



## Performance Evaluation of Supply Chain under Decentralized Organization Mechanism

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### Abstract

Nowadays among many evaluation methods, data envelopment analysis has widely used to evaluate the relative performance of a set of Decision Making Units (DMUs). Data Envelopment Analysis (DEA) is a mathematical tool for evaluating the relative efficiency of a set Decision Making Units (DMUs), with multiple inputs and outputs. Traditional DEA models treat with each DMU as a "black box" thus, the performance measurement may be not effective. So, there are necessities for network DEA models. The objective of this paper is to propose a new network DEA model for measuring the efficiency of two- supplier and one manufacturer chains under the decentralized organization mechanism. In this mechanism, each section of supply chain is controlled under unique decision maker with his/her own interest. We proposed that, in comparison with CCR model, for the supply chain under decentralized organization mechanism, it is not appropriate to ignore the internal structure and treat as a "black box", while there is more than one decision maker with different interests. Furthermore, the relation between the supply chain efficiency and division efficiency is investigated. Numerical example demonstrates the application of the proposed model.

### Keywords:

DEA  
Network DEA  
Supply chain  
performance evaluation  
decentralized organization  
mechanism

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## INTRODUCTION

A supply chain takes the form of a network with multiple divisions and relationships. The supply chain performance measurement that only considers the initial inputs and the final outputs is generally inadequate since it ignores the interactions among the divisions. Thus, an appropriate performance measurement for supply chain should be designed that considers the network characteristics of the chain and interactions in it. Generally, the larger and more complex the supply chain is, the more challenging it becomes to be measured effectively (Beamon, 1999; Chen & Yan, 2011). Data envelopment analysis is a powerful mathematical tool for measuring the relative efficiency of a set of decision making units (DMUs) which utilize multiple inputs to produce multiple outputs. This methodology has been widely used to evaluate the relative performance of a set of production processes, or Decision Making Units (DMU), because DEA models need not to recourse to the exact production function regarding multiple inputs and outputs (Charnes et al., 1978).

Classical DEA makes no assumptions concerning the internal operations of a DMU. Rather, DEA treats each DMU as a "black box" by considering only the inputs consumed and outputs produced by each DMU. This perspective is often appropriate for a simple production process (Lewis and Sexton, 2004; Mirhedayatian et al., 2013). For the complex nature of supply chain, such an approach neglects the internal linking activities and essentially assumes that all the divisions in supply chain are under one single decision maker and each internal operation is absolutely effective. It thus cannot properly characterize the performance of supply chain, the "black Box" approach cannot provide underlying diagnostic information potentially available to the management (Lewis & Sexton, 2004; Chen & Yan, 2011). To estimate the efficiency of complex network system, several authors proposed Network DEA models.

Fare and Grosskopf (2000) proposed a Network DEA model for measuring efficiency for DMUs with multiple production stage. They first build division production possibility set satisfying the standard axioms found in Banker and Charnes(1984) including no "free lunch", strong

free disposability of inputs and outputs, etc. Then the production possibility set of the supply chain is formed by combining its divisional production possibility sets. Supply chain efficiency is calculated according to Farrel radical projection on the efficient frontier of supply chain (Chen & Yan, 2011). Seiford and Zhu (1999) and Chen and Zhu (2004), provide two approaches in modeling efficiency as a two-stage proces. Tone and Tsutsui(2009) proposed a slack based network DEA model, called Network SBM, which could deal with intermediate products. Also, Kao (2009) considered two parallel and series structures for internal parts of DMU. Liang et al. (2006) develop several DEA-based approaches for characterizing and measuring supply chain efficiency based upon the concept of non-cooperative and cooperative games. Fan et al. (2003) provided a new approach in developing a supply chain information system with a decentralized decision making process. Lee and Wang (1999) presented a performance measurement scheme involving transfer pricing, consignment, shortage reimbursement, and an additional backlog penalty at the decentralized multi-echelon supply chain. Wang et al. (2004) analyzed non-cooperative behavior in a two-echelon decentralized supply chain, composed of one supplier and n retailers.

