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Data Envelopment Analysis from Simulation on the Lattice QCD Using CCR Model

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Abstract

One of the most serious principles in production theory in economic is the principle of "efficiency". Simply put, efficiency can be defined as the demand that the desired goals (outputs) are achieved with the minimum use of the available resources (inputs). In order to, distinguish the relative efficiency of organizational units with multiple inputs to produce multiple outputs, "Data Envelopment Analysis" (DEA) method was introduced by Charnes, Cooper and Rhodes. In fact, DEA is a linear mathematical programming which calculates the efficiency of an organisation within a group relative to observed best practice within that group. Unlike common statistical analysis which are based on central tendencies, it is a methodology directed at the frontier. Recently, DEA has become one of the most favorite fields in operations research. The background was a motivation for us to in this paper, via running the CCR model in "DEA-Solver Software", present data envelopment analysis from simulation on the lattice QCD with temporal extent N_τ=4,6, respectively. Astonishingly, results are derived for both cases, indicating the fact that efficient data set belong to the areas of high temperature (deconfinement phase). It is very interesting to highlight that even an efficient data has not reported at low temperature (confinement phase). Note that the data obtained at the critical temperature is also efficient. As expected from practical lattice QCD, the DEA-CCR model presented in this paper also confirms the fact which the best data set arises from simulation in continuum limit $a \to 0$. Indeed, by taking the limit of vanishing lattice spacing, the efficiency of algorithms can be further.

Keywords:
Data Envelopment Analysis
CCR model
ranking model
Lattice QCD
continuum limit

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INTRODUCTION

Quantum chromodynamics or QCD is the fundamental theory to demonstrate of the strong nuclear force in hadrons in which quarks interacting with non-Abelian *SU(3)* gauge fields (gluons) (mandls & shaw, 2010; peskin et al., 1996). At low-energy, QCD has tow separate property: the confinement and the spontaneous breaking of the chiral symmetry. Confinement will cause closure quarks and gluons in hadrons and the origin of large effective masses of quarks and light pseudoscalar mesons come back to spontaneous breaking of the chiral symmetry. At high-energy, QCD has an important feature, the asymptotic freedom. In other words, the QCD coupling decreases at short length. So, the perturbation theory is applicable to high energy phenomena with momentum transfer $q \gg \Lambda_{QCD}$, where $\Lambda_{QCD} \approx 200\text{-}300MeV$ is a typical energy scale of QCD. Clearly, as the energy scale approaching down to Λ_{QCD}, the coupling becomes very strong and result in the failure of perturbation theory. So, the real question is what is the alternative approach?

In order to investigate on the QCD in low-energy, one must first define the theory beyond perturbation theory. The lattice regularization provides a clean way of doing this by replacing the space-time continuum with a discrete mesh of lattice points. Notethat one should not be confused with the notion that lattice is as an approximation to the continuum theory. Indeed, it provides a definition of a theory that is undefined directly in the continuum. Of course, in order to regain the continuum limit, the theory must be renormalized through sending the lattice spacing to zero while adjusting the bare coupling constants suitably. In the beginning, Lattice QCD (LQCD) was offered approximately 40 years ago by wilson (Ginsarg & Wilson, 1982) as a powerful quantitative approach to the analyze the nonperturbative nature of QCD. Indeed, LQCD is a gauge field theory on 4D Euclidean lattice spacetime, and quantized along the Feynman's path integral formalism. The quantized theory can be regarded as a statistical mechanics system, which accredit us to fulfill numerical simulations by the Monte Carlo method. With development of computational equipments as well as numerical algorithms, lattice QCD simulations as a technique, is used to compute many quantities of phenomenological interest such as hadron masses, hadronic matrix elements, the finite temperature phase transition and so forth. Also one of its major applications is study the spontaneous breaking of chiral symmetry. Overall, a non-perturbative lattice calculation proceeds in 3 steps, for more details refer to (Fodor & Hoelbling, 2012). Simulations of LQCD is involve six unfamiliar input parameters such as the coupling constant α_s and the masses of the all quarks except top quark (since the t quark is very short livedto form bound states i.e. 0.4×10^(-24) s, thus not worthy for lattice simulations.) (Gupa, 1998). Overall, the lattice QCD simulation is performed as three step: first of all, generating gauge configurations. second, measuring physical observables and finally, analyzing measured quantities. To learn more details on each of these steps, we recommend refer to Refs (Gupa, 1998؛ Richards, 2001; Shifman, 2001; Creutz, 2011).

Data Envelopment Analysis (DEA) is an more and more popular management tool. Usually, it use to evaluate the efficiency of a number of producers. A typical statistical approach is according to evaluates producers relative to an average producer. But contrary, DEA compares each producer with only the "best" producers. Meantime, in DEA literatures, a producer is usually referred to as a "decision making unit" (DMU). In DEA, there are a number of producers so that production process for each producer is to take a set of inputs and produce a set of outputs. Of course, each producer has a different level of inputs and gives a different level of outputs. As example, imagine a set of producers so that each producers has a certain number of inputs and outputs. Ingenerally, DEA attempts to determine which of the producers are most efficient, and to point out specific inefficiencies of the other producers. In other words, DEA is a methodology for evaluating and measuring the relative efficiencies of a set of decision making units (DMUs) that use multiple inputs to produce multiple outputs) Shokuhi et al., 2010; Cooper et al., 2006).This method is based on the assumption that if a given producer A is able of producing *Y(A)* units of output from *X(A)* units of input, then other producers should also be able to do the same if they were to operate efficiently. In a similar way, if producer B is able of producing *Y(B)* units of output

