Optimization
Iranian Journal of Optimization
Volume 9, Issue 2, 2017, 107-117
Research Paper
Online version is available on: www.ijo.iaurasht.ac.ir
Islamic Azad University
Rasht Branch
E-ISSN:2008-5427

A Fuzzy Multi-Objective Model for a Project Management Problem

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Received: 20 June 2017
Accepted: 18 July 2017

Abstract

In this research, the multi-objective project management decision problem with fuzzy goals and fuzzy constraints are considered. We constitute α-cut approach and two various fuzzy goal programming solution methods for solving the Multi-Objective Project Management (MOPM) decision problem under fuzzy environments. The Interactive fuzzy multi-objective linear programming (i-FMOLP) and Weighted Additive approaches are proposed for solving multi-objective PM decision problem where fuzzy information are demonstrated by using linear membership functions (LMF). The proposed approaches effort contemporarily to minimize the total project costs, total completion time and total crashing costs and the several constraints such as the time between events i and j, the crashing time for activity (i,j) and the total budget capital. The weight of criteria for each objective function base on project DM preference degree computed with Fuzzy AHP technique. The performance analysis calculated with a set of distance metric for i-FMOLP and Weighted Additive solution methods that represent uncertainty goals and constraints in PM decision problem with ideal solution in an industrial case study is compared.

Keywords:
multi-objective Project management decision problem
α-cut approach
Interactive fuzzy multi-objective linear programming (i-FMOLP)
Weighted Additive
linear membership functions
Fuzzy AHP

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INTRODUCTION

Project-based management is becoming more and more significant as a tool for improving the performance of organizations. In the real-world Project Management (PM) decisions, model inputs and environmental coefficients, such as operating costs, activities duration, available resources and total cost budget, are typically fuzzy/imprecise owing to incomplete and unobtainable information over the project planning horizon. Conventional deterministic techniques described above obviously cannot solve practical PM decision problems in a vague environment (Liang, 2010). The decision maker (DM) must normally handle conflicting goals in term of the use of organizational resources, and these conflicting objectives are required to be optimized simultaneously by the project managers (Liang, 2009). Solutions to fuzzy multi-objective PM optimization problems profit from assessing the vagueness of the DM's judgments such as "the objective function of project duration should be substantially less than or equal to 267 days," and "total project costs should be substantially less than or equal to 1.5 million". Conventional deterministic PM decision techniques cannot obviously solve the fuzzy multi-objective PM programming problems (Liang, 2009). Therefore, fuzzy sets theory was offered by Bellman and Zadeh (1970) has been extensive in different fields such as PM decision model and more applications in project scheduling.

The main goal of this paper is developing multi-objective project management under fuzzy goal and fuzzy constraint using α-cut approach and various solution methods basis on fuzzy programming techniques. We represent four different solution method consist of interactive fuzzy multi-objective linear programming (i-FMOLP) and Weighted Additive. The weighted of criteria in Weighted Additive solution method calculate by AHP Fuzzy technique. Finally, we will compare different solution method and the best solution method select using performance analysis functions.

We arrange the rest of the paper as follows. Section 2, provides the problem description. Solution methodology is described in Section 3. In Section 4, we give the model implementation and the performance analysis. Finally, we provide conclusions and directions for future research.

PROBLEM DESCRIPTION

We adopt a fuzzy multi-objective PM (MOPM) problem describe in Liang (2009; 2010). The author assumes that a project encompasses interrelated activities that must be executed in a certain order before the entire task can be completed (Liang, 2009). The principle fuzzy MOLP model designed in this study aims to simultaneously minimize total project costs, total completion time and total crashing costs (Liang, 2010).

The proposed fuzzy mathematical programming model is based on the following assumptions (Liang, 2009; 2010):

(1) All of the objective functions are fuzzy with imprecise aspiration levels.

(2) All of the objective functions and constraints are linear equations.

(3) Direct costs increase linearly as the duration of activity is reduced from its normal time to its crash value.

(4) The normal time and shortest possible time for each activity and the cost of completing the activity in the normal time.

(5) The available total budget is known over the planning horizons.

(6) The linear membership functions are adopted to specify fuzzy goals and fuzzy constraints the minimum operator and the average operator are sequentially used to aggregate fuzzy sets.

(7) The total indirect costs can be divided into fixed costs and variable costs, and the variable costs per unit time are the same regardless of project completion time.

Set of indices, parameters and decision variables for the MOLP model are defined in the nomenclature (see Table 1).

Objective functions

Fuzzy multi-objective linear programming for Project Management decisions problem formulated as Table 1.