In previous studies, performance evaluation of supply chain was performed with one supplier and one manufacturer. In this paper, we provide an output-oriented network DEA model as one of the most powerful performance evaluation models to investigate a supply chain with two suppliers and one manufacturer based on the concept of decentralized organizational mechanisms. One common reason for decentralized of supply chain is that local managers hold local information not available to the headquarters, and therefore they are relied upon to make some decisions. Under such information asymmetry, a decentralized policy with an appropriate incentive mechanism to coordinate the local managers can strictly dominate centralized decision-making without access to local information (Lee & Wang, 1999). In fact, this paper focuses on how to performance evaluation in a supply chain under decentralized organization mechanism.

Moreover we analyze the relation between the supply chain efficiency and division's efficiency

and then we compare the results of the proposed model with the results of CCR model. The rest of the paper is organized as follows; Section 2 is the review of network DEA models based on different organizational mechanisms, in section 3 we proposed a Network DEA model for measuring the efficiency of the supply chain. In section 4 we have investigated the efficiency of supply chain from two different perspectives. In section 5 a numerical example is provided and section 6 includes the conclusion and suggestion for further research.

### BACKGROUND

For performance evaluation of supply chain by using Network DEA models, two fundamental issues should be regarded. First: definition of supply chain efficiency, second: considering deal with the interactions in supply chain.

As for the first issue, the technical efficiency is used. Technical efficiency depicts the capability of production units to transform inputs into outputs. That is, by allowing adjustments to intermediate products, a supply chain is to be rated as fully efficient if and only if none of its inputs or outputs can be improved without worsening some of its other inputs or outputs. This definition is known as the Pareto–Koopmans condition of technical efficiency, or called Extended Pareto–Koopmans definition of technical efficiency. For the second issue, the perspective of organization mechanism to deal with the interactions in supply chain is used. Consider a multiple stage supply chain with several divisions as shown in fig.1. The modeling process of organizational mechanisms based on centralized, decentralized and mixed concepts is as follow:

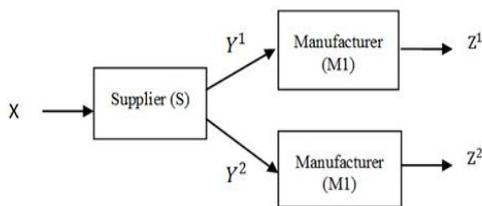


Fig.1. one-supplier and two manufacturer chain

### Centralized control model

In a centralized control mechanism, the supply chain is supervised by a single decision maker who can arrange the supplier and manufacturer's operations to maximize the whole supply chain efficiency. The critical issue of model construction is to incorporate intermediate product in DEA model. Thus, for intermediate product, so we should show that the level taken as the input of division M should not be larger than that taken as the output of division S. With regard to the initial input X and final outputs Z 1 and Z2, assume that they all satisfy the strong free-disposability, as it is common in conventional DEA models. Thus, the production possibility set corresponding to centralized control supply chain is expressed as,

$$T_{\text{central}} = \left\{ \begin{array}{l} \sum_{j=1}^n \lambda_j^1 x_j \leq \theta_{\text{central}} x_o, \\ \sum_{j=1}^n \lambda_j^1 y_j^1 \geq \sum_{j=1}^n \lambda_j^2 y_j^1 \\ \sum_{j=1}^n \lambda_j^1 y_j^2 \geq \sum_{j=1}^n \lambda_j^3 y_j^2 \\ \sum_{j=1}^n \lambda_j^2 z_j^1 \geq z_o^1, \quad \sum_{j=1}^n \lambda_j^3 z_j^2 \geq z_o^2 \end{array} \right\}$$

