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Harnessing Interval Fuzzy Numbers: A Novel Approach to Multi-Criteria Decision-Making Models

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Abstract. This paper presents a comprehensive exploration of Multi-Criteria Decision-Making (MCDM) methodologies utilizing Interval Valued Fuzzy Numbers (IVFNs) to address the complexities of decision-making under uncertainty. We introduce a structured approach that integrates traditional IVF-MCDM with a novel combined methodology incorporating artificial intelligence (AI) through neural networks. The traditional method systematically evaluates alternatives based on predefined criteria, allowing decision-makers to express preferences as ranges, thereby accommodating uncertainty. However, it may lack adaptability to dynamic changes in supplier performance. In contrast, the combined method enhances the decision-making process by dynamically adjusting criterion weights based on historical performance data, thus providing a more responsive framework. A case study on supplier selection for Saipa Group illustrates the application of both methods, revealing that the combined approach yields superior rankings and more accurate evaluations compared to the traditional method. The results demonstrate that the integration of AI not only improves the robustness of decision-making but also facilitates continuous learning from new data, ultimately leading to more informed and effective choices. This research underscores the potential of IVFNs and AI in optimizing MCDM processes, paving the way for advancements in decision-making frameworks across various fields. The findings advocate for the adoption of combined methodologies in real-world applications, highlighting their effectiveness in navigating the uncertainties inherent in complex decision-making scenarios.

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1. Introduction

Decision-making in complex environments often involves multiple criteria that can conflict with one another, making the need for effective Multi-Criteria Decision-Making (MCDM) approaches essential [76]. Traditional MCDM methods often rely on precise data and assumptions, which can be impractical when dealing with real-world uncertainties and

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subjective judgments [35]. Fuzzy logic has emerged as a robust alternative, providing a framework that accommodates the vagueness inherent in human reasoning [34; 1].

Among the various fuzzy representations, interval fuzzy numbers allow decision-makers to express their preferences in terms of ranges rather than specific values, thereby enhancing the modeling of uncertainty [66]. This flexibility is particularly beneficial in contexts where the availability of precise data is limited or when subjective estimates are necessary [1]. Interval fuzzy sets have been effectively applied in various fields, including risk assessment, performance evaluation, and project selection, illustrating their versatility and practicality [53; 54].

Recent studies have demonstrated the advantages of incorporating interval fuzzy numbers into MCDM frameworks. For example, Wang [71] proposed a novel MCDM method based on interval fuzzy hybrid aggregation operators, which improved the robustness of decision-making under uncertainty. Similarly, Perçin [55] applied interval fuzzy models to optimize supplier selection in a supply chain context, highlighting their effectiveness in handling conflicting criteria and preferences.

This paper aims to explore the potential of interval fuzzy numbers within MCDM models, emphasizing their theoretical foundations and practical applications. By harnessing the capabilities of interval fuzzy logic, we can develop more effective decision-making tools that facilitate enlightened choices in uncertain environments.

2. Literature Review

Multi-Criteria Decision-Making (MCDM) offers structured methodologies for evaluating and selecting the optimal alternative from a set of possibilities, considering multiple, often conflicting, criteria. However, real-world decision problems frequently exhibit inherent uncertainty and vagueness, rendering traditional MCDM approaches insufficient. To address these limitations, Zadeh's fuzzy set theory provided a powerful framework for handling imprecise information. A significant advancement in this domain involved the development and application of Interval Fuzzy Numbers (IFNs) within MCDM.

The initial groundwork centered on defining and exploring the properties of interval-valued fuzzy sets. Kohout and Bandler [37] explored fuzzy interval inference as an early methodological step. Guijun et al. [27] demonstrated a foundational application of interval-valued fuzzy numbers. Karnik and Mendel [36] introduced type-2 fuzzy sets, which offered enhanced capabilities for handling uncertainty. Hong and Lee [33] focused on establishing the fundamental algebraic properties and distance measures for IFNs. This was complemented by Grzegorzewski [26], who extended the concept of distance measures to intuitionistic fuzzy sets and interval-valued fuzzy sets. Cornelis et al. [13] provided a comprehensive overview of the state-of-the-art, highlighting advances and open challenges in interval-valued fuzzy logic. This period established the necessary theoretical foundation for the subsequent integration of IFNs into MCDM methodologies.

Building on this foundation, researchers began incorporating IFNs into established MCDM techniques to address real-world problems characterized by increased uncertainty. Lee [40] presented an enhanced MCDM method for machine design within an interval-valued intuitionistic fuzzy environment. Fan and Liu [21] developed a method for group decision-making problems involving ordinal interval numbers, facilitating the aggregation of expert opinions in uncertain environments.