■ Minimize total project costs

$$Min\,Z_1 \cong \sum_i \sum_j C_{D_{ij}} + \sum_i \sum_j k_{ij}\,Y_{ij} + [C_I + m(E_n - T_{nc})] \quad (1)$$

■ Minimize total completion time

$$Min\,Z_2 \cong E_n - E_1 \quad (2)$$

Table 1: Nomenclature (fuzzy parameters are shown with tilde ~)

Sets of indices	
(i, j):	Activity between events i and j
g:	Index for objective function (1,2,…,K)
Decision variables	
t_{ij}:	Crashed duration time for activity *(i,j)*
Y_{ij}:	Crash time for activity *(i,j)*,
E_i:	Earliest time for event *i*
E_j:	Earliest time for event *j*
Objective functions	
Z_1:	Total project costs
Z_2:	Total completion time
Z_3:	Total crashing costs
Parameters	
D_{ij}:	Normal time for activity *(i,j)*
d_{ij}:	Minimum crashed time for activity *(i,j)*
C_{Dij}:	Normal (direct) cost for activity *(i,j)*
C_{dij}:	Minimum crashed (direct) cost for activity *(i,j)*
K_{ij}:	Incremental crashing costs for activity *(i,j)*
E_i:	Project start time
E_n:	Project completion time
T_{nc}:	Project completion time under normal conditions
T:	Specified project completion time
C_I:	Fixed indirect costs under normal conditions
m:	Variable indirect costs per unit time
B:	Available total budget

■ Minimize total crashing costs

$$Min\, Z_3 \cong \sum_i \sum_j k_{ij} Y_{ij} \tag{3}$$

2.2. Constraints

■ Constraints on the time between events *i* and *j*

$$E_i + t_{ij} - E_j \le 0 \quad \forall i, \forall j \tag{4}$$

$$Y_{ij} \le \tilde{D}_{ij} - \tilde{d}_{ij} \quad \forall i, \forall j \tag{5}$$

■ Constraints on the crashing time for activity *(i,j)*

$$Y_{ij} \le \tilde{D}_{ij} - \tilde{d}_{ij} \quad \forall i, \forall j \tag{6}$$

■ Constraint on the total budget

$$Z_1 \le \tilde{B} \tag{7}$$

■ Non-negativity constraints on decision variables

$$t_{ij}, Y_{ij}, E_i, E_j \ge 0 \ \forall i, \forall j \tag{8}$$

SOLUTION METHODOLOGY

In this section, some approaches transform the fuzzy multi-objective linear programming model (FMOLP) into an equivalent auxiliary crisp mathematical programming model for PM problem is defined. These approaches adapt to linear membership functions to represent all fuzzy objective functions and constraints for the DM making of Bellman and Zadeh (1970) and interactive fuzzy multi-objective linear programming (i-FMOLP) (Liang, 2006) and Weighted Additive (Amid et al., 2009) solution methods.

Linear membership functions

The linear membership functions for minimizing fuzzy objective functions defined by

$$\mu_k\left(Z^k(x)\right) = \begin{cases} 1, & if\ Z^k(x) \le L_k \\ \frac{U_k - Z^k(x)}{U_k - L_k}, & if\ L_k < Z^k(x) < U_k \\ 0, & if\ Z^k(x) \ge U_k \end{cases} \tag{9}$$

Where U_k and L_k are the upper and lower bounds of the kth objective function $(Z^k(x))$ respectively. The linear membership function can be determined by asking the DM to select the object value interval $[L_k, U_k]$. In practical situation, a possible value for vague objective function can be determined based on the experience of experts.

α-cut approach

The α-cut of a fuzzy set A of X is a crisp set characterize by A^α defined by a subset of all ingredient $x \in X$ such that their membership functions transgress or equal to a real number $\alpha \in [0, 1]$, as follows:

$$A^\alpha = \left\{x \middle| \mu_A\left(Z^k(x)\right) \geq \alpha, \alpha \epsilon[0,1], \forall x \in X\right\} \quad (10)$$

The triangular fuzzy number is exert by $\tilde{R} = (r_1, r_2, r_3)$ which r_1, r_2, r_3 are crisp numbers and $r_1 < r_2 < r_3$.

Hence, the α-cut of $\tilde{R}$ can be indicated using the following interval:

$$(\tilde{R})^\alpha = \left[(\tilde{R})_L^\alpha, (\tilde{R})_U^\alpha\right] = [(r_2 - r_1)\alpha + r_1, r_3 - (r_3 - r_2)\alpha] \quad (11)$$