When the joint supply chain or DMU, called DMU<sub>0</sub> in short, is under evaluation, according to Farrell radical projection, the whole supply chain efficiency is calculated by:

$$\begin{array}{ll} \min \theta_o & \\ \text{s. t.} & \sum_{j=1}^n \lambda_j^1 x_j \leq \theta_{\text{central}} x_o \quad ; j = 1, \dots, n \quad (1) \\ & \sum_{j=1}^n \lambda_j^1 y_j^1 \geq \sum_{j=1}^n \lambda_j^2 y_j^1 ; j = 1, \dots, n \\ & \sum_{j=1}^n \lambda_j^1 y_j^2 \geq \sum_{j=1}^n \lambda_j^3 y_j^2 \quad ; j = 1, \dots, n \\ & \sum_{j=1}^n \lambda_j^2 z_j^1 \geq z_o^1 \quad ; j = 1, \dots, n \\ & \sum_{j=1}^n \lambda_j^3 z_j^2 \geq z_o^2 ; j = 1, \dots, n \\ & \lambda_j^1, \lambda_j^2, \lambda_j^3 \geq 0 \quad ; j = 1, \dots, n \end{array}$$

While allowing the decision maker to adjust the supplier and manufacturers' operations, model (central) derives the minimum proportion that input can be reduced for the given output level.

The first three inequalities in the constraint set correspond to division S, division M1 and division M2's technology, respectively. Note that in centralized control organization mechanism, all the divisions are belong to one decision maker.

Thus, the multiplier, or the importance, associated with intermediate product remains the same, no matter it is used as the output of supplier or the input of manufacturer(Chen & Yan, 2011).

**Definition 1:**

If the optimal value of (central) satisfies  $\theta_{\text{decentralized}}^* = 1$ , then we call  $DMU_0$  weakly efficient corresponding to model (central), or weakly efficient (central) in short.

**Decentralized control model**

In a decentralized control organization, there is no such a “super decision maker” to control all divisions. Each division has its own incentive and strategies. They are selfish and pursue their own interests. Thus, in the course of searching the frontier minimum input, any adjustment to the divisions must be a Pareto solution. Thus, the production possibility set corresponding to decentralized control supply chain is as follows:

$$T_{\text{decentral}} = \left\{ (x, z^1, z^2) \left| \begin{array}{l} \sum_{j=1}^n \lambda_j^1 x_j \leq x, \quad \sum_{j=1}^n \lambda_j^1 y_j^1 \geq \sum_{j=1}^n \lambda_j^2 y_j^1, \\ \sum_{j=1}^n \lambda_j^1 y_j^2 \geq \sum_{j=1}^n \lambda_j^3 y_j^2 \\ \sum_{j=1}^n \lambda_j^2 y_j^1 \leq y^1, \quad \sum_{j=1}^n \lambda_j^2 z_j^1 \geq z^1, \\ \sum_{j=1}^n \lambda_j^3 y_j^2 \leq y^2, \quad \sum_{j=1}^n \lambda_j^3 z_j^2 \geq z^2 \end{array} \right. \right\}$$

Accordingly, its DEA model is expressed as,

$$\begin{aligned} & \min \theta_0 \\ \text{s.t.} \quad & \sum_{j=1}^n \lambda_j^1 x_j \leq \theta_{\text{decentral}} x_0 \quad ; j = 1, \dots, n \quad (2) \\ & \sum_{j=1}^n \lambda_j^1 y_j^1 \geq \sum_{j=1}^n \lambda_j^2 y_j^1 \quad ; j = 1, \dots, n \\ & \sum_{j=1}^n \lambda_j^1 y_j^2 \geq \sum_{j=1}^n \lambda_j^3 y_j^2 \quad ; j = 1, \dots, n \\ & \sum_{j=1}^n \lambda_j^2 y_j^1 \leq y_0^1 \quad ; j = 1, \dots, n \\ & \sum_{j=1}^n \lambda_j^2 z_j^1 \geq z_0^1 \quad ; j = 1, \dots, n \\ & \sum_{j=1}^n \lambda_j^3 y_j^2 \leq y_0^2 \quad ; j = 1, \dots, n \\ & \sum_{j=1}^n \lambda_j^3 z_j^2 \geq z_0^2 \quad ; j = 1, \dots, n \\ & \lambda_j^1, \lambda_j^2, \lambda_j^3 \geq 0 \quad ; j = 1, \dots, n \end{aligned}$$