with X(B) inputs, then other producers should also be capable of the same production syllabus. Producers A, B, and others can then be combined to form a mixture producer with mixture inputs and outputs, respectively. As regards, this mixture producer does not necessarily exist, it is known as the "virtual producer". Mainly,the heart of the DEA method lies in finding the best virtual producer for each real producer. If the virtual producer result in making more output with the same input or making the same output with less input then the original producer is inefficient. The DEA method is an alternative non-parametric approach. In other words, no need to adopting specific production functions for relate inputs with outputs, theevaluation of the efficiency of the system is done with empirical data. The DEA in particular can be carried out either with the assumption of Constant Returns to Scale (CRC) according to the model of (Charnes et al., 1978). CRS means that the producers are able to linearly scale the inputs and outputs without increasing or decreasing efficiency, i.e. if for example the number of the X (productive factors or inputs) is doubled, then the quantity of the Y (outputs(is doubled as well. It is an important point that classical model presented by Cooper et al. in 1978 is on the basis of Farrellâ€™s work in (Farrell, 1957) and so-called CCR model. Also DEA can be carried out either with the assumption of Variable Returns to Scale (VRS) according to the model of (Banker et al., 1984). DEA dedicate a grade of 1 to a unit only when comparisons with other relevant units do not provide evidence of inefficiency in the use of any input or output. Hence, DEA dedicate an efficiency grade less than one to (relatively) inefficient units. To understand what is necessary to explain that these grades indicates the radial distance from the estimated production frontier to the DMU under consideration.

Generally, the main body of this paper devoted to the analysis of the LQCD data (rather with temporal extent $N_\tau=4,6$) using DEA method assuming Constant Returns to Scale (CRC) according to the classical model of CCR (Charnes et al., 1978). We also according to AP method (Khodadi & Seangi, 2014) rankings DMUs obtained from simulation on the LQCD, separately. Finally, presented a brief discussion on the results extracted from the DEA-CCR model in this paper and the main framework of the LQCD.

OVERVIEW OF THE CCR MODEL

In this section, we plan to overview one of the most basic DEA models, the CCR model, which was initially proposed in (Charnes et al., 1978). There, for each DMU, we formed the virtual input and output by weights α_r and β_r, respectively

$$I_V=\alpha_1 x_{1o}+\alpha_2 x_{2o}+.....+\alpha_m x_{mo};$$
$$O_V=\beta_1 y_{1o}+\beta_2 y_{2o}+.....+\alpha_s x_{so}, \quad (1)$$

Where I_V, O_V refer to virtual input and output, respectively. Then we tried to characterize the weight, using linear programming so as to maximize the ratio

$$\frac{\Sigma_{r=1}^{s}\beta_r y_{ro}}{\Sigma_{i=1}^{m}\alpha_i x_{io}} \quad (2)$$

Note that the optimal weights may vary from one DMU to another. Suppose there are n DMUs: $\{DMU_1,DMU_2,.....DMU_n\}$. Input and output items for each of DMUs should have included two common features. First of all, numerical data are available for each input and output, with the data assumed to be positive for all DMUs. Latter, the items, inputs, outputs and choice of DMUs, should reflect an analyst's in the components that will enter into the relative efficiency evaluations of the DMUs. Now, assume m input items and s output items are selected by preserving these two features. So, the input and output data for DMUj be $\{x_{1j},x_{2j},.....,x_{mj}\}$ and $\{y_{1j}, y_{2j}, ..., y_{mj}\}$, respectively. Accordingly, the input data matrix $X_{m\times n}$ and the output data matrix $Y_{s\times n}$ can be arranged as follows

$$X_{m\times n}=\begin{pmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ x_{31} & x_{32} & \dots & \vdots \\ \vdots & \vdots & \dots & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{pmatrix};$$

$$Y_{m\times n}=\begin{pmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ y_{31} & y_{32} & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ y_{s1} & y_{s2} & \cdots & y_{sn} \end{pmatrix} \quad (3)$$

For instance, x_{mj} refer to mth input of jth DMU or y_{mj} refer to mth output of jth DMU. Given the data, we measure the efficiency of each DMU once and hence need n optimizations, one for each DMUj to be evaluated. So, to obtain valuesfor the input "weights" α_i and the output "weights" β_r as variables, must be solved the following fractional programming (FP_o)

$$Max\theta_0=\frac{\beta_1 y_{10}+\beta_2 y_{20}+\ldots\ldots+\alpha_s x_{so}}{\alpha_1 x_{1o}+\alpha_2 x_{2o}+\ldots\ldots+\alpha_m x_{mo}} \quad (4)$$

subject to

$$\frac{\Sigma_{r=1}^{s}\beta_r y_{rj}}{\Sigma_{i=1}^{m}\alpha_i x_{ij}}\leq 1 \quad j=1,2,\ldots,n, \quad \alpha_i,\beta_r\geq 0. \quad (5)$$

