Optimization
Iranian Journal of Optimization
Volume 13, Issue 4, 2021, 293-304
Research Paper

Online version is available on: www.ijo.rasht.iau.ir

Islamic Azad University
Rasht Branch
ISSN: 2588-5723
E-ISSN:2008-5427

A Branch and Bound for single machine stochastic batch scheduling with delivery costs. A Chance Constraint approach (Case study in Iran)

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Revise Date 12 March 2022
Accept Date: 16 April 2022

Abstract

This paper study a single machine stochastic scheduling problem based on an Iranian real case study in Saipa company in which the objective is to minimize total completion times and delivery costs. According to existence data sets of this company the processing times follows a Normal distribution and based on the managerial decisions two objectives have to be achieved simultaneously including total completion times and controlling tardiness values so that their values are lower than an specified penalty. So, a Chance Constraint Programming (CCP) approach is employed and a mathematical model is presented. In order to solve the problem a Branch and Bound (B&B) method is used to solve the problem optimally and a Particle Swarm Optimization (PSO) metaheuristic is used to find near optimal solutions for large scales. The results show that using the proposed mathematical model and solution approach, could increase the effectiveness and efficiency of production line as 16 to 40 percent. Computational experiments validate the accuracy of proposed method.

Keywords:
Chance Constraint Programming
batch scheduling
Single machine
Branch and Bound

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INTRODUCTION

This paper tackles a problem of single machine batch scheduling so as to minimize the total tardiness and delivery costs where all of jobs are delivered to some customers in some batches. Each job might be sent to the customer just after its completion or be held to be sent with more jobs as a batch. Batching could accelerate the job process and decrease the delivery costs to customers. Tang et al. (2017) studied a two-agent scheduling problem with deteriorating jobs on a single machine when jobs are processed on machine within some batches. They considered two objective functions including minimizing makespan for an agent and number of tardy jobs for another agent. They considered some bounded and unbounded cases for the problem and proposed some optimal algorithms so as so solve them. Zhou et al. (2020) presented a single machine batch scheduling when the arrival times for jobs were dynamic and the time of use policy for electricity was considered for proposing a model with efficient energy consumption. Based on their research the mismatch between demand and supply is a challenging issue. In this regard a demand response approach was used for aligning the demand and supply of electricity. Rostami (2021) proposed a new problem in this field involving capacitated batch delivery and pickup systems. In this regard a comprehensive mathematical model was presented for production scheduling, delivering the products to multiple customers and picking up the end of life products in order to recycle and reused in manufacturing plants. Zhang et al. (2021) studied a generalized single machine batch scheduling problem with just in time consideration so as to minimize the earliness and tardiness costs simultaneously. They investigated some intrinsic properties of optimal solution and proposed a span limited tree search for achieving near optimal solutions. Zheng et al. (2021) studied the problem of single machine batch scheduling with dual setup times. They stated that setup time depends not only on batch size also on technological characteristics. They considered two types of setup costs including quantitative and technological inspired from autoclave modeling production. They presented a MILP model with objective of makespan minimization. Then a two stage approximation algorithm was established for solving the problem. Another related research in this field is Pei et al. (2017) in which the sequence setup time and learning effect were considered.

Single machine scheduling problem have a rich body of knowledge and could be classified by variant concepts. For instance, based on how the batches are processed on the machine, the problem would considered as serial or parallel batching. In serial batching, the processing time of each batch is equal to the completion time of the last job assigned to it. Ng et al. (2002 b) considered a single machine serial batch scheduling to minimize maximum lateness with precedence constraints between the batches and proposed an $o(n^2)$ polynomial time algorithm. They (2003 a) also considered the same problem with objective of total completion time, release date and precedence constraints and presented an $o(n^5)$ polynomial algorithm.