The first, the fifth and the seventh inequalities in constraint set represent minimizing the initial input X at given level of final out puts  $z_0^1$  and  $z_0^2$

.The second, the third, the fourth and the sixth inequalities in constraint set are corresponding to intermediate products. For division M1, the adjustment to Y1 should be no large than its current level  $Y_0^1$ , since otherwise, division M1 would not agree to proceed. That is the fourth inequality in constraint set The same is to division M 2 as represented in the sixth inequality of model (decentral). Thus, the selfishness of division M1 and M 2 are reacted in the fourth and sixth inequalities in constraint set.

Since each division belongs to different decision maker, the multiplier associated with intermediate product as output of supplier is different from that as the input of manufacturer.

**Mixed control model**

Consider that the supplier and manufacturer M2 are under one decision maker, and manufacturer M1 is under another decision maker. According to the analysis in Sections 2.1 and 2.2, the production possibility set corresponding to supply chain under mixed control organization is characterized as:

$$T_{\text{mix}} = \left\{ (x, z^1, z^2) \left| \begin{array}{l} \sum_{j=1}^n \lambda_j^1 x_j \leq x, \quad \sum_{j=1}^n \lambda_j^1 y_j^1 \geq \lambda_j^2 y_j^1, \\ \sum_{j=1}^n \lambda_j^1 y_j^2 \geq \sum_{j=1}^n \lambda_j^3 y_j^2 \\ \sum_{j=1}^n \lambda_j^2 y_j^1 \leq y^1, \quad \sum_{j=1}^n \lambda_j^2 z_j^1 \geq z^1, \\ \sum_{j=1}^n \lambda_j^3 z_j^2 \geq z^2 \end{array} \right. \right\}$$

Its DEA model is represented as,

$$\begin{aligned} & \min \theta_{\text{mix}} \\ \text{s.t.} \quad & \sum_{j=1}^n \lambda_j^1 x_j \leq \theta_{\text{mix}} x_0 \quad ; j = 1, \dots, n \quad (3) \\ & \sum_{j=1}^n \lambda_j^1 y_j^1 \geq \sum_{j=1}^n \lambda_j^2 y_j^1 \quad ; j = 1, \dots, n \\ & \sum_{j=1}^n \lambda_j^1 y_j^2 \geq \sum_{j=1}^n \lambda_j^3 y_j^2 \quad ; j = 1, \dots, n \\ & \sum_{j=1}^n \lambda_j^2 y_j^1 \leq y_0^1 \quad ; j = 1, \dots, n \\ & \sum_{j=1}^n \lambda_j^2 z_j^1 \geq z_0^1 \quad ; j = 1, \dots, n \\ & \sum_{j=1}^n \lambda_j^3 z_j^2 \geq z_0^2 \quad ; j = 1, \dots, n \\ & \lambda_j^1, \lambda_j^2, \lambda_j^3 \geq 0 \quad ; j = 1, \dots, n \end{aligned}$$

## THE RELATIONSHIP OF SUPPLY CHAIN EFFICIENCY UNDER DIFFERENT ORGANIZATION MECHANISMS

Theorem 1. The relationship of supply chain efficiency under centralized, decentralized and mix organization mechanisms can be expressed as;  
 $\theta_{\text{central}}^* \leq \theta_{\text{mix}}^* \leq \theta_{\text{decentralized}}^*$

### Proof:

First it is prove  $\theta_{\text{mix}}^* \leq \theta_{\text{decentralized}}^*$ . suppose  $\theta_{\text{central}}^*$ ,  $\lambda_j^{1*}, \lambda_j^{2*}, \lambda_j^{3*}, j=1, \dots, n$  be an any optimal solution of decentralize model. So based on model 3 in context, obviously, it is also a feasible solution to model (mix). Thus,  $\theta_{\text{mix}}^* \leq \theta_{\text{decentral}}^*$ : Similarly, it is proved that any optimal solution of (Mix) is also feasible to (Central). Hence, we have  $\theta_{\text{central}}^* \leq \theta_{\text{mix}}^* \leq \theta_{\text{decentralized}}^*$  this Theorem implies that if a supply chain is weakly efficient in the decentralized organization mechanism, then it must be weakly efficient in both the centralized and mixed organization mechanisms.  $\theta_{\text{central}}^* \leq \theta_{\text{mix}}^* \leq \theta_{\text{decentralized}}^*$  (Chen & Yan, 2011).