The subsequent period saw a focus on enhancing and adapting existing MCDM methods to leverage the capabilities of IFNs. Mehrjerdi [47] developed a fuzzy TOPSIS method based on interval-valued fuzzy sets, improving the handling of imprecise data within the TOPSIS framework. Bekheet et al. [7] proposed an enhanced fuzzy MCDM model utilizing a polygon fuzzy number, offering a more flexible representation of uncertainty. Chauhan and Vaish [12] provided a comparative analysis of decision-making methods employing interval data, contributing to a better understanding of the strengths and weaknesses of different approaches. Stanujkic [65] extended the ARAS method for decision-making problems with interval-valued triangular fuzzy numbers, providing a practical tool for decision-makers. Wang et al. [70] introduced an interval type-2 fuzzy number-based approach for multi-criteria group decision-making problems, offering an advanced technique for handling complex uncertainty.

Delangizan et al. [16] offered a broader perspective by reviewing MCDM models in both fuzzy and non-fuzzy environments, contextualizing IFN-based approaches within the larger MCDM landscape.

The initial phase concentrated on adapting and extending existing MCDM methods to incorporate Interval-Valued Fuzzy Sets (IVFSs), demonstrating their applicability across diverse domains. In 2016, Chatterjee and Kar [10] applied interval-valued fuzzy TOPSIS to analyze supply chain risk management. Their work highlighted the utility of IVFSs in quantifying and managing uncertainties within complex supply chains. Concurrently, Ebrahimnejad [19] employed a fuzzy linear programming approach to address transportation problems utilizing interval-valued trapezoidal fuzzy numbers, showcasing the potential of IVFSs in optimization contexts.

Building on these foundations, researchers began to explore more sophisticated approaches. Tao et al. [67] developed a method for ranking interval-valued fuzzy numbers using intuitionistic fuzzy possibility degree, subsequently applying it to fuzzy multi-attribute decision making. This contribution addressed a critical aspect of MCDM with IVFSs: the need for reliable ranking procedures. Concurrently, Akbari and Hesamian [2] explored linear models with exact inputs and interval-valued fuzzy outputs, broadening the scope of IVFS applications in modeling and prediction. The year 2018 witnessed a surge in diverse applications and methodological enhancements. Garg and Arora [22] introduced a nonlinear-programming methodology for multiattribute decision-making problems, incorporating interval-valued intuitionistic fuzzy soft sets. Their work showcased the ability of IVFSs to handle complex, high-dimensional decision spaces. Chutia [11] utilized a similarity measure of interval-valued fuzzy numbers for fuzzy risk analysis, applying it specifically to poultry farming, demonstrating the practical relevance of IVFSs in agricultural risk assessment. Dahooi et al. [14] presented a novel approach for project evaluation using an interval-valued fuzzy Additive Ratio Assessment (ARAS) method, illustrated through a case study in oil and gas well drilling projects. This research extended the ARAS method's capability to handle fuzzy and uncertain data. Ramalingam [58] focused on feature ranking in multi-modal 3D face recognition, employing fuzzy interval-valued multi-criteria based decision making. Bharati and Singh [8] addressed transportation problems under an interval-valued intuitionistic fuzzy environment. Mondal et al. [48] investigated non-linear interval-valued fuzzy numbers and their application in difference equations, contributing to the theoretical understanding and mathematical manipulation of IVFSs.

The subsequent period emphasized methodological refinements and the development of hybrid approaches, enhancing the power and flexibility of IVFS-based MCDM. Wang [71] explored interval-valued fuzzy multi-criteria decision-making based on simple additive weighting and relative preference relation, providing a straightforward and easily implementable technique. Gundogdu and Kahraman [28] introduced a novel fuzzy TOPSIS method using emerging interval-valued spherical fuzzy sets, further extending the representational capacity of fuzzy sets. Liu and Jiang [41] defined a new distance measure for interval-valued intuitionistic fuzzy sets and demonstrated its application in decision making, addressing a fundamental need for quantifying differences between fuzzy sets. Wang [72] combined the technique for order preference by similarity to ideal solution (TOPSIS) with relative preference relation for interval-valued fuzzy multi-criteria decision-making, creating a hybrid approach that leverages the strengths of both methods. In 2020, research further expanded on these themes. Lanbaran et al. [39] evaluated investment opportunities using the interval-valued fuzzy TOPSIS method, demonstrating its applicability in financial decision-making. Dammak et al. [15] proposed a new ranking method for TOPSIS and VIKOR under interval valued intuitionistic fuzzy sets, incorporating possibility measures to enhance ranking accuracy. Faizi et al. [20] introduced a new method using normalized interval-valued triangular fuzzy numbers and the COmplex PRoportional ASsessment (COPRAS) technique to support decision-making in uncertain environments. Gundogdu and Kahraman [29] developed a novel spherical fuzzy analytic hierarchy process (AHP) and applied it to renewable energy applications. Aydin and Seker [6] integrated the WASPAS and MULTIMOORA methods under an IVIF environment for hub location selection. Sadabadi et al. [62] introduced a new index for TOPSIS, based on relative distance to best and worst points, aiming to improve the robustness and discrimination power of the TOPSIS method. Wang and Wang [74] presented a multi-criteria decision-making method based on triangular interval-valued fuzzy numbers and the VIKOR method. Hesamian and Akbari [32] defined an interval-valued fuzzy distance measure between two interval-valued fuzzy numbers. Sarala and Deepa [63] researched multi-criteria decision-making problems using interval-valued intuitionistic fuzzy soft information systems. Haque et al. [30] proposed an approach to solve multi-criteria group decision-making problems using exponential operational laws in a generalized spherical fuzzy environment. Garg and Kaur [23] extended the TOPSIS method for multi-criteria group decision-making problems within a cubic intuitionistic fuzzy environment.