Yuan et al. (2007) studied a serial batch scheduling when each batch contained exactly k jobs and the objective was to minimize the total weighted completion time. They proved that the problem is NP-hard and presented a polynomial algorithm with some simplified rules. Another research is related to Ishii et al. (2010) where the size of each batch was considered as a fuzzy number and three objectives including makespan, maximum flow time and the satisfaction level were considered. A Lagrange relaxation was proposed for the problem that generated non-dominant solutions. Melouk et al. (2004) proposed a simulation annealing approach for minimizing the makespan with limited machine capacity and compared their results with CPLEX solver. Based on their comparison, SA overcame the results of CPLEX in all instances. Cheng and Kovalyov (2000) accomplished an extensive research in the field of regular cost functions in single machine batch scheduling and considered several objectives including number of late jobs, maximum lateness, total tardiness, total weighted completion time and total weighted tardiness and classified the complexity of them. Furthermore, a

dynamic programming approach was proposed for solving the objectives optimally.
On the other hand, the processing time of a parallel batch is calculated by the maximum processing time of jobs that are apparent to that batch. Nong et al. (2008 a) studied a single machine parallel batching problem to minimize makespan with consideration of family setup costs and release date. A polynomial approximation scheme was developed by some simplification rules and an algorithm with worst case ratio of 2.5 was proposed.

An extensive review in the field of parallel batching literature were studied by Potts and kovalyov (2000). In recent years, researchers have considered the delivery cost as a secondary objective called "combined optimization batch delivery problem" According to literature of scheduling, the problems those combine scheduling and delivery costs are complex and achieving global optimal solutions is impossible using traditional optimization methods (Mazdeh et al. (2011 a)). Mahdavi et al. (2011 a) studied the problem of minimizing the number of weighted tardy jobs with delivery cost and used a simulation annealing approach to solve the problem. Tian et al. (2007) studied the problem of minimizing the sum of total weighted flow times with delivery cost. They proved that the problem is NP-hard and presented an optimal algorithm. Mahdavi et al. (2011 c) presented a branch and bound approach to solve the problem of minimizing the weighted sum of flow times with delivery cost. Another research was related to Mahdavi et al. (2007 b) which presented a branch and bound to minimize flow time and delivery cost simultaneously.

In order to extend the results of researches for real industries, some researchers have employed the stochastic and uncertain programing in scheduling problems. Shen and Zhu (2018) studied a single machine with periodic maintenance activities when processing and repairing times were uncertain. The objective function was to minimize makespan. They proposed three uncertain programing models and solved them in polynomial run times. Zhou and Ning (2021) proposed and opportunistic maintenance scheduling model for a multi-unit scheduling problem with random production waits. The time for executing maintenance activities was considered as a stochastic parameter and the objective was to minimize maintenance costs. Aschauer et al. (2020) studied a problem with controllable stochastic processing times in a no-wait job shop environment. The problem was consisting of timetabling and sequencing and the objective function was considered as makespan and robustness optimization. Another research with uncertain processing times were related to Novak et al. (2019). They considered various processing times for each task based on its criticality so as to overcome the uncertainty and converted the stochastic model to a deterministic one. They show that the problem is NP-hard and an approximation algorithm and a branch-and-price decomposition approach were used to solve the problem.

In some researches, the arrival time or release date of each job is considered as a uncertain parameter. Hsing Chung et al. (2012) proposed a family-based scheduling rule so as to reduce capacity loss on a single machine with stochastic arrival times. Another important researches in the field of consideration of uncertainty in scheduling problems are those considered an integrated model of lot sizing and scheduling. Hu and Hu (2019) adopted a hybrid stochastic and robust optimization approach in such problem in which suppliers had the flexibility for satisfying a fraction of demand based on the market and their policies. They considered two types pf uncertainties simultaneously consisting of demand and over time processing costs and a sample average approximation technique was used to solve the problem. Another research for mentioned authors is Hu and Hu (2018) in which demand was considered as a stochastic parameter. The objective function was minimizing the overall system costs including production, setup, inventory and backlog costs.

In this paper a real case study in Saipa company is considered in which the the objective is to minimize the total completion time and delivery costs simultaneously. The real data sets are used for solving the problem based on the information from that company. Based on the existence statistical data from production line, the processing times of each jobs follows a Normal distribution. So a CCP approach is used for modeling the problem in uncertain environment. Afterwards, a B&B algorithm is employed so as to achieve the global optimal solutions and a particle swarm optimization meta heuristic is also presented to solve the problem in large scales. In order to check the verification of proposed method, some data sets of problem are solved optimally, using Lingo global solver and the results are compared with each other. The remaining parts of this paper are as follows: in section 2 the proposed model is presented and the variables and parameters are introduced. In section 3 the solution approach is offered and in section 4 the computational studies are presented. The main highlights of the current research could be mentioned as follows:

- Reviewing and modelling a real case study in Iran (Saipa company) so as to improving the effectiveness of production line
- Presenting a stochastic mathematical model based on the Chance Constraint Programming approach
- Proposing a B&B algorithm for solving the problem optimally

problem description and formulation

Saipa is one of the greatest companies in Iran for making vehicles that manufactures and montage several type of cars like Pride, Peugeot and etc. In this research the production line of Peugeot 206 have been studied and a bottleneck was identified in the place of a cutting machine. This machine processes n jobs with no preemption and all of the jobs are available for processing at time zero. The completed jobs can be delivered to the customer immediately after completion or be awaited for next jobs to be delivered as a batch. According to the number of jobs, N batches are considered and the jobs are assigned to the batches. Clearly, any batch that does not have any jobs will be omitted and then the sequence of batches is determined so that the proposed objective function is minimized. In order to study the problem in real application environments, the processing times are considered as stochastic values and they will be determined based on the existence statistical data arise from Saipa company. For doing this objective an Anderson-Darling statistical test is run so as to identify their probability distribution. This action also could be accomplished based on DEA approach which is presented in lotfi et al. (2020), Moghaddas et al. (2021), Lotfi f. (2013) and (2017)

The results are presented in fig. 1 and 2.

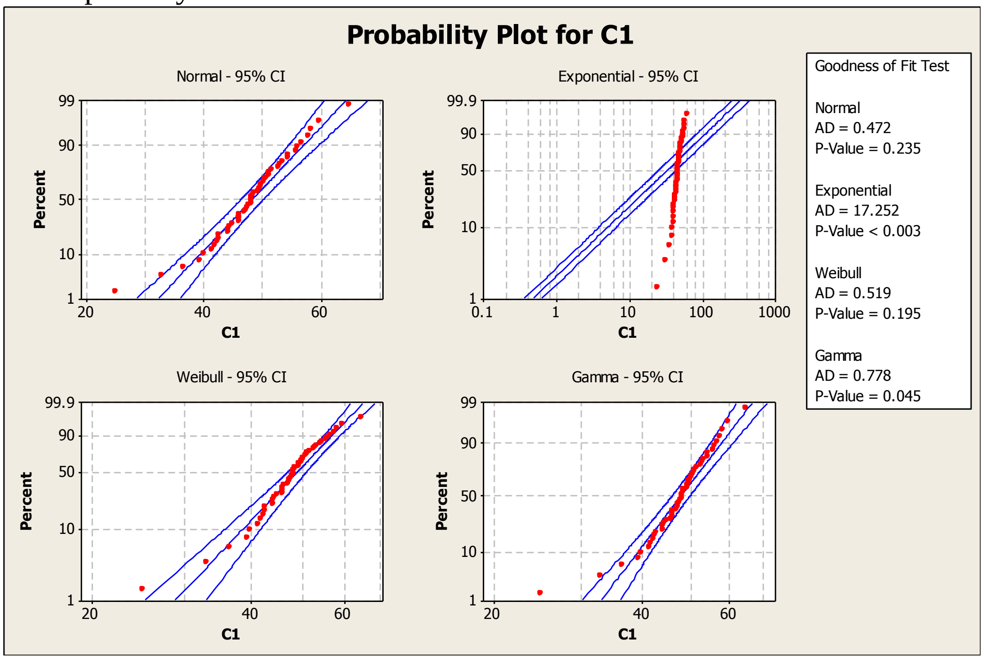

Fig.1. Anderson Darling output for distribution identification

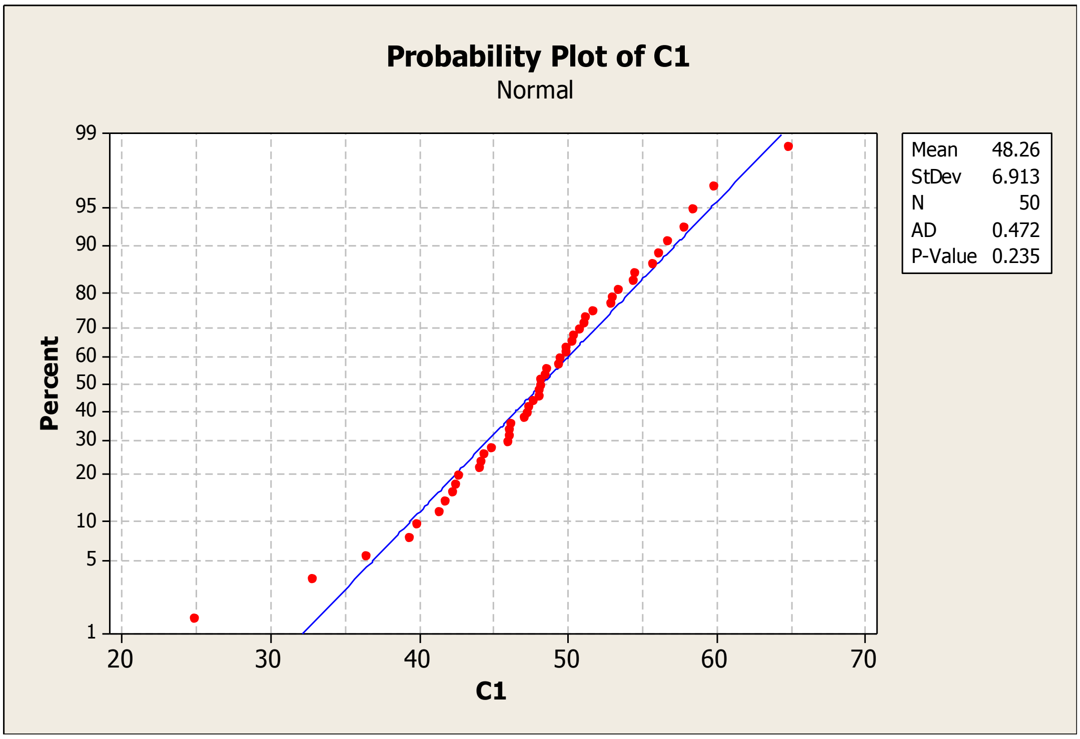

Fig. 2. Anderson Darling output for distribution identification for Normal distribution

