this case, the total number of decision units will be equal to $N = nT$. Model (5) is used to evaluate the efficiency of $DMU_k (k \in J)$ in the input-based state:

$$
\begin{aligned}
&\theta_k^3 = Min\,\{\beta_1 + \beta_2\} \\
&s.t. \ \sum_{j=1}^{n} \lambda_j^1 \left(\frac{{x_{ij}}^{1t}}{{z_{dj}}^{1t}} \right) \le \beta_1 \left(\frac{{x_{ik}}^{1t}}{{Z_{dk}}^{1t}} \right), \ 1 \le \\
&i \le m_1, \ 1 \le d \le p, 1 \le t \le T \\
&\sum_{j=1}^{n} \lambda_j^1 \left(\frac{{x_{ij}}^{1t}}{{y_{rj}}^{1t}} \right) \le \beta_1 \left(\frac{{x_{ik}}^{1t}}{{y_{rk}}^{1t}} \right), \ 1 \le i \le \\
&m_1, 1 \le r \le s, 1 \le t \le T \\
&\sum_{j=1}^{n} \lambda_j^2 \left(\frac{{x_{ij}}^{2t}}{{y_{rj}}^{2t}} \right) \le \beta_2 \left(\frac{{x_{ik}}^{2t}}{{y_{rk}}^{2t}} \right), 1 \le i \le \\
&m_2, 1 \le r \le f, 1 \le t \le T \\
&\sum_{j=1}^{n} \lambda_j^2 \left(\frac{{z_{dj}}^{1t}}{{y_{rj}}^{2t}} \right) \le \beta_2 \left(\frac{{z_{dk}}^{1t}}{{y_{rk}}^{2t}} \right), 1 \le r \le \\
&f, 1 \le d \le p, 1 \le t \le T \\
&\sum_{j=1}^{n} \lambda_j^1 = K_1, \quad \sum_{j=1}^{n} \lambda_j^2 = K_2, \\
&K_1 + K_2 = 1, K_1 \ge 0, K_2 \ge 0
\end{aligned} \tag{5}
$$

$\lambda_j^1 \geq 0, \lambda_j^2 \geq 0 \quad j = 1, \dots, n$

In this model, β1 and β2 are the values of the efficiencies of stages 1 and 2, respectively. On the other hand, model (5) is always possible because assuming that θ_k^{3*} is the optimal value of the objective function of model (5), the optimal value does not exceed one. And is always greater than zero.

Ranking of units

Using the three models (2), (4) and (5) mentioned in the previous section, the three efficiency values resulting from the three methods are combined using the Shannon entropy method and a general efficiency criterion for each unit is defined. The steps of Shannon algorithm will be as follows:

Step 1: First, the Shannon entropy matrix is formed, in the rows of which are decision units and in its columns are the efficiencies obtained from three methods θ_k^1 ، θ_k^2 and θ_k^3. Therefore, the above matrix is $A = [e_{kl}]$ that is $n * 3$matrix and $e_{kl} = \theta_k^l, l = 1,2,3 , k = 1, \dots, n$ where $k = 1, \dots, n$ is the number of DMUs.

Step 2: Normalizing the matrix. Normalizes the above matrix by dividing the value of each column by the sum of the values in that column, and each normalized value is denoted byP_{kl}in other words

$$P_{kl} = \frac{e_{kl}}{\sum_{k=1}^{n} e_{kl}}, \forall k, l$$

Step 3: Calculating the entropy value

In this step, the entropy E_l is calculated by the values of P_{kl}using the following formula.

$$E_l = -\frac{1}{Ln3} \sum_k P_{kl}\, Ln\, P_{kl} \;\; , l = 1,2,3$$

Step 4: Calculating d_l(deviation value).

The values of $d_l$are calculated using the following formula

$$d_l = l - E_l, l = 1,2,3$$

Step 5: Calculating the weight w_l.

In this step, the weight of w_l is calculated using the following formula:

$$w_l = \frac{d_l}{\sum_{l=1}^{3} d_l} \;\; , l = 1,2,3$$

Step 6: Obtaining the combined efficiency ($\boldsymbol{\tau_k}$).