So that, o ranges over $1,2,\ldots,n$. The constraints (5) mean that the ratio of "virtual output" to "virtual input" should not exceed 1 for every DMU. Indeed, the aim is to obtain weights β_r and α_i that maximize the ratio of DMU_o. By virtue of the constraints, the optimal (maximum) objective value θ^* is more than 1. Next, we replace the above FP_o by the following linear program LP_o

$$Max\theta=\beta_1 y_{1o}+\beta_2 y_{2o}+\ldots\ldots+\beta_s x_{so}, \quad (6)$$

subject to

$$\alpha_1 x_{1o}+\alpha_2 x_{2o}+\ldots\ldots+\alpha_m x_{mo}=1,$$
$$\beta_1 y_{1j}+\ldots\ldots+\beta_s x_{sj}\leq\alpha_1 x_{1j}+\ldots\ldots+\alpha_m x_{mj}; \quad (j=1,2,\ldots,n),$$
$$\alpha_1,\alpha_2,\ldots,\alpha_m\geq 0; \quad \beta_1,\beta_2,\ldots,\beta_s\geq 0. \quad (7)$$

Based on the two Theorem in (Anderson & Petersen, 1933; Khezrimotagh et al., 2012), the FP_o is equivalent to LP_o and are units invariant. In other words, them have the same optimal objective value θ^* and are independent of the units of measurement used in the sense that, multiplication of each input by a constant $\lambda_i>0, i=1,\ldots,m$ and each output by a constant $\gamma_r>0, r=1,\ldots,s$ does not change the obtained solution. In here we can then identify that DMU_0 is CCR-efficient if $\theta^*=1$ and there is at least one optimal (α^*,β^*) with $\alpha^*>0$ and $\beta^*>0$. Otherwise, DMU_0 is CCR-inefficient. Thus, CCR-inefficiency means that either $\theta^*<1$ or $\theta^*=1$ and at least oneelement of (α^*,β^*) is zero for every optimal solution of LPo. As a result, (α^*,β^*) are the set of most favorable weights for the DMUo in the sense of maximizing the ratio scale.

AP RANKING MODEL

In the DEA, one may be faced with a set of efficient DMUs. On the other, CCR is not capable of ranking technical efficient DMUs. Hence, ranking efficient DMUs is become as a most interesting discussion for large number of DEA researches. So far, diverse DEA models have been formulated for ranking efficient DMUs. As example in (Charnes et al., 1973; Charnes et al., 1962) a brief review is made â€‹â€‹by authors, so that to learn more about the details, one can refer to the references. In the present paper, particular focus is on the classical modified model formulated by Andersen and Petersen (AP) in) Khodadi & Sepangi, 2014). In short, it is based upon comparison of technical efficient DMUs than a reference technology spanned by other DMUs (Cheng et al., 2008). More precisely, this method is based on the position of each removed efficient DMU in relation to its corresponding new Production Possibility Set (PPS) (Charnes et al., 1973). AP modified model proposed as

$$Max \quad \sum_{r=1}^{s} u_r y_{r0}$$
$$S.t \quad \sum_{i=1}^{m} v_r x_{i0}=1$$
$$\sum_{r=1}^{s} u_r y_{rj}-\sum_{i=1}^{m} v_r x_{ij}\leq 0, \quad j=1,\ldots,n, j\neq 0 \quad (8)$$

So that, $u_r\geq\epsilon$, $r=1,2,\ldots,m$ and $v_i\geq\epsilon$, $i=1,2,\ldots,m$. and its dual is

$$Min \quad \theta-\left(\sum_{i=1}^{m} s_i^- - \sum_{r=1}^{s} s_r^+\right)$$
$$S.t \quad \sum_{j=1,j\neq 0}^{n} \lambda_j x_{ij}+s_i^-=\theta x_{i0}, \quad i=1,\ldots,m$$
$$\sum_{j=1,j\neq 0}^{n} \lambda_j y_{rj}+s_r^+=y_{r0}, \quad r=1,\ldots,s \quad (9)$$

So that $\lambda_j \geq 0, \quad j=1,.....,n, j\neq 0, s_i^- \geq 0, \quad i=1,.....,m$ and $s_r^+ \geq 0, \quad r=1,.....,s$.

It should be emphasized that, efficiency obtained in this method, does not change the performance of the inefficient DMUs in DEA-CCR model. In AP method removing any of the DMUs on the efficient frontier, the new PPS will be constructed on the basis of remain efficient DMUs. So, production frontier will be shifted so that the efficiency of the DMU will be determined based on the distance of the new frontier.

LATTICE QCD DATA WITH N_τ=4,6

At the beginning of this section, we would prefer recall some basic thermodynamic relations, to start our discussion of QCD thermodynamics on the lattice. Usually all thermodynamic quantities follow from pressure, however in lattice QCD it is convenient to introduce a dimensionless quantity called the "interaction measure" (Cheng et al., 2006). Indeed, the interaction measure is the trace of the energy momentum tensor $\Theta^{\mu\mu}$ (T) divided by T^4 which in terms of a derivative of the pressure with respect to temperature, is defined as

$$\frac{\Theta^{\mu\mu}(T)}{T^4} \equiv \frac{\varepsilon - 3p}{T^4} = T\frac{\partial}{\partial T}(p/T^4) \qquad (10)$$