Based on the results of proposed test, the value for P_value of Normal distribution is greater than others. So it can be concluded that the processing times follow Normal distribution with mean of μ and standard deviation of σ. Therefore the completion and tardiness values are Normal too. Based on the managerial view of Saipa company, two objectives have to be achieved as follows:

- The total completion times and delivery costs have to be minimized.
- The total tardiness costs should not avoid a predefined penalty

Index:

i	jobs
j	positions on machine
k	batches

Decision variables and parameters:

N	Number of jobs ready to be scheduled
$\tilde{p}_i$	The stochastic processing time of job i
d_i	The due date of i-th job
$\tilde{p}_{bat}(k)$	The stochastic processing time of batch k
$D_{bat}(k)$	The due date of batch k
n_k	Number of jobs assigned to batch k
$\tilde{t}_i$	The tardiness of job where scheduled on j-th position
c_i	The completion time of job where scheduled on j-th position
s_k	The shipping cost of batch k
$y_{ik} = 0 \text{ or } 1$	It is 1 if job i is assigned to batch k and otherwise it is 0
$x_{kj} = 0 \text{ or } 1$	It is 1 if batch k is located on j-th position in the sequence and Otherwise it is 0
$a_k = 0 \text{ or } 1$	It is 1 if there is at least one job assigned to batch k and Otherwise it is 0

And the proposed model is presented as follows:

$$\min z = \sum_k s_k a_k + \sum_k \check{c}_k \qquad (1)$$

St:

$$\tilde{c}_j = \tilde{c}_{j-1} + \sum_k x_{kj}(\tilde{p}_{bat}(k) \qquad j=1,2,\dots,N \quad (2)$$

$$c_0 = 0 \qquad (3)$$

$$\tilde{p}_{bat}(k) = \max_i(y_{ik}.\tilde{p}_i) \qquad K=1,2,\dots,N \quad (4)$$

$$n_k = \sum_i y_{ik} \qquad K=1,2,\ldots,N \quad (5)$$

$$\tilde{t}_j = max\left\{0, \tilde{c}_j - \sum_k D_{bat}(k)\, x_{kj}\right\} \qquad j=1,2,\ldots,N \quad (6)$$

$$pr\left\{\left\{\tilde{t}_j < t_{penalty}\right\}\right\} \geq \alpha \quad (7)$$

$$D_{bat}(k) = \max_i(y_{ik}. d_i) \qquad n_k \neq 0,\ K=1,2,\ldots,N \quad (8)$$