In this step, using the weights obtained for the first, second and third methods in the previous step, the "combined efficiency" for each unit is obtained as follows, which can be used to rank decision-making units based on their efficiency in different time periods.

$$\tau_k = \sum_{l=1}^{3} w_l \theta_k^l \;\; , k = 1,2,3, \dots, n$$

NUMERICAL EXAMPLE

In this section, the algorithm proposed in the previous section is implemented on a real example. This example is taken from the paper of Tohinia and Tohidi (2019). The number of decision making units in the example is ten units, each unit has a two-stage structure and in three time periods, input and output data are collected. Each unit under evaluation has three inputs in the first stage, which are represented byx_1^t،x_2^t ،x_3^t respectively. The two intermediate products, which are in fact the output of the first stage and the input of the second stage, are introduced with the vectorsz_1^t،z_2^t. And the three final outputs or the output of the second stage are displayed withy_1^t،y_2^t،y_3^t. The mean and standard deviation of input, middle and output data of decision units in three time periods are given in Tables 1, 2 and 3.

Table 1: Mean and standard deviation of the data of the first time period

	A	B	C	D	E	F	G	H	I	J
Mean	7.2500	6.6250	8.0000	8.7500	7.3750	7.7500	7.3750	6.8750	5.5000	7.7500
Std.Deviation	4.83292	3.37797	5.12696	5.33854	3.73927	4.74342	3.37797	2.85044	3.16228	4.33425

Table 2: Mean and standard deviation of the data of the second time period

	A	B	C	D	E	F	G	H	I	J
Mean	8.3375	9.0000	11.0375	12.0125	9.6500	9.6375	9.4375	8.9125	7.5125	10.0500
Std.Deviation	4.97822	4.79434	8.03473	7.82221	5.13893	5.65229	4.47786	3.97076	4.61006	5.75872

Table 3: Mean and standard deviation of the data of the third time period

	A	B	C	D	E	F	G	H	I	J
Mean	9.9862	13.5000	16.5562	20.3588	14.4562	12.5938	12.5125	12.4375	10.3562	12.4857
Std. Deviation	5.39055	7.19151	1.20521	1.21523	7.24527	6.56552	5.78111	4.58661	6.03806	7.02197

Based on the data of three time periods, three efficiency values are calculated using the proposed algorithm. First, using model (2), we calculate the efficiency in three time periods separately and then we obtain the average of these efficiencies for each decision unit. In the second method, the data of all three periods are collected together and an overall efficiency is obtained for each unit (Model 4). In the third method, each time period is considered as a unit under evaluation (Model 5). In the example, according to the number of units (10 units) and three time periods, we will have a total of thirty units. Using the three efficiency values obtained, the results are shown in the second to fourth columns of Table (4). The results of combining these three efficiency values using the Shannon algorithm are shown in the fifth column. For more on the Shannon algorithm, consider unit number six. The value of the combined efficiency τ_k for the sixth unit is obtained by following the steps of the Shannon algorithm: In the first step, we form the Shannon entropy matrix, which is

A	0.831176	0.835179	0.875000
B	0.717573	0.695821	0.848747
C	0.969017	1.000000	1.000000
D	0.868265	0.844873	0.961538
E	0.833001	0.836386	0.852484
F	0.650835	0.650679	0.684649
G	0.768127	0.775510	0.849563
H	0.917949	0.949972	1.000000
I	0.749760	0.752464	0.791398
J	0.939394	0.973925	1.000000

That is a matrix of 10×3, in the sixth row and its first column the efficiency obtained in the first method for this unit is 0.650835, in the sixth row and its second column the efficiency obtained in the second method for this unit is 0.650679 and in the sixth row and the third column of this matrix are the efficiency obtained by the third method, 0.684649. In the second step, with the method mentioned, we normalize the matrix of the first step, which is the normalized matrix as follows

A	0.100808	0.100444	0.098721
B	0.087030	0.083684	0.095759
C	0.117526	0.120267	0.112824
D	0.105306	0.101610	0.108484
E	0.101029	0.100589	0.096180
F	0.078936	0.078255	0.077244
G	0.093161	0.093268	0.095851
H	0.111332	0.114250	0.112824
I	0.090934	0.0904960	0.089288
J	0.113933	0.117131	0.112824