All other bulk thermodynamic observables, e.g. p/T^4 and ε/T^4 are derived from (10). As example be obtained the pressure from $\Theta^{\mu\mu}(T)$ through integration of (10)

$$\frac{p(T)}{T^4} - \frac{p(T_0)}{T_0^4} = \int_{T_0}^{T} dT' \frac{\theta^{uu}(T')}{T'^5} \qquad (11)$$

$[T_0]$ is the temperature for the lower integration limit. Conveniently, $[T_0]$ be set in hedonic phase of QCD. In lattice QCD thermodynamics, temperature is given in terms of the temporal α, $T=1/(N_\tau\, \alpha(\beta))\ [N_T]$ and lattice spacing

It is very interest that lattice spacing is controlled via lattice gauge coupling

$\beta=6/g^2$

In generally, all observables that are derived on the lattice, are functions of the gauge coupling β With avoid mentioning details, (10) in terms of observables compute in lattice calculations at zero and non-zero temperature with three flavor quarks (β), be written as (Cheng et al., 2008)

$$\frac{\Theta^{\mu\mu}(T)}{T^4} = \frac{\Theta_G^{\mu\mu}(T)}{T^4} + \frac{\Theta_F^{\mu\mu}(T)}{T^4} + \frac{\Theta_h^{\mu\mu}(T)}{T^4},$$

So that

$$\begin{aligned}
\frac{\Theta_G^{\mu\mu}(T)}{T^4} &= R_\beta[< S_G >_0 - < S_G >_\tau]N_\tau^4, \\
\frac{\Theta_F^{\mu\mu}(T)}{T^4} &= -R_\beta R_m[2\hat{m}_l(< \bar{\psi}\psi >_{l,0} - < \bar{\psi}\psi >_{l,\tau}) + \hat{m}_s(< \bar{\psi}\psi >_{s,0} - < \bar{\psi}\psi >_{s,\tau})]N_\tau^4 \\
\frac{\Theta_h^{\mu\mu}(T)}{T^4} &= -R_\beta R_h\hat{m}_s[< \bar{\psi}\psi >_{s,0} - < \bar{\psi}\psi >_{s,\tau})]N_\tau^4.
\end{aligned}$$

(11,12)

In here, < :::: >_;0 refers to expectation values evaluated on _nite temperature and zero tem-premature lattices of size N3 _N_ , respectively. Of course zero temperature lattices, corresponds on lattices with large temporal extent, i.e. N_ & N_. Term h (T) T4 = 0 since Rh vanishes on the line of constant physics (LCP)[1]. For further review and understanding more accurate computational details, one can refer to Refs (Cheng et al., 2008 Cheng et al., 2006; Aoki et al., 2006; Huovinen & Petreczky, 2010) . But our main purpose in this section (overall the whole paper) present a DEA from data on lattice QCD (with temporal extent N_ = 4; 6) reported in (Cheng et al., 2008).

Lattice QCD data with N_ = 4; 6

In (Cheng et al., 2008) authors reported on a calculation of bulk thermodynamics in QCD with a tree level

Symanzik improved gauge action and an improved staggered fermions action. They in a wide range of temperatures (140MeV _ T _ 800MeV), carry out simulations with two degenerate light quark masses (u; d) and a heavier strange quark (s) on lattices with temporal extent N_ = 4; 6, respectively. Preferably, in this subsection focus on lattice QCD data with N_ = 4. After sorting data reported in Tables III and IV in (Cheng et al., 2008) , let us consider 19 DMUs with eight inputs and two outputs in Table 1. Needless to say that inputs x1::::::x8 refer to the values obtained in the simulation for quantities such as _, ms=ml, < SG >0, < _ >l;0, < _ >s;0, < SG >_

The LCP is defined as dependence between the bare lattice parameters such as _ and lattice bare quark masses mu;d;s. In fact, these relationships are insisting on the principle that the physics(as example mass ratios) remains constant, Whereas varying any of the parameters

Table 1:19 DMUs (eight inuts and two outputs) obtained from simulation on the LQCD whit N4=4

DMU	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	Y_1	Y_2
1	3.460	0.313	4.04471	0.0573	0.11734	4.00931	0.007148	0.06878	3.57	3.5917
2	3.760	0.13	3.580005	0.00578	0.027805	3.57727	0.001998	0.019966	0.404	4.6498
3	3.540	0.240	3.89302	0.0281	0.07513	3.87812	0.004627	0.045837	1.678	4.1974
4	3.277	0.765	4.5166	0.20232	0.2983	4.5001	0.17784	0.28688	3.18	0.3208
5	3.570	0.212	3.84392	0.02176	0.062829	3.83212	0.003904	0.038807	1.378	4.3116
6	3.335	0.62	4.36041	0.1542	0.24425	4.28541	0.04964	0.19082	10.77	1.0757
7	3.351	0.591	4.3188	0.1417	0.23045	4.2453	0.03744	0.17423	9.68	1.4748
8	3.382	0.52	4.23499	0.1151	0.19922	4.16623	0.01875	0.10229	5.56	2.8435
9	3.410	0.412	0.19922	0.09013	0.16256	4.10463	0.011657	0.13797	7.7	2.2418
10	3.150	1.1	4.82564	0.28727	0.392677	4.82413	0.28165	0.39082	0.54	0.0639
11	3.490	0.29	3.9845	0.04424	0.10072	3.95941	0.006156	0.060172	2.668	3.8864
12	3.510	0.259	3.94694	0.03657	0.08764	3.92564	0.0052568	0.05183	2.249	4.0322
13	3.240	0.9	4.61441	0.23156	0.33395	4.60904	0.21962	0.3292	1.23	0.206
14	3.920	0.11	3.396477	0.002967	0.019635	3.39538	0.001544	0.015434	0.188	4.7156
15	3.630	0.17	3.75291	0.013176	0.045175	3.74581	0.002912	0.029047	0.896	4.4751
16	3.690	0.15	3.669908	0.00874	0.035734	3.66559	0.002431	0.024276	0.592	4.5789
17	3.210	1	4.68944	0.25284	0.358813	4.68525	0.24357	0.35522	1.03	0.1492
18	3.820	0.125	3.508124	0.004467	0.024666	3.5062	0.001628	0.016268	0.273	4.683
19	3.290	0.65	4.47696	0.18807	0.27506	4.45142	0.15132	0.25654	4.61	0.4037