$$\sum_j x_{kj} = 1 \qquad k=1,2,\ldots,N \quad (9)$$

$$\sum_k x_{kj} = 1 \qquad j=1,2,\ldots,N \quad (10)$$

$$\sum_k y_{ik} = 1 \qquad i=1,2,\ldots,N \quad (11)$$

$$x_{kj} = 0 \; or \; 1 \quad (12)$$

$$y_{ik} = 0 \; or \; 1 \quad (13)$$

Eq. 1 introduces the objective function where the first term corresponds to the delivery cost and the second term corresponds to the sum of completion times. Constraint (2) states that the completion time of each batch is equal to the completion time of prior batches plus the sum of processing times of jobs assigned to it. Constraint (3) mentions that the machine and all the jobs are available at time zero. Constraint (4) declares that the completion time of each batch is equal to the maximum of processing times of jobs assigned to it. Constraint (5) determines how many jobs are located on each batch. Constraint (6) clarifies that the tardiness of each batch is equal to the gap between the time that the processing of that batch has been completed and its due date. Clearly, the value of tardiness must be positive. Equation (7) is related to CCP constraint. Equation (8) shows how the due date of each batch is calculated. Constraint (9) states that each batch is processed only once at each sequence and constraint (10) determines each position can be assigned to just one batch. Constraint (11) mentions each job can be assigned to exactly one batch to be processed and constraint (12) and (13) shows that x and y are binary variables.

In order to solve the proposed model, first it is required to convert the stochastic parameters to determinist values. For this purpose the values of processing and completion have to be replaced by their means and constraint 7 should be determined based on a CCP approach which be clarified in next section.

Chance constraint programming

In literature of production scheduling, deterministic approaches have been widely employed in the process industries. The challenging task for researchers currently is to solve large-scaled , complex optimization problems under various uncertainties. Generally in scheduling problems uncertainty could be divided by internal and external ones. Internal uncertainties are those which could be controlled in production line like machine failure while external uncertainties are out of control by production decision makers like price of markets, demand and etc.

The chance-constrained method is one of the major approaches for solving optimization problems under various uncertainties. It is a formulation of an optimization problem that ensures that the probability of meeting a certain constraint is above a certain level. In other words, it restricts the feasible region so that the confidence level of the solution is high. The chance-constrained method is a relatively robust approach, however, it is often difficult to solve. Chance constrained optimization is especially important in engineering and finance where uncertainties in price, demand, supply, currency exchange rate, recycle and feed rate, and demographic condition are common. Some classical applications of the chance-constrained method include water reservoir management and financial risk management. More recently, the method has been used in unmanned autonomous vehicle navigation as well as optimal renewable energy generation

The basic model for CCP approach could be addressed as follows:

$\min f(x)$

St:

$pr\{g(u) < b\} \geq \alpha$

Where x and u are deterministic and stochastic parameters respectively and α is a real number ranged in [0,1].

If u follows a Normal distribution with mean of µ and standard deviation of σ, then $\frac{g(u)-E(g(u))}{\sqrt{var(g(u)}}$ follows the Normal distribution too. So, the constraint of model could be written as follows:

$$pr\left\{\frac{g(u)-E(g(u)}{\sqrt{var(g(u)}} < \frac{b-E(g(u))}{\sqrt{var(g(u))}}\right\} \geq \alpha$$

$$F\left(\frac{b-E(g(u))}{\sqrt{var(g(u))}}\right) \geq \alpha \rightarrow \left(\frac{b-E(g(u))}{\sqrt{var(g(u))}}\right) \geq z_\alpha$$

And then:

$$b \geq z_\alpha\sqrt{var(g(u))} + E(g(u))$$

So, the stochastic constraint could be converted by a nonlinear deterministic constraint.

For more information about CCP, see Li et al (2008)

Based on the mentioned facts about CCP, the proposed model could be presented as follows where the stochastic parameters are replaced by their average values and constraint 7 is replaced by its equivalent CCP equation.

$$\min z = \sum_k s_k a_k + \sum_k \bar{c_k} \quad (14)$$

St:

$$\bar{c_j} = \bar{c}_{j-1} + \sum_k x_{kj}(\bar{p}_{bat}(k) \quad j=1,2,\ldots,N \quad (15)$$

$$c_0 = 0 \quad (16)$$

$$\bar{p}_{bat}(k) = \max_i(y_{ik}.\bar{p}_i) \quad K=1,2,\ldots,N \quad (17)$$

$$n_k = \sum_i y_{ik} \quad K=1,2,\ldots,N \quad (18)$$

$$t_{penalty} > z_\alpha\sqrt{var(\tilde{t}(j)} + E(\tilde{t}(j) \quad j=1,2,\ldots,N \quad (19)$$