### PROPOSED MODEL

Consider a multiple stage supply chain with several divisions, if all divisions are controlled by a super decision maker with access to available information, this is referred to centralized control supply chain. Nowadays, with the development of global specification, in most cases it is seen that, ever divisions of supply chain are control under unique decision maker. This approach referred to decentralized control supply chain.

In this section, we develop a Network DEA model that can directly evaluate the performance of the supply chain while considering the relationship between suppliers and manufacturers. The modeling processes are based upon the concepts of decentralized control organization mechanisms. Consider a two stage supplier-manufacturer chain as following Figure2. where S and M characterize the supplier and manufacturer, respectively.  $x_i^1, x_i^2$  are its input vectors to the suppliers (S) division and  $y_1^1, y_1^2$  are its output vectors which are also input vectors to the manufacturer division.  $z_1, z_2$  are its the output vectors corresponding to manufacturer. consider n same supply chains called Decision Making Units (DMU<sub>s</sub>) in DEA literatures denoted by

SCM<sub>1</sub>...SCM<sub>2</sub>, in context.

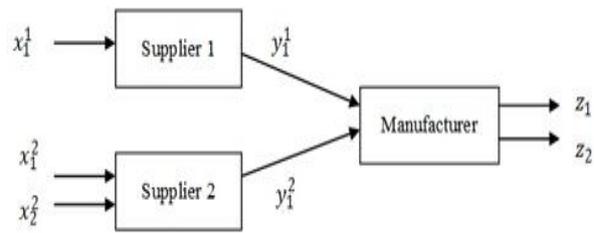


Fig.2. Two –supplier and one manufacturer chain

For SMC belonging to a set n homogenous and independent SCMn with two-suppliers and one manufacturer, output oriented version of the envelopment, with decentralized organization mechanism, can be written as follows, while the Notification used in the mathematical model is presented in Table1.

Thus, the production possibility set corresponding to decentralized control supply chain as follows:

$$T_{\text{decentral}} = \left\{ (x^1, x^2, z) \left| \begin{array}{l} \sum_{j=1}^n \lambda_j^1 x_{ij}^1 \leq x_i^1, \sum_{j=1}^n \lambda_j^2 x_{ij}^2 \leq x_i^2, \\ \sum_{j=1}^n \lambda_j^1 y_{uj}^1 \geq \sum_{j=1}^n \lambda_j^3 y_{uj}^3, \\ \sum_{j=1}^n \lambda_j^2 y_{uj}^2 \geq \sum_{j=1}^n \lambda_j^3 y_{uj}^3, \sum_{j=1}^n \lambda_j^3 z_{rj} \geq z_r \end{array} \right. \right\}$$

Its Network DEA model is presented as,

$$\begin{aligned} \max \phi_0 & \quad (4) \\ \text{s.t.} \quad & \sum_{j=1}^n \lambda_j^1 x_{ij}^1 \leq x_{i0}^1 \quad ; i = 1, \dots, I_1; j = 1, \dots, n \\ & \sum_{j=1}^n \lambda_j^2 x_{ij}^2 \leq x_{i0}^2 \quad ; i = 1, \dots, I_2; j = 1, \dots, n \\ & \sum_{j=1}^n \lambda_j^1 y_{uj}^1 \geq \sum_{j=1}^n \lambda_j^3 y_{uj}^3 ; u = 1, \dots, U_1; j = 1, \dots, n \\ & \sum_{j=1}^n \lambda_j^2 y_{uj}^2 \geq \sum_{j=1}^n \lambda_j^3 y_{uj}^3 ; u = 1, \dots, U_2; j = 1, \dots, n \\ & \sum_{j=1}^n \lambda_j^3 z_{rj} \geq \phi_0 z_{r0} \quad r = 1, \dots, S \quad ; j = 1, \dots, n \\ & \lambda_j^1, \lambda_j^2, \lambda_j^3 \geq 0 \quad ; j = 1, \dots, n \end{aligned}$$