, < _ >l;_ , < _ >s;_ , respectively. Also outputs y1 and y2 refer to the values obtained in the simulation for thermodynamics quantities such as "□3pT4 and p=T 4, respectively. Now hat we've indented the input and output variables, can using CCR model be measured the efficiency and performance of each DMUs. The results of data analysis Table 1, are reported in Table 2. In the last column of Table 2, using the AP model of DEA were able to rank the efficiency of each units of lattice QCD data with N4 in terms of their performance. As can be seen, DMU18 and DMU17 have _rst and last grade, respectively. This indicates that the DMU18 is as a reference for other DMUs, since it has achieved the best possible outcomes. One with refer to linear programming (6) will found that there constraints (7) is included. On the other hand, in mathematics, a constraint is a condition of an optimization problem that the solution must satisfy. There are several types of constraints, equality constraints and inequality constraints.

In an optimization problem, introducing a slack variable substitute an inequality constraint with an equality constraint)Boyd & Vandenberghe, 2004). Indeed, a slack variable is a variable that is added to an inequality

constraint to transform it to an equality. Should be explained that the values of these variables

must be greater than or equal to zero. In generally, If inequality and equality constraint holds

(7) together at the optimal point, the constraint (7) is known as the binding, as the optimal point cannot be varied in the direction of the constraint even though doing so would improve the value of the linear programming (6). Explicitly, binding constraint is associated with the zero value of slack variable. According to the data presented in Table 1, the values of slack variables for each

Table 2: performance score and ranking data in Table 1

DMU	Score	Rank
1	1	2
2	1	3
3	0.90874	13
4	0.301149	17
5	1	6
6	1	7
7	1	8
8	1	9
9	1	10
10	0.406675	16
11	0.732022	14
12	1	12
13	0.134939	18
14	1	11
15	1	5
16	1	4
17	0.109477	19
18	1	1
19	0.433896	15

Table 3: The values of slack variable for 19 DMUs obtained from simulation on the LQCD with Nr = 4(Tabl)

DMU	Score	$S_{-(1)}$	$S_{-(2)}$	$S_{-(3)}$	$S_{-(4)}$	$S_{-(5)}$	$S_{-(6)}$	$S_{-(7)}$	$S_{-(8)}$	$S_{+(1)}$	$S_{+(2)}$
1	1	0	0	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0	0
3	0.908	1.174	0.968	0.372	0	0.014	1.282	0	0.006	0	0
4	0.301	0	0.047	0.07	0.015	0.017	0.087	0.038	0.003	0	0
5	1	0	0	0	0	0	0	0	0	0	0
6	1	0	0	0	0	0	0	0	0	0	0
7	1	0	0	0	0	0	0	0	0	0	0
8	1	0	0	0	0	0	0	0	0	0	0
9	1	0	0	0	0	0	0	0	0	0	0
10	0.406	0.685	0.306	0.405	0.0832	0.0985	0.425	0.109	0.118	0	0
11	0.732	0	0.307	0.013	0	0.009	0.7467	0	0.004	0	0
12	1	0	0	0	0	0	0	0	0	0	0
13	0.134	0	0.046	0.061	0.013	0.015	0.0703	0.024	0.022	0	0
14	1	0	0	0	0	0	0	0	0	0	0
15	1	0	0	0	0	0	0	0	0	0	0
16	1	0	0	0	0	0	0	0	0	0	0
17	0.1094	0	0.0478	0.0612	0.0128	0.0151	0.0684	0.0224	0.0206	0	0
18	1	0	0	0	0	0	0	0	0	0	0
19	0.4338	0	0.0166	0.0761	0.0156	0.0148	0.0971	0.044	0.0296	0	0.0567

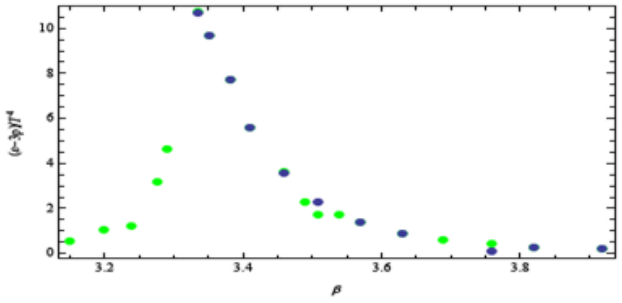

Fig.1. Trace anomaly " □ 3p in unite of T4 versus gauge coupling constant _ obtained from calculations on lattices with temporal extent N_ = 4 (according to Tables 1 and 2): efficient data (Solid balls), mix data (Green solid balls). Some of solid balls and green solid balls are inseparable.