Fuzzy decision Bellman and Zadeh

Let X be a given set of all possible solutions to a decision problem. A fuzzy goal G is a fuzzy set on X denoted by its member function

$$\mu_G : X \rightarrow [0,1] \quad (12)$$

A fuzzy constraint C is a fuzzy set on X denoted by its membership function

$$\mu_C : X \rightarrow [0,1] \quad (13)$$

Then G and C incorporate to generate fuzzy decision D on X, which is a fuzzy sets resulting from intersection of G and C, denoted by its membership function as follows:

$$L = \mu_D(x) = \mu_G(x) \wedge \mu_C(x) = Min(\mu_G(x), \mu_C(x)) \quad (14)$$

and corresponding maximizing decision is defined by

$$Max\ L = Max\mu_D(x) = Max\ (Min(\mu_G(x), \mu_C(x))) \quad (15)$$

More generally, assume that the fuzzy decision D is the results of k fuzzy goals $G_1, G_2, G_3, \dots, G_k$ and m constraint $C_1, C_2, C_3, \dots, C_m$. Then, the fuzzy decision D is intersection of $G_1, G_2, G_3, \dots, G_k$ and $C_1, C_2, C_3, \dots, C_m$, and is denoted by its membership function as below:

$$\begin{aligned} L = \mu_D(x) = {} & \mu_{G_1}(x) \wedge \mu_{G_2}(x) \wedge \dots \wedge \mu_{G_k}(x) \\ & \wedge \mu_{C_1}(x) \wedge \mu_{C_2}(x) \dots \wedge \mu_{C_m}(x), \\ = {} & Min\ (\mu_{G_1}(x) \wedge \mu_{G_2}(x) \wedge \dots \wedge \mu_{G_k}(x) \\ & \wedge \mu_{C_1}(x) \wedge \mu_{C_2}(x) \dots \wedge \mu_{C_m}(x)), \end{aligned} \quad (16)$$

and the corresponding maximizing decision is defined in (Bellman & Zadeh, 1970) by

$$\begin{aligned} Max\ L = {} & Max\ \mu_D(x) \\ = {} & Max\ Min\ (\mu_{G_1}(x), \mu_{G_2}(x), \dots, \\ & \mu_{G_k}(x), \mu_{C_1}(x)\mu_{C_2}(x) \dots \mu_{C_m}(x)). \end{aligned} \quad (17)$$

Fuzzy AHP approach

AHP, firstly proposed by Saaty (1980), is a mathematical based method which is recognized as a powerful tool in hand of decision makers and practitioners for analyzing information. It can be employed to solve unstructured problems in various area of decision-making analysis such as political, social, and economic and management sciences (Lee et al., 2005; 2009). AHP is constructed based on decomposing a complex problem into several small sub-problems providing hierarchical framework. Utilizing nine-point numerical scale, elements of each level are compared regarding their impact on the solution of their higher hierarchy element and forms comparison matrix. After checking the inconsistency of decision maker's judgments in each comparison matrix the relative weights of decision elements are determined. Then the relative weights of hierarchies are integrated and lead to obtaining final result.

Besides its usefulness and widely apply to solve the multi-criterion decision making problems, AHP method is often criticized due to its use of unbalanced scale of judgments and its inability to adequately handle the inherent uncertainty and imprecision in the pair-wise comparison process (Deng, 1999; Ertugrul & Karakasoglu, 2009). During AHP implementation, decision makers may not reflect their subjective opinion about comparison element accurately using crisp scales.

To overcome AHP mentioned inadequacy, Fuzzy AHP (FAHP) was developed to tolerate the inherent uncertainty and imprecision of the human decision making process. Incorporation of the fuzzy theory in AHP (using Linguistic variables as triangular fuzzy number) can provide the robustness and flexibility needed for explicitly capturing of decision makers preferences.

There are many FAHP method proposed in the literature. In this paper, the extent analysis method (EAM) is applied, which was which was originally proposed by Chang (1992). The steps of Chang's (1992) analysis can be given as in the following (Kahraman et al., 2004; Cheng, 1992):

(1) The value of fuzzy synthetic extent with respect to ith object is defined as

$$S_i = \sum_{j=1}^{m} M_{g_i}^{j} \otimes [\sum_{i=1}^{n}\sum_{j=1}^{m} M_{g_i}^{j}]^{-1} \tag{18}$$

To obtain $\sum_{j=1}^{m} M_{g_i}^{j}$, perform the fuzzy addition operation of m extent analysis values for a particular matrix such that

$$\sum_{j=1}^{m} M_{g_i}^{j} = \left(\sum_{j=1}^{m} l_j , \sum_{j=1}^{m} m_j , \sum_{j=1}^{m} u_j\right) \tag{19}$$

and to obtain $[\sum_{i=1}^{n}\sum_{j=1}^{m} \mathrm{M}_{\mathrm{g_i}}^{\mathrm{j}}]^{-1}$, perform the fuzzy addition operation of M_{gi}^{j} $(j=1, 2,..., m)$ values such that

$$\sum_{i=1}^{n}\sum_{j=1}^{m} M_{g_i}^{j} = \left(\sum_{i=1}^{n} l_i , \sum_{i=1}^{n} m_i , \sum_{i=1}^{n} u_i\right) \tag{20}$$

and then compute the inverse of the vector in Eq.20 such that

$$[\sum_{i=1}^{n}\sum_{j=1}^{m} \mathrm{M}_{\mathrm{g_i}}^{\mathrm{j}}]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i}\right) \tag{21}$$