Table1: Notification used in the mathematical model

$x_{ij}^1$	Input amount of ith corresponds to the 1supplierof jth SCM
$x_{ij}^2$	Input amount of ith corresponds to the 2 supplier of jth SCM
$y_{uj}^1$	Intermediate measures of uth corresponds to the 1supplierof jth SCM
$y_{uj}^2$	Intermediate measures of uth corresponds to the 2 supplier of jth SCM
$z_{rj}$	Output amount rth of jth SCM
$\lambda_j^1$	Intensity vector corresponds to the supplier 1 division of jth SCM
$\lambda_j^2$	Intensity vector corresponds to the supplier 2 division of jth SCM
$\lambda_j^3$	Intensity vector corresponds to the manufacturer division of jth SCM
$\theta_{decentral}$	Overall efficiency Score under decentralized organization mechanism
$i$	Numeratorindex correspond to number of inputs of suppliers
$u$	Numerator index correspond to number of Intermediate measures
$r$	Numerator index correspond to number of outputs of manufacturer
$j$	Numerator Index correspondent to SCMs
$n$	Number of Supply Chains

The first, and the second inequalities in constraint set represent the initial inputs  $x$  at given level of final output  $y$ . For division S1, the adjustment to  $x^1$  should be no larger than its current level  $x_o^1$ . The same is to division S2 is represented in second inequality of model. Thus, the selfishness of division S1 and S2 are reacted in first and second inequalities in constraint set.

The third and the fourth, inequalities in constraint set are corresponding to intermediate products. They represent that the input level of the supplier division is no less than that as the output level of the supplier division. The fifth inequality in constraint set is corresponding to outputs of manufacturer.

**Efficiency analysis**

Efficiency analysis on the supply chain can be investigated from many different aspects. In this section, the relationship of supply chain efficiency and division efficiency is investigated and analyzed. Also its relation between efficiency of CCR model for a supply chain with two suppliers and one manufacturer is investigated.

**Supply chain efficiency vs. division efficiency**

Let  $\theta_{decentral}^*, \theta_{S1}^*, \theta_{S2}^*, \theta_M^*$  be the optimal values corresponding to model decentral, S1, S2, M, respectively.

Theorem 2. In decentralized organization mechanism, the supply chain efficiency and the division efficient have the following relationship.

$$\theta_{decentral}^* \leq \max\{\theta_{S1}^*, \theta_{S2}^*\} * \theta_M^*$$

**Proof:**

Suppose input and output vectors are one dimension. Denote  $\theta_{S1}^*, \lambda_j^{1*}; \theta_{S2}^*, \lambda_j^{2*}; \theta_{S3}^*, \lambda_j^{3*}$ ,  $j=1, \dots, n$  as the optimal pair of solutions corresponding to model (S1), (S2) and (M), respectively. That is:

$$\begin{aligned} \sum_{i=1}^n \lambda_j^{1*} x_i^1 &\leq x_o^1 && ; j = 1, \dots, n \\ \sum_{i=1}^n \lambda_j^{2*} x_i^2 &\leq x_o^2 && ; j = 1, \dots, n \\ \sum_{i=1}^n \lambda_j^{1*} y_j^1 &\geq \theta_{S1}^* y_o^1 \geq y_o^1 && ; j = 1, \dots, n \\ \sum_{i=1}^n \lambda_j^{2*} y_j^2 &\geq \theta_{S2}^* y_o^2 \geq y_o^2 && ; j = 1, \dots, n \\ \sum_{i=1}^n \lambda_j^{3*} y_j^3 &\leq y_o^3 && ; j = 1, \dots, n \\ \sum_{i=1}^n \lambda_j^{3*} y_j^2 &\leq y_o^2 && ; j = 1, \dots, n \\ \sum_{i=1}^n \lambda_j^{3*} z_j &\geq \theta_M^* z_o && ; j = 1, \dots, n \end{aligned} \tag{5}$$