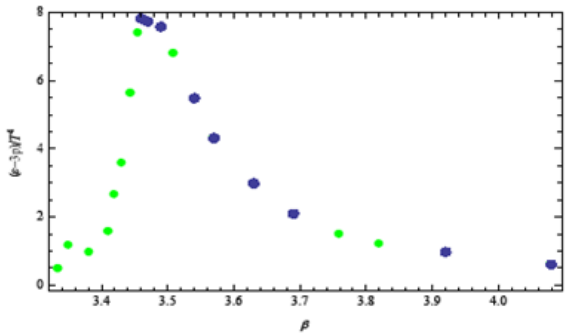

Fig.3. Trace anomaly "□3p in unite of T4 versus gauge coupling constant _ obtained from calculations on lattices with temporal extent N_ = 6 (according to Tables 4 and 5): efficient data (Solid balls), mix data (Green solid balls).

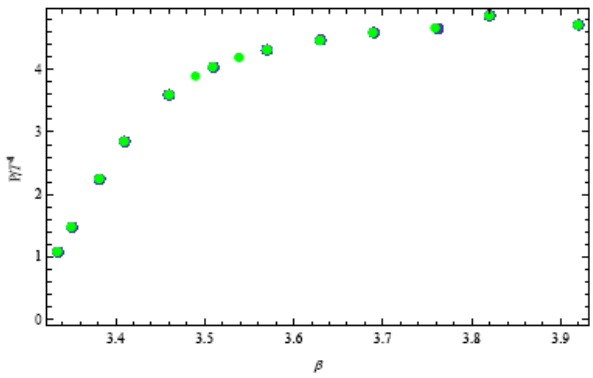

Fig.2. pressure p in unite of T4 versus gauge coupling constant _ obtained from calculations on lattices with temporal extent N_ = 4 (according to Tables 1 and 2): efficient data (Solid balls), mix data (Green solid balls). Some of solid balls and green solid balls are inseparable.

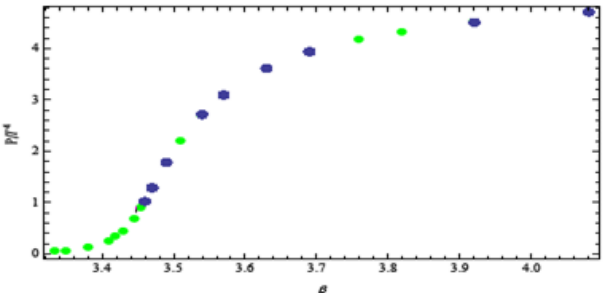

Fig.4. pressure p in unite of T4 versus gauge coupling constant _ obtained from calculations on lattices with temporal extent N_ = 6 (according to Tables 4 and 5): efficient data (Solid balls), mix data (Green solid balls).

inputs and outputs is listed in Table 3. According to Table 3, if S variables For each of the units is zero then that particular DMU is efficient. Oth-

Table 4: 20 DMUs (eight inputs and two outputs) obtained from simulation on the LQCD with N_r=6

DMU	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	Y_1	Y_2
1	3.335	0.62	4.36044	0.1542	0.24425	4.3598	0.15242	0.2436	0.51	0.048
2	4.08	0.081	3.234961	0.001546	0.012779	3.23433	0.001140	0.011397	0.599	4.7085
3	3.382	0.52	4.23499	0.1151	0.19922	4.2333	0.11103	0.1977	0.97	0.1393
4	3.690	0.15	3.669908	0.00874	0.035734	3.66697	0.002765	0.02753	2.09	3.936
5	3.420	0.39	4.13616	0.08303	0.15304	4.13075	0.07214	0.1481	2.68	0.334
6	3.430	0.37	4.11217	0.07606	0.14364	4.10498	0.06110	0.1367	3.57	0.44
7	3.445	0.344	4.07770	0.06650	0.130718	4.06634	0.04231	0.1193	5.64	0.676
8	3.820	0.125	3.5088124	0.004467	0.02466	3.506568	0.002035	0.020324	1.23	4.313
9	3.760	0.13	3.580005	0.00578	0.027805	3.57801	0.002225	0.022203	1.49	4.168
10	3.470	0.295	4.02346	0.05237	0.109388	4.00834	0.01715	0.0911	7.73	1.288
11	3.490	0.29	3.9845	0.04424	0.10072	3.97023	0.01187	0.0818	7.58	1.778
12	3.510	0.259	3.94649	0.03657	0.08764	3.93393	0.008204	0.0682	6.80	2.2005
13	3.540	0.240	3.89302	0.0281	0.07513	3.88347	0.006247	0.0574	5.48	2.712
14	3.570	0.212	3.84392	0.02176	0.062829	3.83671	0.004923	0.0473	4.31	3.092
15	3.630	0.17	3.75291	0.013176	0.045175	3.74830	0.0034263	0.03389	2.98	3.613
16	3.410	0.412	0.19922	0.09013	0.16256	4.15710	0.08251	0.1594	1.58	0.260
17	3.460	0.313	4.04471	0.0573	0.11734	4.02913	0.02374	0.1006	7.82	1.024
18	3.455	0.329	4.05605	0.06098	0.12314	4.04126	0.02928	0.1078	7.39	0.898
19	3.920	0.11	3.396477	0.002967	0.019635	3.395328	0.001676	0.016750	0.973	4.505
20	3.351	0.591	4.3188	0.1417	0.23045	4.31701	0.13865	0.2292	1.19	0.068