$$D_{bat}(k) = \max_i(y_{ik}.d_i) \quad K=1,2,\ldots,N \quad (20)$$

$$\sum_j x_{kj} = 1 \quad k=1,2,\ldots,N \quad (21)$$

$$\sum_k x_{kj} = 1 \quad j=1,2,\ldots,N \quad (22)$$

$$\sum_k y_{ik} = 1 \quad i=1,2,\ldots,N \quad (23)$$

$$x_{kj} = 0 \; or \; 1 \quad (24)$$

$$y_{ik} = 0 \; or \; 1 \quad (25)$$

SOLUTION APPROACH

Branch and Bound

In this section a B&B approach is presented in order to possibly achieve the global optimal solutions. In this regard the solution space is searched by conducting a depth bi-level tree search in which the first level corresponds the batching procedure and the second level is related to assigning job to positions. The constructed tree search is shown in figure 3 for an instance with three jobs.

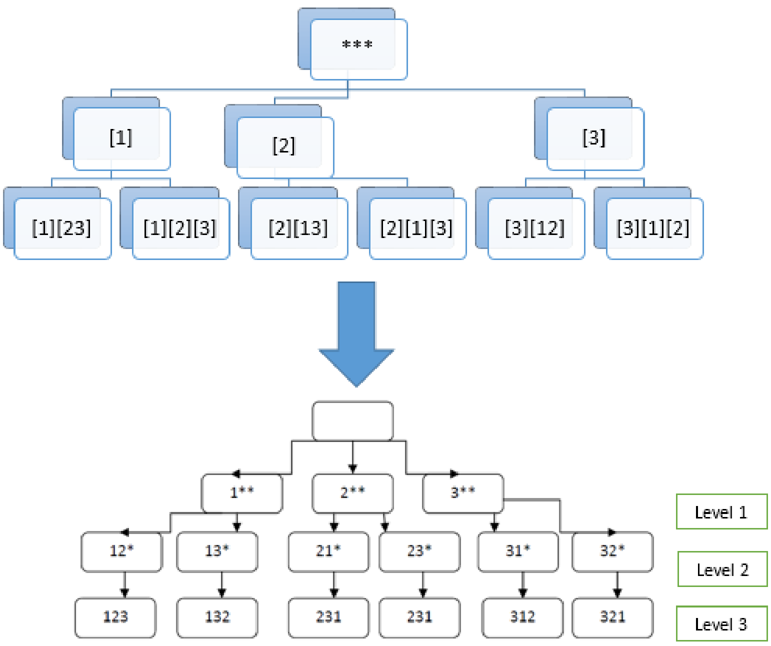

Fig. 3. the constructed tree search

During the searching in depicted tree, a node is fathomed if:

- It is a leaf node which means all the jobs have been scheduled.
- In this node the value of considered lower bound is equal or greater than upper bound.

The search of solution space continues till all nodes are fathomed. When a node is fathomed, backtracking happens. If there are no nodes that must be generated, then the branching procedure is terminated and the last upper bound would be considered as the optimal solution. It should be noted that the proposed branching method provides the guarantee that in the most pessimistic case it will generate all possible feasible solutions. Therefore, the optimality of the developed B&B algorithm could be claimed.

Upper bound

During searching the solution space, for each node an upper bound is achieved by a PSO meta heuristic.

PSO is a powerful meta heuristic presented in 1995 by Kennedy and Eberhart and is inspired of social behavior of birds flocking to a place for searching food. PSO starts with M initial solutions represents each particle and update the best achieved solution in each iteration until a termination condition is met. In every iteration of algorithm two factors consisting of local and global bests which are calculated. Two factors also considered for determination the status of each particle named velocity and its position.

$$v_i(t) = w(t)v_i(t-1) + c_1r_1\left(x_i^l - x_i(t-1)\right) + c_2r_2(X^G - X_i(t-1))$$

$$X_i(t) = v_i(t) + X_i(t-1)$$

Xi represents a local best of i-th particle after t-1 iterations. For every iteration, there will be one local best per iteration. while XG represents the global best among all the swarm of particles (M) achieved among all iterations. c1 and c2 are considered as positive learning factors and r1 and r2 are random numbers, value ranges between 0 and 1; w(t) is the inertia weight, and it is used to minimize the impact of the previous velocities on the current velocity.