Without loss of generality, assume that  $\theta_{S1}^* \geq \theta_{S2}^*$ . By multiplying  $\theta_{S1}^*$  on both sides of the last three inequalities (5), and let:

$$\lambda_j^{3*} = \theta_{S1}^* \lambda_j^{3*}, \quad j=1, \dots, n, \quad \text{we get:}$$

$$\sum_{j=1}^n \lambda_j^{3'*} y_j^1 \leq \theta_{S1}^* y_o^1 \quad (a)$$

$$\sum_{j=1}^n \lambda_j^{3'*} y_j^2 \leq \theta_{S1}^* y_o^2 \quad (b)$$

$$\sum_{j=1}^n \lambda_j^{3'*} z_j \geq \theta_{S1}^* \theta_M^* z_o \quad (c)$$

From (a) and third in equality in (5), we have:

$$\sum_{j=1}^n \lambda_j^{1'*} y_j^1 \geq \theta_{S1}^* y_o^1 \geq \sum_{j=1}^n \lambda_j^{3'*} y_j^1 \quad (d)$$

Similarly, (b) and the fourth inequality in (5) can be represented as:

$$\sum_{j=1}^n \lambda_j^{2'*} y_j^2 \geq \theta_{S1}^* y_o^2 \geq \sum_{j=1}^n \lambda_j^{3'*} y_j^2 \quad (e)$$

Note that (c), (d), (e), the first, second and seventh inequality in (5), together imply that  $(\theta_{S1}^*, \theta_M^*), \lambda_j^{1'*}, \lambda_j^{2'*}, \lambda_j^{3'*}, j=1, \dots, n$  is feasible to model (decentral). thus  $\theta_{decentral}^* \leq \theta_{S1}^* \theta_M^*$ , viz.  $\theta_{decentral}^* \leq \theta_M^* \max\{\theta_{S1}^*, \theta_{S2}^*\}$ .

**Corollary 1.** If a supply chain is weakly efficient, then there must exist a path from the original inputs to the final outputs along which every division is weakly efficient.

**Proof:**

This can be directly obtained by Theorem 1. Corollary 1 provides a necessary condition for a supply chain to be weakly efficient. But it is not sufficient, examples listed in the next section. As is proved in Theorem 1, which

implies that the efficiency of whole supply chain is affected by all divisions. Since the interactions of internal divisions are complex, the affection is thus complex.

## Network efficiency vs. CCR efficiency

### Theorem 3.

For the supply chain organized under centralized control mechanism, the supply chain effi-

ciency evaluated by model (central) is no larger than that calculated by “black box” model (CCR), that is,  $\theta_{central}^* \leq \theta_{CCR}^*$ .

**Proof:**

Suppose  $\theta_{CCR}^*, \lambda_j^*, j=1, \dots, n$  is an optimal solution to model (CCR). let  $\lambda_j^{1'} = \lambda_j^{2'} = \lambda_j^{3'} = \lambda_j^*$ ,  $j=1, \dots, n$ . It is obvious that  $\theta_{CCR}^*, \lambda_j^{1'}, \lambda_j^{2'}, \lambda_j^{3'}, j=1, \dots, n$

is also a feasible solution to model (central). Thus, we have  $\theta_{central}^* \leq \theta_{CCR}^*$ . However, the relationship between  $\theta_{central}^*$  and  $\theta_{CCR}^*$  is different. One could be larger than another one depending on parameter.