Table 5: performance Score and ranking data in Table 4

DMU	Score	Rank
1	0.067	20
2	1	3
3	0.127531	19
4	1	5
5	0.346719	16
6	0.460515	15
7	0.724368	13
8	0.999156	10
9	0.998026	11
10	1	9
11	1	8
12	0.6288	14
13	1	7
14	1	6
15	1	4
16	0.207738	17
17	1	2
18	0.94638	12
19	1	1
20	0.157124	18

erwise, even if one of the variables is not zero, it DMU is inefficient. It is very interest that the values of Table 3 are indicative the strengths and weaknesses each of inputs or outputs a septic DMU. The data in Table 2 can based on efficiently be divided into two categories, E = 1 and E < 1. We prefer to call these two categories, efficient and inefficient data, respectively. In follow we plan to based on the two categories of data, consider the behavior of two QCD thermo dynamical quantity, trace anomaly "□3p T4 and pressure p. In gores 1 and 2 using data Tables 1 and 2, is drawn the behavior of "□3p T4 and p in term of _ for set of gauge couplings, _ 2 [3:15; 3:92]. As can be seen, in bothgure, efficient data (or reference data) overlap with mix data for values _c _ 3:335.

In other words, data obtained in Tc; :::::; Thigh (phase transition to quark - gluon plasma) are efficient and reference. Surprisingly, among the fteen data (_ _ _c), twelve are efficient but even a low-temperature data (hadron phase)_ < _c is not efficient.

Let us here, repeat the same data envelopment analysis that was done in above, but this time using the data reported in Tables III and V in (Cheng et al., 2008) on lattices with temporal extent N_ = 6.

After sorting , preferably consider 20 DMUs with eight inputs and two outputs in Table 4. The results of CCR model and slack variable also are summarized in Tables 5 and 6, respectively.

Similar to Table 2, the last column of Table 5, indicates the ranking of each of the DMUs. The results reported in Table 5 refers to the fact that, DMU19 and DMU1 are The most efficient and inefficient data, respectively. From data Ta-

bles 4 and 5, used for drawing the behavior of "□3p T4 and p in term of _ for set of gauge couplings, _ 2 [3:335; 4:08], in _gures 3 and 4. Clearly, efficient data (or reference data) overlap with mix data for values _c _ 3:445. We observe similar behavior with the _gures 1 and 2, hence can conclude that here also, data obtained in Tc; ::::; Thigh (phase transition to quark column plasma) are efficient and reference. In here also, even a low-temperature data (hadrons phase) is not efficient.

DISCUSSION AND CONCLUS1ONS

Data envelopment analysis (DEA) is a non-parametric method in operations research for the estimation of production frontiers. It is used to empirically measure productive efficiency of decision making units (DMUs). Despite the fact that DEA has a strong relationship to pro-duction theory in economics, the tool is also used for benchmarking in operations management, where a set of measures is selected to test the performance of manufacturing and service op-elations. In the condition of benchmarking, the efficient DMUs, as denned by DEA, may not necessarily form a production frontier, but rather lead to a best-practice frontier. Hence we prefer in this paper, using DEA method, study on the best data set (the most efficient data set) from simulations on the lattice QCD with extent temporal N_ = 4; 6, respectively(Cheng et al., 2008).

After analyzing the data table, each of these two cases, efficient data sets are reported as: fDMU1;DMU2;DMU5;DMU6;DMU7;DMU8 ;DMU9;DMU12;DMU14;DMU15;DMU16;DM U18g and fDMU2; DMU4; DMU10; DMU11; DMU13;DMU14;DMU15;DMU17;DMU19g, respectively.

This result is very interesting to note that none of these DMUs does not belong to the low temprelature (_ < _c i.e. conferment phase). To better understand the issue, we prefer data in terms of efficiency divided into three categories as: e = 1, 0:9 _ e < 1 and e < 0:9. With refers to Tables 2 and 5, one will realize that all low temperature data belong to the third category i.e. e < 0:9. This means that the low-temperature data in terms of efficacy, are the worst. Here we would like to _nd a connection between efficient data set and main characteristic of LQCD.

It is importance for note that free asymptotic in high temperature (_ ! 1) is corresponds to the continuum limit i.e. a ! 0. To be more precise, in this limit conferment region of the coupling connected to the weak coupling region. Of course if this connection is smoothly then the transition will not happen. Otherwise we are dealing with rest or second order phase transition. As annual

Table 6: The values of slack variable for 20 DMUs obtained from simulation on the LQCD with Nr=6 (Table 4)