(2) The degree of possibility of $M_2=(l_2,m_2, u_2) \geq M_1=(l_1, m_1, u_1)$ is defined as $V(M_2\geq M_1)=Sup_{y\geq x}[min(\mu_{M_1}(x), \mu_{M_2}(y))]$

and can be equivalently expressed as follows:

$$V(M_2 \geq M_1) = hgt(M_1 \cap M_2) = \mu_{M_2}(d) = \begin{cases} 1, & if\ m_2 \geq m_1 \\ 0, & if\ l_1 \geq u_2 \\ \frac{u_1 - l_2}{(m_2 - u_2) - (m_1 - l_1)} & otherwise, \end{cases}$$

where d is the ordinate of the highest intersection point D between μ_{M_1} and μ_{M_2}. In Fig.1. the intersection between M_1 and M_2 can be seen. To compare M_1 and M_2 we need both the values of $V(M_1\geq M_2)$ and $V(M_2\geq M_1)$.

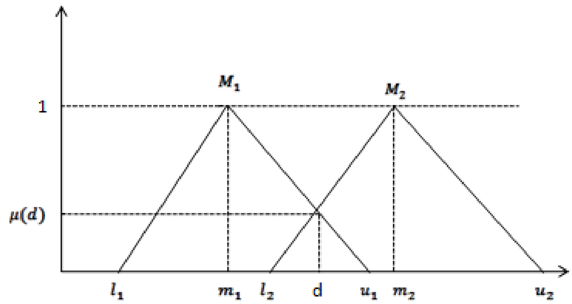

Fig. 1. The intersection between M_1 and M_2

(3) The degree of possibility for a convex fuzzy number to be greater than k convex fuzzy numbers M_i $(i=1, 2,..., k)$ can be defined by
$V(M \geq M_1, M_2,...,M_k)=V[(M \geq M_1$ and $(M \geq M_2),..., (M \geq M_k)]$

Assume that $d'(A_i)=min\ V(S_i \geq S_k)$. For $k = 1, 2,..., n$; $k \neq i$ then the weight vector is given by

$W'=(d'(A_1), d'(A_2),..., d'(A_n))^T$

where $A_i(i=1, 2,...n)$ are elements.

(4) Via normalization, the normalized weight vectors are

$W=(d(A_1), d(A_2),..., d(A_n))^T$

where W is a non-fuzzy number. This gives the priority weights of elements.

Formulation of fuzzy programming with fuzzy constraints

Firstly, the multi-objective linear programming model with fuzzy constraints consider as follows:

$$\text{Minimize} \quad Z_k(x_{ij}) = \sum_{i=1}^{n} \sum_{j=1}^{\alpha(i)} (C_k)\ x_{ij}$$

$$\text{Subject to } X \in S = \{x_{ij} \in X \mid \tilde{A}_{ij} * x_{ij} \le \tilde{b}_i\} \qquad (22)$$

where $\tilde{A}_{ij}$ is a constraint coefficient and $\tilde{b}_i$ is right hand sight coefficients, the entire of co-efficients represented by fuzzy numbers. In this research trapezoidal fuzzy number will survey. Suppose that x_{ij} is the solution of Eq.22, where $\alpha \in [0, 1]$ exert the level of possibility at which all fuzzy coefficients are feasible. Let $(\tilde{R})^{\alpha}$ be the α-cut of a fuzzy number $\tilde{R}$ descript by (Pramanik & Roy, 2008):

$$(\tilde{R})^{\alpha} = \left\{ r \in S(\tilde{R}) \middle| \mu_{\tilde{R}}\left(Z^k(x)\right) \ge \alpha, \alpha \in [0,1] \right\} \qquad (23)$$

where $S(\tilde{R})$ is the protect of $\tilde{R}$. Let $(\tilde{R})_L^{\alpha}$ and $(\tilde{R})_U^{\alpha}$ be the lower and upper bounds of the α-cut of, regularly, Such that

$$(\tilde{R})_L^{\alpha} \le r \le (\tilde{R})_U^{\alpha}, r \in \left[(\tilde{R})_L^{\alpha}, (\tilde{R})_U^{\alpha}\right] \qquad (24)$$

The α-cut close interval of constraint coefficients consist of upper and lower bounds can be presented in two senses:

First Sense: for disparate fuzzy constraints

$$\sum_{i=1}^{n} \sum_{j=1}^{\alpha(i)} (\tilde{a}_{ij})\ x_{ij} \le \tilde{b}_i, i = 1,2,\dots, n \text{ and } j = 1,2,\dots,\alpha(i) \qquad (25)$$

$$\sum_{i=1}^{n} \sum_{j=1}^{\alpha(i)} (\tilde{a}_{ij})\ x_{ij} \ge \tilde{b}_i i = 1,2,\dots, n \text{ and } j = 1,2,\dots,\alpha(i) \qquad (26)$$

The α-cut technique for defuzzify above fuzzy constraints implemented as follow:

$$\sum_{i=1}^{n} \sum_{j=1}^{\alpha(i)} (\tilde{a}_{ij})_L^{\alpha}\ x_{ij} \le (\tilde{b}_i)_U^{\alpha}, i = 1,2,\dots, n \text{ and } j = 1,2,\dots,\alpha(i) \qquad (27)$$

$$\sum_{i=1}^{n} \sum_{j=1}^{\alpha(i)} (\tilde{a}_{ij})_U^{\alpha}\ x_{ij} \ge (\tilde{b}_i)_L^{\alpha}, i = 1,2,\dots, n \text{ and } j = 1,2,\dots,\alpha(i) \qquad (28)$$

Second sense: for parity fuzzy constraint

$$\sum_{i=1}^{n} \sum_{j=1}^{\alpha(i)} (\tilde{a}_{ij})\ x_{ij} = \tilde{b}_i, i = 1,2,\dots, n \text{ and } j = 1,2,\dots,\alpha(i) \qquad (29)$$

In this sense, fuzzy constraint should be commute two constraints as follows:

$$\sum_{i=1}^{n} \sum_{j=1}^{\alpha(i)} (\tilde{a}_{ij})\ x_{ij} \ge \tilde{b}_i, i = 1,2,\dots, n \text{ and } j = 1,2,\dots,\alpha(i) \qquad (30)$$

$$\sum_{i=1}^{n} \sum_{j=1}^{\alpha(i)} (\tilde{a}_{ij})\ x_{ij} \le \tilde{b}_i, i = 1,2,\dots, n \text{ and } j = 1,2,\dots,\alpha(i) \qquad (31)$$

Likely, the α-cut approach perform base on Eqs.27 and 28 in order defuzzify parity fuzzy constraint. In next process, in order the α-cut approach implemented for multi-objective linear programming problem with fuzzy constraints can be Eq.22 commuted to deterministic multi-objective linear programming problem as follows:

$$\text{Minimize } Z_k(x_{ij}) = \sum_{i=1}^{n} \sum_{j=1}^{\alpha(i)} C_k\ x_{ij} \qquad (32)$$

Subject to:

$$\sum_{i=1}^{n} \sum_{j=1}^{\alpha(i)} (\tilde{a}_{ij})_L^{\alpha}\ x_{ij} \le (\tilde{b}_i)_U^{\alpha}, i = 1,2,\dots, n \text{ and } j = 1,2,\dots,\alpha(i) \qquad (33)$$

$$\sum_{i=1}^{n} \sum_{j=1}^{\alpha(i)} (\tilde{a}_{ij})_U^{\alpha}\ x_{ij} \ge (\tilde{b}_i)_L^{\alpha}, i = 1,2,\dots, n \text{ and } j = 1,2,\dots,\alpha(i) \qquad (34)$$

Interactive fuzzy multi-objective linear programming (i-FMOLP) solution method

Fuzzy multi-objective project management (FMOPM) problem can be solved using fuzzy decision-making of Bellman and Zadeh (1970). The linear membership functions are denoted to show the fuzzy sets encompassed. Therefore, by introducing the auxiliary variable β the crisp MOPM problem can be transformed to equivalent single-objective LP problem. The MOPM problem can be formulated as follows (Liang, 2006):

$$\begin{aligned}
&\max \beta \\
&S.t. \\
&\beta \leq \mu_k\left(Z^k(x)\right), \\
&E_i + t_{ij} - E_j \leq 0 \quad \forall i, \forall j \\
&t_{ij} = D_{ij} - d_{ij} \quad \forall i, \forall j \\
&Y_{ij} \leq D_{ij} - d_{ij} \quad \forall i, \forall j \\
&Z_1 \leq B \\
&t_{ij}, Y_{ij}, E_i, E_j \geq 0 \; \forall i, \forall j
\end{aligned} \tag{35}$$

where, the auxiliary variable is the overall degree of DM's satisfaction with the specified multiple-objective values. The interactive solution procedure of the proposed i-FMOLP for solving multi-objective project management (MOPM) problem can be described as follow (Liang, 2006):

Step 1: Formulate the initial fuzzy multi-objective project management (FMOPM) problem.

Step 2: Determine corresponding Linear Membership Function (LMF) for entire of the objective functions according to Eq.9.

Step 3: Represent the auxiliary variable , and aggregate the fuzzy multi-objective project management (FMOPM) problem in to an equivalent ordinary single objective LP model using the minimum operator.