## ILLUSTRATIVE EXAMPLE

Consider two-supplier and one manufacturer supply chains as depicted in Fig 2. Input vectors to S1 and S2 are one and two dimension, respectively. Data are listed in Table 2. We have shown in Table 3 the supply chain efficiency calculated by model decentralized and CCR, respectively.

Corresponding to decentralized model, If the optimal value of decentralized model satisfies  $\theta_{decentralized}^* = 1$  then we call DMUo weakly efficient, and there must exist a path from the initial input to the final output along which all divisions are weakly efficient. The CCR model fails to properly characterize the performance of supply chain, since it only considers the initial inputs and the final outputs of the supply chain and ignores intermediate products associated with supply chain members.

Table 2: Data for 10 two-supplier and one manufacture chain (SCM).

NO.	$x_1^1$	$x_1^2$	$x_2^2$	$z_1^1$	$z_1^2$	$y_1$	$y_2$
SCM1	1.0168	1.221	1.2215	166.9755	8.3098	122.1954	3.579
SCM2	0.5915	0.611	0.4758	50.1164	1.7634	19.4829	0.6600
SCM3	0.7237	0.645	0.6061	48.2831	3.4098	34.4120	0.7713
SCM4	0.5150	0.486	0.3763	35.0704	2.3480	15.2804	0.3203
SCM5	0.4775	0.526	0.3848	49.9174	5.4613	34.9897	0.8430
SCM6	0.6125	0.407	0.3407	23.1052	1.2413	32.5778	0.4616
SCM7	0.7911	0.708	0.4407	39.4590	1.1485	30.2331	0.6732
SCM8	1.2363	0.713	0.5547	37.4954	4.0825	20.6013	0.4864
SCM9	0.4460	0.443	0.3419	20.9846	0.6897	8.6332	0.1288
SCM10	1.2481	0.638	0.4574	45.0508	1.7237	9.2354	0.3319

Table 3: Overall efficiency scores of network DEA under decentralized mechanism and CCR model.

NO.	$\theta_{\text{decentralized}}^*$	$\theta_{\text{CCR}}^*$
SCM1	0.7935	0.9937
SCM2	0.3534	0.0734
SCM3	0.4457	0.5675
SCM4	0.2476	0.4059
SCM5	0.7083	0.9089
SCM6	0.9429	0.9558
SCM7	0.5754	0.6857
SCM8	0.3224	0.3712
SCM9	0.2470	0.2524
SCM10	0.2238	0.2476

### CONCLUSION

In supply chain management (SCM), strengthening the partnership with the suppliers is a significant factor for enhancing competitiveness. The main aim of supply chain management is to integrate various suppliers to satisfy the market demand because multi-supplier manufacturer chains performance evaluation is important for companies. In this paper we propose two-supplier and one manufacturer chain performance evaluation model using network DEA that includes intermediate products between the sub-units of DUMs.

If all divisions of supply chain are supervised by a single decision maker that can arrange the supplier and manufactures, operations maximize

the whole supply chain efficiency. Thus, we only need to ensure that the inputs of division Supplier should not be larger than the outputs of division supplier.

In a decentralized control organization, there is no "super decision making unit" to control all divisions; we also need to guarantee that any optimal adjustment strategy to a division is a Pareto solution; otherwise, the adjustment cannot be conducted. Furthermore, we propose that for the supply chain under decentralized control mechanisms, it is not appropriate to ignore the internal structure and treat it as a "black box", since there are more than one decision makers with their own interests. The CCR model fails to properly characterize the performance of supply chain,

since it only considers the initial inputs and the final outputs of the supply chain and ignores the intermediate products associated with the supply chain members.

This problem can also be applied for firms, some factories or supermarkets that provide goods from different plants. The presented model has important applications in areas such as the performance evaluation of complex network supply chain. Future research paths might focus on the following issues: developing powerful performance measures for supplier evaluation, articulating the criticality of supplier performance, selecting suppliers for a supplier evaluation program, extending the current research to the fuzzy environments; also, using other organization mechanism to deal with the complex interactions such as mixed organization mechanism in supply chain.

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