The steps of PSO algorithm is presented as follows:

1. Generation M random swarm of particles
2. Evaluation the fitness for each particle
3. Updating the individual bests when an iteration is over
4. Updating global best when a better individual best is achieved
5. Calculation the velocity and position for each particle when an iteration is over
6. Go to step 2 until the equilibrium condition occurs

Lower bound

According to basic operational research theories, when a constraint is removed from the mathematical model, the value of objective function get not worst. So a lower bound for this problem might be considered by removing the constraints 12 and 13 when it is assumed that the binary variables could have a value ranged in [0,1].

Based on the proposed upper and lower bounds For each node if the value of proposed lower bound is greater or equal than upper bound, the searching is stop on that node and it continues from next node.

In this regard a lower bound is generated as follows:

- Assign the related job to the position based on tree search map
- For all un-assigned jobs, considered them as a perpetrate bath and schedule all jobs based on SPT (Shortest Processing Times) order
- Calculate the value for completion times
- Add the lowest delivery cost to reach job

Computational studies:

All the required computations were coded using Matlab 2013 software and were run on a CORE I 7 intel with 16 GB of Ram. The parameters including processing times, due dates and etc. were utilized from existence data of Saipa company. Table 1 presents the results of solving problem for small instances. For small data scales the problem could be solved using LINGO 10 global solver in considerable running times.

Table 1: comparison the solving the small instances for Lingo, PSO and B&B

Instance number	Number of jobs	Lingo solver		PSO		% Gap	B&B		% Gap
		Objective function	Run time	Objective function	Run time		Objective function	Run time	
1	3	4188	5	4188	2	0	4188	3	0
2	4	7779	5	7779	2	0	7779	3	0
3	5	3202	6	3202	2	0	3202	3	0
4	6	4676	145	4683	3	0.14	4676	56	0
5	7	7015	354	7054	2	0.56	7015	82	0
6	8	5437	896	5515	3	1.43	5437	93	0
7	9	7381	2568	7460	3	1.07	7381	95	0
8	10	-	-	8998	3	-	3232	115	-
9	11	-	-	8778	3	-	7082	145	-
10	12	-	-	9565	4	-	3841	202	-

Based on the results of table 1 the proposed B&B method could achieve the optimal solution for all instances in shorter run times in comparison to Lingo solver. On the other hand, Lingo is not capable for solving the problem for instances larger than 10 jobs. The presented PSO also has a considerable performance to find near optimal solution in economical run times. So, the proposed B&B is useful for solving the real data sets even for large scales.

Table 2 shows the results for solving the problem in medium and large scales.

Table 2: comparison the results of large scales for PSO and B&B

Instance number	Number of jobs	B&B		PSO			
		Objective function	Run time	Objective function (Best)	Objective function (average)	Objective function (worst)	Run time
1	15	12161	122	12161	12202	12301	3
2	30	8874	265	8874	8882	8890	3
3	50	9286	378	9293	9296	9301	4
4	100	12871	465	12871	12912	12955	4
5	120	8606	875	8615	8622	8650	4

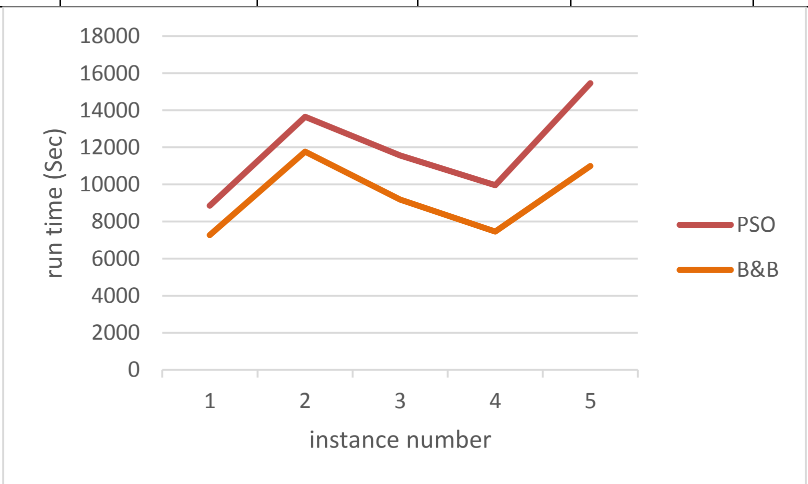

Fig.4. running time comparison for B&B and PSO

For medium and large data scales, the Lingo solver is not usable because of enormous number of solution nodes, But B&B could solve the problem optimally. However the required running times grow exponentially because of high complexity of problem. PSO also have good compatibility for solving medium and large data sets and it could achieve desirable solutions in economic run times.