Step 4: Solve the LP problem and acquire initial compromise solutions.

Step 5: Execute interactive decision process. If DM not satisfied with the initial compromise solution, the model must be changed until satisfactory solution found.

Weighted Additive solution method

The weighted additive model is greatly used in vector objective optimization problems; the initial of the overall precedence of DM to draw out the relative importance of criteria (Lai & Hwang, 1994). In this case, a linear weighted utility function is attained by multiplying each membership function of fuzzy goals by their Proportionate weights and then aggregating the out comes together (Amid et al., 2009).

Tiwari et al. (1987) proposed a weighted additive model which uses flexibility to determine the priority of the fuzzy goals. The model is defined as follows:

$$\begin{aligned}
&\mu_k \leq \frac{Z^k(x) - L_k}{U_k - L_k} \left(or \mu_k \leq \frac{U_k - Z^k(x)}{U_k - L_k}\right), \text{(For all objectives)} \\
&Ax_i \leq b_i,\; i = 1,2,\dots,n \\
&\mu_k \in [0,1], \\
&\textstyle\sum_{k=1}^{K} W_k = 1,\; k = 1,2,\dots,K \\
&x_i, \geq 0 \quad k \in K
\end{aligned} \tag{37}$$

where W_k the weighting coefficients indicate relative significance the between fuzzy goals. That aspiration levels obtains in fuzzy goals. In this condition, DM achievement degrees intensify and MOPM model close to ideal solution. To exploit weighted or preference between goals from a DM is very chief basic process to solve this model. In order delineating the weighted of goals in this research the Fuzzy AHP (FAHP) approach is considered.

MODEL IMPLEMENTATION

The FMOLP model proposed in this paper has been examined by using the case study defined in Liang (2009; 2010). Thus it is possible to compare the results of interactive fuzzy multi-objective linear programming (i-FMOLP) and Weighted Additive solution methods representing fuzzy goal and fuzzy constraints with regard to the two phase fuzzy goal programming (TPFGP) solution method employed by Liang (2009). Now, we give a case study description for our study. The firm, where the model was tested is the Daya Technologies Corporation. The Daya Technology is the leading producer of precision machinery and transmission components in Taiwan, and is the main manufacturer producing the super precision ballscrew, linear stage, linear bearing, guide ways, and aerospace parts. Its

Table 2: Preliminary information of the Daya case

(i, j)	D_{ij}	d_{ij}	C_{Dij} *($)*	C_{dij} *($)*	k_{Dij} *($/ day)*
1-2	14	10	1000	1600	150
1-5	18	15	4000	4540	180
2-3	19	19	1200	1200	-
2-4	15	13	200	440	120
4-7	8	8	600	600	-
4-10	19	16	2100	2490	130
5-6	22	20	4000	4600	300
5-8	24	24	1200	1200	-
6-7	27	24	5000	5450	150
7-9	20	16	2000	2200	50
8-9	22	18	1400	1900	125
9-10	18	15	700	1150	150
10-11	20	18	1000	1200	100

products are distributed throughout Asia North America and Europe and have been in high demand for several years. The real-life PM decision examined here involves expanding a metal finishing plant owned by Daya. The deterministic CPM technique currently used by Daya suffers from the limitation owing to the fact that the project manager does not have sufficient information over the planning horizon. The case study focuses on expanding i-FMOLP and Weighted Additive solution methods to expand an appropriate PM plan for the metal finishing plant in a fuzzy environment. The PM decision of Daya target to simultaneously minimize total project costs, total completion time and total crashing costs in terms of direct costs, indirect costs, activity and crash durations, and the constraint of available budget Table 3.0 list of the preliminary information of the case. Other correlate information as follows: fixed indirect cost 12000$ saved daily variable indirect costs 150$. Total budget 38500$ and project completion duration under normal conditions 125 days. The project start time is set to zero. The critical path is 1-5-6-7-9-10-11. Fig.2. displays the activity-on-arrow network diagram.

Now using -cut technique and interactive fuzzy multi-objective linear programming (i-FMOLP) solution method, we consider different -cut levels, the lower and upper bounds of the -cut the optimal solution goal values represented in Table 3. In this sense, FMOPM decision problem convert to FMOPM decision problem with fuzzy goals and deterministic constraints. More ever, the interactive fuzzy multi-objective linear programming (i-FMOLP) solution procedures proposed for solve FMOPM decision problem with fuzzy goals. The solution procedure using the proposed

i-FMOLP for the Daya case is described can be found in (Liang, 2009; 2010).

Table 3 represents the results obtained by α-cut approach and i-FMOLP solution method which adds the satisfaction degree of the objective functions. Similar results obtain for the total project costs (Z_1), total completion time (Z_2) and total crashing costs (Z_3) for various α-cut levels in 0.1 to 0.9. The best results, according to DM preferences are obtained when the α-cut level is lower.