For example, for unit number six in row six and the first column, the normalized matrix will be P_{61} = 0.078936, in row six and the second column, the normalized matrix will be P_{62} = 0.078255, and in row six and column three, P_{63} = 0.077244. After the third, fourth and fifth steps by calculating the values ofE_l , d_l and $w_l$for l = 1, 2, 3, respectively as follows

	$\boldsymbol{E_l}$	$\boldsymbol{d_l}$	$\boldsymbol{w_l}$
$l = 1$	2.08945	1.08945	0.384908
$l = 2$	1.855867	0.855867	0,3.2382
$l = 3$	1.885098	0.885098	0.31271

We calculate the combined efficiency of number six $\tau_6 = \sum_{l=1}^{3} w_l \theta_6^l$which is equal to 0.661363. This algorithm also applies to other units. Table (4) shows the results of the implementation of models (2), (4) and (5). Also, the combined efficiency numbers are shown in the fifth column of Table (4).

Table 4: Efficiencies obtained by three methods, combined efficiency, mean and standard deviation and rating and the results of Tohidnia and Tohidi model (2019)

DMUS	$\boldsymbol{\theta_k^1 = \frac{1}{T}\sum_{t=1}^{T}\theta_k^t}$	$\boldsymbol{\theta_k^2}$	$\boldsymbol{\theta_k^3}$	$\boldsymbol{\tau_k}$	rank	Tohidi (2019)	rank
A	0.831176	0.835179	0.875000	0.84609	5	0.571	3
B	0.717573	0.695821	0.848747	0.752016	9	0.6567	1
C	0.969017	1.000000	1.000000	0.988074	1	0.3545	9
D	0.868265	0.844873	0.961538	0.890359	4	0.356	8
E	0.833001	0.836386	0.852484	0.840117	6	0.425	7
F	0.650835	0.650679	0.684649	0.661363	10	0.4305	5
G	0.768127	0.775510	0.849563	0.79582	7	0.5195	4
H	0.917949	0.949972	1.000000	0.95329	3	0.6135	2
I	0.749760	0.752464	0.791398	0.763599	8	0.42825	6
J	0.939394	0.973925	1.000000	0.9687	2	0.571	3
Mean	0.843104	0.863742	0.837583	0.832089	------	0.475	
Standard Deviation	0.100502	0.099068	0.11093	0.097574		0.104571	

As mentioned, units can be ranked using the combined efficiency value orτ_k. According to the obtained numbers, unit C has the first rank and unit F has the last rank among these ten units. These results are shown in the sixth column of Table (4). The last two columns of the table show the results of the calculations of Tohidi and Tohidnia model (2019) on the data set. As it is obvious, the proposed algorithm makes a greater distinction between decision making units. But in Tohidi and Tohidnia model (2019), the first and last two units have the same efficiency numbers and therefore the model cannot distinguish much between these two units. But in the proposed algorithm, the distinction between the units according to the efficiency number obtained is quite clear. The last two rows of the table show the mean and standard deviation of the resulting efficiency. The highest mean value of 0.863742 is related to the second method in the proposed algorithm, which according to the logic of the model is quite obvious. The lowest average value of 0.475 is related to Tohidi and Tohidnia method (2019). The standard deviation of the third method of the proposed algorithm or the comprehensive efficiency of 0.11093 is the highest among the proposed models and the lowest standard deviation of the combined efficiency is 0.097574.

CONCLUSION

Despite their many advantages in evaluating the efficiency of decision making units, standard data envelopment analysis models have weaknesses. They ignore the internal structure of the units and use accurate input/output data. Due to the fact that in the real world, real data is not available and sometimes ratio data is available, ratio data envelopment analysis models provide a more realistic evaluation than standard models. Another point that is not considered in the evaluation with standard models is the non-involvement of the time element in the evaluation. In this paper, based on the structure of ratio data envelopment analysis, systems with multi-period two stage structure were evaluated and a method for ranking these multi-period two stage systems was proposed. The proposed algorithm evaluates the efficiency in different time periods with three different perspectives and corresponding to each method, an efficiency number is obtained. Then, the three efficiency values resulting from the three methods are combined using Shannon entropy method and a general efficiency criterion is defined for each unit. This criterion is finally considered as the main indicator for ranking the units. Comparison of this method with similar methods reveals the advantages of this method. Due to the uncertainty of the data (when the data is fuzzy) in the real world, it is suggested that data ranking in this case be considered in future research.

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