Table 3: reduction visited nodes when PSO is used as upper bound

Instance number	Number of jobs	Run time		% improvement in visited nodes
		Simple B&B	B&B + PSO	
1	15	160	125	20
2	30	299	264	32
3	50	455	379	25
4	100	502	466	28
5	120	1025	882	32

Using PSO as a upper bound leads a considerable improvement in terms of reducing run times and decreasing the total visited nodes in solution space. This fact has been depicted in table 3 where the last column demonstrates the percentage of improvements in total visited nodes for each instance.

Table 4: improvement in production costs

Instance number	Number of jobs	B&B	Real operational cost	% improvement
		Value of objective function		
1	15	7262	8856	22
2	30	11773	13658	16
3	50	9175	11568	26
4	100	7460	9965	36
5	120	10999	15456	41

Finally the main objective of this research is to improve the production planning and decreeing the production costs. Table 4 demonstrates that employing the proposed method could improve the total production cost dramatically between 16 to 41 percentages.

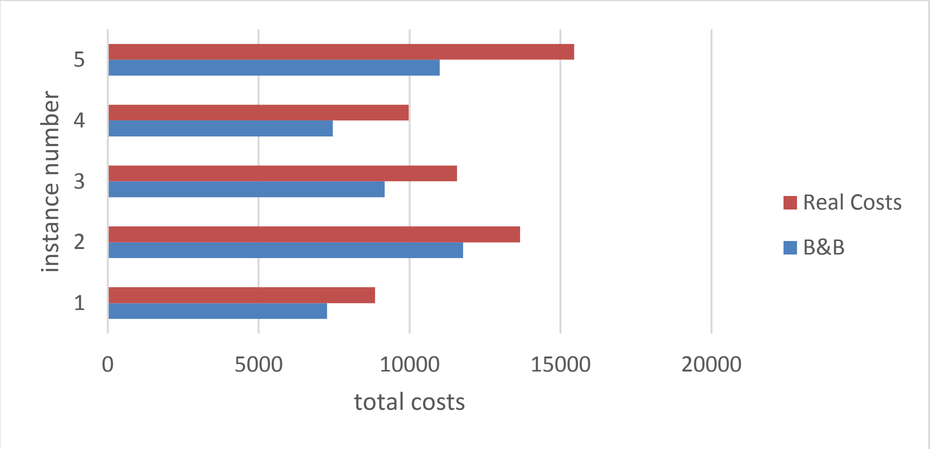

Fig. 5. improvements in production costs

CONCLUSION AND FUTURE STUDIES

This paper studied a single machine stochastic scheduling problem based on a real case study in Saipa company in Iran in which a bottleneck was identified in production line of Peugeot 206 as a single machine. According to existence data sets of this company the processing times follows a Normal distribution and based on the managerial decisions two objectives had to be achieved simultaneously including total completion times and controlling tardiness values so that their values be lower than an specified penalty. So, a CCP approach was used to model the problem. In order to solve the problem a B&B method was used to solve the problem optimally and a PSO metaheuristic was used to find near optimal solutions for large scales of problem. In order to check the validity and accuracy of proposed method, some small instances of problem was solved using LINGO global solver and the results show that the presented B&B was able to achieve the global optimal solutions in all studied instances. Since the proposed problem is NP-Hard the required run time for solving enlarges exponentially with growing the dimension. So, a PSO metaheuristic was also presented to solve the problem for larger scales in reasonable running times. The results show that using the proposed mathematical model and solution approach, could increase the effectiveness and efficiency of production line as 16 to 40 percent. So this research is a good choice for managers in similar companies to reduce the production costs. The main limitations of current study could be summarized as follows:

- Because of immense complexity related to the nature of scheduling problems (even in deterministic mode) the required run time for running the instances increases exponentially. So, it is hard to prepare a production scheduling planning framework for number of jobs greater than 100.
- The existence data set was limited. So it was vital to fit the data to a unique distribution and this fact could increase the error of estimations.

For future studies the following items are offered:

- Solving the problem with consideration of arrival time for each job
- Using the basic of queuing theory to model the problem

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