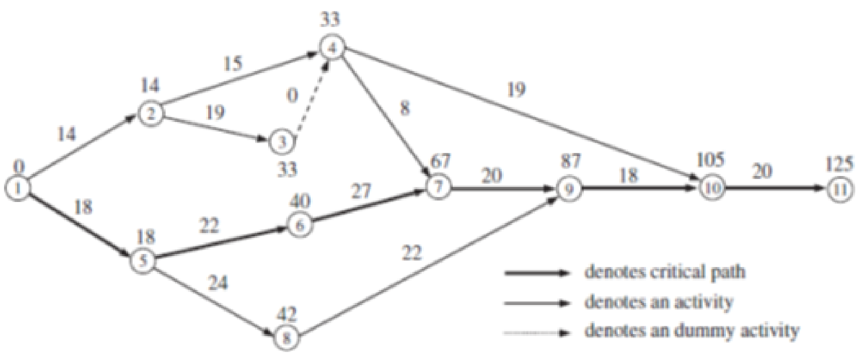

Fig.2. The project network of the Daya case

Table 3: Optimal solutions α_1 to α_9 relate to α-cut level in i-FMOLP method

α-cut level	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Y_{79} *(days)*	6.9	6.8	6.7	6.6	6.5	6.4	6.3	6.2	6.1
Y_{1011} *(days)*	0.678	1.011	1.345	1.687	2.098	2.510	2.921	3.332	3.744
E_{11} *(days)*	56.721	59.788	62.854	65.912	68.901	71.880	74.878	77.867	80.855
Z_1	26571.05	27059.39	27547.75	28035.62	28520.05	29004.49	29488.92	29973.34	30454.65
Z_2	56.72	59.78	62.85	65.91	68.90	71.88	74.87	77.86	80.85
Z_3	412.82	441.19	469.50	498.74	534.88	571.02	607.15	673.29	679.4
μ_{Z1}	0.969	0.966	0.963	0.960	0.957	0.954	0.951	0.948	0.946
μ_{Z2}	0.967	0.964	0.962	0.960	0.958	0.956	0.953	0.951	0.949
μ_{Z3}	0.967	0.964	0.962	0.960	0.957	0.954	0.951	0.948	0.946

As mentioned before, a low α value means that the model attempts to find a solution by focusing more on obtaining a better satisfaction degree for the objective functions. A high value of α means that the model attributes less important to DM achievement level for the objective functions. For this reason, when α-cut level increases, the achievement level of the objective functions Z_1, Z_2 and Z_3 decreases. On the other hand, when α-cut level increases the total project costs, total completion time and the total crashing costs are higher and hence the achievement level for each one (μ_{Z1}, μ_{Z2} *and* μ_{Z3}) is lower. Fig. 3. exhibit Crash time for activity (7, 9) and (10, 11) according to α-cut level approach. When the -cut level increases, the crash time for activity (7, 9) decreases. Moreover, when the α-cut level increases, the crash time for activity (10, 11) is increases. Fig.4. display project completion time base on α-cut level approach. As shown in Fig.4. when the α-cut level increases, the project completion time values are increase.

Now, to assess the performance of the proposed solution approaches, we survey the solution of computational example in special sense *α=0.5* using i-FMOLP and Weighted Additive solution methods. Table 4 demonstrates the obtained results of two various solution methods. The interactive fuzzy multi-objective linear programming (i-FMOLP) involves the following results: Z_1=$28520.05, Z_2=$68.90 days and Z_3=$534.88. The proposed Weighted Additive approach gave the following consequences: Z_1=$29275, Z_2=$77.5, days and Z_3=$0. To specify the degree of closeness of the three solution approach results to the desired solution, we remark the following family of distance functions (El-wahed & Lee, 2006):

$$D_P(\lambda, K) = \left[\sum_{k=1}^{K} \lambda_k^p \,(1 - d_k)^p\right]^{1/p} \tag{37}$$

where d_k shows the degree of closeness of the preferred compromise solution vector X^* to the

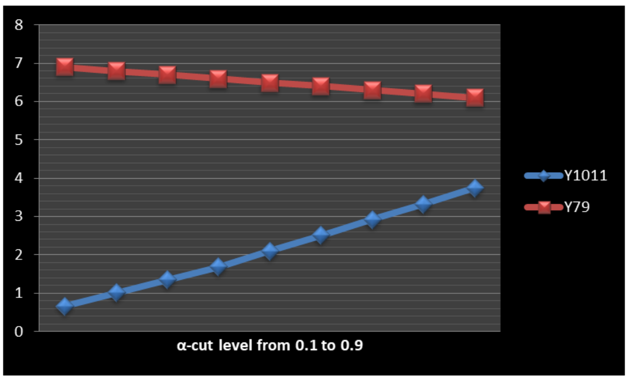

Fig. 3. Crash time for activity (7, 9) and (10, 11)