DMU	Score	$S_{-(1)}$	$S_{-(2)}$	$S_{-(3)}$	$S_{-(4)}$	$S_{-(5)}$	$S_{-(6)}$	$S_{-(7)}$	$S_{-(8)}$	$S_{+(1)}$	$S_{+(2)}$
1	0.067	0	0.021	0.031	0.006	0.008	0.032	0.0087	0.009	0	0.018
2	1	0	0	0	0	0	0	0	0	0	0
3	0.127531	0	0.027	0.37	0.007	0.011	0.039	0.0114	0.013	0	0
4	0	0	0	0	0	0	0	0	0	0	0
5	0.346719	0	0.028	0.047	0.0091	0.012	0.051	0.016	0.016	0	0.0162
6	0.460515	0	0.027	0.047	0.008	0.012	0.051	0.017	0.017	0	0.0275
7	0.724368	0	0.023	0.036	0.0068	0.010	0.039	0.013	0.0139	0	0.061
8	0.999156	0	0.0061	0.084	0.0001	0.0013	0.084	0.0001	0.0011	0	0
9	0.998026	0	0.0023	0.096	0..001	0.0007	0.096	0.000005	0.0005	0	0
10	1	0	0	0	0	0	0	0	0	0	0
11	1	0	0	0	0	0	0	0	0	0	0
12	0.6288	0	0.059	0.201	0.015	0.027	0.196	0.030	0.022	0	0
13	1	0	0	0	0	0	0	0	0	0	0
14	1	0	0	0	0	0	0	0	0	0	0
15	1	0	0	0	0	0	0	0	0	0	0
16	0.207738	0	0.023	0.042	0.007	0.011	0.045	00.013	0.0139	0	0
17	1	0	0	0	0	0	0	0	0	0	0
18	0.94638	0	0.015	0.016	0.0035	0.0056	0.017	0.0052	0.007	0	0.069
19	1	0	0	0	0	0	0	0	0	0	0
20	0.1571	0	0.045	0.063	0.013	0.018	0.065	0.018	0.020		0.087

Table 7: Results derived from DEA-CCR model for the most efficient and inefficient DMUs of LQCD data with N_r=4

DMU.No18	**Data**	**Projection**	**Difference**	**Percent**
x_1	3.82	3.82	0	0.00
x_2	0.125	0.125	0	0.00
x_3	3.508124	3.508124	0	0.00
x_4	0.004467	0.004467	0	0.00
x_5	0.024666	0.024666	0	0.00
x_6	3.5062	3.5062	0	0.00
x_7	0.001628	0.001628	0	0.00
x_8	0.01628	0.016268	0	0.00
y_1	0.273	0.273	0	0.00
y_2	4.683	4.683	0	0.00
Score	1	-	-	-
DMU.NO17	-	-	-	-
x_1	3.2	0.350327	-2.849673	-89.05
x_2	1	0.0617	-0.938292	-93.83
x_3	4.68944	0.452166	-4.237274	-90.36
x_4	0.25284	0.0149	-0.23794	-94.11
x_5	0.358813	0.0241	-0.33468	-93.27
x_6	4.68525	0.444515	-4.240735	-90.51
x_7	0.24357	0.00422	-0.239349	-98.27
x_8	0.35522	0.0183	-0. 336922	-94.85
y_1	1.03	1.03	0	0.00
y_2	0.1492	0.1492	0	0.00
Score	0.109477	-	-	-

Table 8: Results derived from DEA-CCR model for the most efficient and inefficient DMUs of LQCD data with N_r=6

DMU.No19	**Data**	**Projection**	**Difference**	**Percent**
x_1	3.92	3.92	0	0.00
x_2	0.11	0.11	0	0.00
x_3	3.396477	3.396477	0	0.00
x_4	0.002967	0.002967	0	0.00
x_5	0.019635	0.019635	0	0.00
x_6	3.395328	3.395328	0	0.00
x_7	0.001676	0.001676	0	0.00
x_8	0.01675	0.01675	0	0.00
y_1	0.973	0.973	0	0.00
y_2	4.5057	4.5057	0	0.00
Score	1	-	-	-
DMU.NO1	-	-	-	-
x_1	3.335	0.225652	-3.109348	-93.23
x_2	0.62	0.0204	-0.599587	-96.71
x_3	4.3604	0.263785	-4.096615	-93.95
x_4	0.15429	0.00374	-0.150553	-97.58
x_5	0.24425	0.00765	-0.236597	-96.87
x_6	4.3598	0.262769	-4.097031	-93.97
x_7	0.15242	0.00153	-0.237108	-97.31
y_1	0.51	0.51	0	0.00
y_2	0.048	0.0668	0.0188	39.13
Score	0.0677	-	-	-

word, results derived from DEA-CCR model, implies that efficient data set appear in continuum limit a ! 0.

APPENDIX

In this section, the certain outputs by running the CCR model for the most efficient and inefficient DMU obtained from simulation on the lattice QCD with extent temporal N_ = 4; 6, listed in Tables 7, 8, respectively. For a deeper understanding of the numbers reported in each of the two tables, it is necessary explanations are given. In "Data column" is included the information of input and output a DMU. "Projection column" indicate that each of the inputs and outputs have spent much of their initial data and how much output is produced on the basis of their consumption. "Difference column" numbers mean the difference between the initial amount and the amounts are consumed. value of zero means complete production and consumption.

Non-zero value indicates the amount of shortages and surpluses. The lack of input in order to produce the desired output. The "Percent column" indicates the decency of values according to the percentage. In other words, the numbers given in this column indicate weakness in the process of making data. Consequently, if all the numbers listed in the Difference and Percent columns are zero, then the DMU under consideration would be efficient.

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