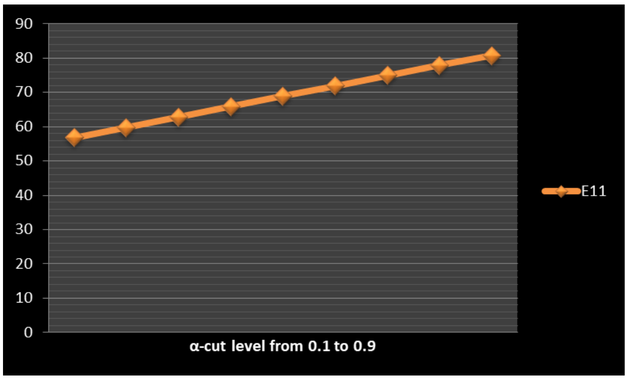

Fig. 4. Project completion time

optimal solution vector with respect to the kth objective function $\lambda=(\lambda', \lambda_2, ..., \lambda_k)$ is the vector of objectives aspiration levels. The power p represents a distance parameter $1<p<\infty$ supposing, $\sum_{k=1}^{K} \lambda_k = 1$, We can $Dp(\lambda, k)$ with $p=1, 2$ and ∞ as follow (El-wahed & Lee, 2006):

$$D_1(\lambda, K) = 1 - \sum_{k=1}^{K} \lambda_k d_k, \tag{38}$$

(The Manhattan distance)

$$D_2(\lambda_k, K) = [\sum_{k=1}^{K} \lambda_k^2 (1-d_k)^2]^{\frac{1}{2}} \tag{39}$$

(The Euclidean distance)

$$D_\infty(\lambda_k, K) = max_k\{\lambda_k(1-d_k)\}, \tag{40}$$

(The Tchebycheff distance)

where in a minimization problem, d_k takes the form: d_k= (the optimal solution of Z^k) / (the preferred compromise solution). Also, in a maximization problem d_k obtain as follows: d_k= (the preferred compromise solution Z^k) / (the optimal solution of Z^k). We assume $\lambda_1=0.5$, $\lambda_2=0.3$, $\lambda_3=0.2$ in above equations. Table 4 summarizes the results of the two approaches (three solutions).

We analogy the degree of closeness of two employed approach with the desired solution summarized in Table 4. In this table, the preferred compromise solution of the proposed i-FMOLP solution method which is better than the solution by Weighted Additive solution method for all distance functions D_1, D_2, D_∞. Hence, the i-FMOLP approach is better solution method than Weighted Additive for solving FMOPM decision problem with fuzzy goals and fuzzy constraints in Daya case study. Similar results obtain for all α-cut level from 0.1 to 0.9.

SUMMERY AND CONCLUSION

In the real world project management decision problem, the project decision maker encounter with two basic topics: 1) The multiple conflicting goals because observe the consumption of constraints resource, 2) The polarize project parameters like as completion time, crashing costs and total available capital in order vague and imprecise information reach in project DM for virtual project environments. Thus, we must conquest in PM decision uncertainty problems; one of the important techniques is Fuzzy Sets Theory (FST) that this theory can contrast with incomplete information for project DM pylon practical PM decision environments. This research aim to extent Fuzzy Multi-Objective Project Management (FMOPM) decision problem in fuzzy goal and fuzzy constraints that can contrast with imprecise data face to DM in real-world project environments. We mixture α-cut technique and two various solution methods for solving FMOPM decision problem. Two solution methods consist of (1) the interactive fuzzy multi-objective linear programming

(i-FMOLP) and (2) Weighted Additive fuzzy goal programming are adopted using linear membership functions (LMF). The weighted of criteria for each goals computed by Fuzzy AHP (FAHP) approach. The advantage of the applied methods is that provides a systematic framework that facilitates the fuzzy decision making process to obtain a satisfactory solution. Therefore, the interactive and non-interactive solution methodology presented here yields an efficient compromise solution and serves the overall DM satisfaction with the determined goal values in FMOPM decision problem.

Finally, the performances of the proposed solution methods are evaluated by using a set of metric distance respectively ideal solution. The

Table 4: Comparison of the degree of closeness (special case α=0.5)

	(1) i-FMOLP	(2) Weighted Additive	Desire solutions
Z_1	28520.05	29275	21350
Z_2	68.90	77.5	12
Z_3	534.88	0	0
D_1	0.573	0.588	-
D_2	0.342	0.350	-
D_∞	0.247	0.253	-

obtained results demonstrate that the i-FMOLP solution method is more efficient and power tools than Weighted Additive solution method for solving FMOPM decision problem in Daya case study.

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