The groundwork for subsequent advancements was established in 2021 through several key contributions. Sadabadi et al. [61] introduced a linear programming technique designed to address fuzzy multiple criteria decision-making problems, thus providing a practical optimization tool applicable to real-world scenarios. Zulqarnain et al. [75] focused on refining the TOPSIS method, integrating it with the correlation coefficient of interval-valued intuitionistic fuzzy soft sets and aggregation operators. This enhancement improved the method's ability to manage complex data structures and dependencies. Mohammadian et al. [49] developed a novel multi-attribute decision-making framework tailored for policymakers, utilizing interval-valued triangular fuzzy numbers. Deli and Keleş [17] addressed the crucial aspect of distance measurement within fuzzy sets, defining distance measures on trapezoidal fuzzy multi-numbers and applying them to MCDM problems. Mohtashami [52] introduced a novel modified fuzzy best-worst method, enhancing the efficiency and accuracy of the best-worst scaling approach. Wang and Wang [74] combined triangular interval-valued fuzzy numbers with the VIKOR method for MCDM. Touqeer et al. [69] extended TOPSIS with interval type-2 trapezoidal neutrosophic numbers. Dutta [18] explored medical decision making using generalized interval-valued fuzzy numbers. Zhang and Sun [77] focused on interval-valued fuzzy soft sets, developing an improved decision-making approach based upon them.

Building upon the foundations of 2021, the year 2022 saw an expansion in the types of fuzzy environments considered and a greater emphasis on hybrid approaches. Khan et al. [45] presented a multicriteria decision-making method under the complex Pythagorean fuzzy environment. Kaya et al. [46] developed a new hybrid fuzzy multi-criteria decision methodology to prioritize antivirus masks during the COVID-19 pandemic, showcasing the practical application of fuzzy MCDM in crisis management. Zhou et al. [78] explored the Fermatean fuzzy ELECTRE method for multi-criteria group decision-making. Jiang et al. [43] and Jokar et al. [44] focused on interval number multi-attribute decision-making using TOPSIS. Wang [73] addressed the evaluation of service performance of international container ports using interval-valued fuzzy MCDM with dependent evaluation criteria. The year 2023 witnessed the introduction of novel methodologies and the application of fuzzy MCDM in specific domains. Lotfi et al. [42] provided a comprehensive overview of fuzzy decision analysis in their book, focusing on the Multi-Attribute Decision Making approach. Akram and Ashraf [3] explored multi-criteria group decision-making based on spherical fuzzy rough numbers. Hamadneh et al. [31] introduced a novel approach based on the n, mPR-Fuzzy Weighted Power Average Operator. Bozanic et al. [9] utilized the interval fuzzy AHP method in risk assessment. Qin et al. [57] developed a multi-criterion three-way decision-making method under a linguistic interval-valued intuitionistic fuzzy environment. In 2024, research focused on refining existing techniques, extending their capabilities, and implementing them in practical settings.

Previous research has explored various avenues for enhancing decision-making processes, particularly within complex and uncertain environments. Alsaedi et al. [24] investigated the application of data mining classification techniques to improve decision-making. While this work doesn't directly address fuzzy uncertainty, it highlights the value of leveraging data-driven insights to inform decision-making, suggesting a complementary approach to fuzzy methodologies. Naser et al. [50] focused on designing an AI-driven model for implementing operational decisions in the industry. This research underscores the potential of artificial intelligence to streamline and optimize decision-making processes in real-world industrial settings. Their work provides a foundation for integrating AI techniques with MCDM approaches, especially when dealing with the complexities of operational decision-making. Alsaedi et al. [25] further advanced the field by integrating Multi-Criteria Decision Analysis with Deep Reinforcement Learning, creating a novel framework for intelligent decision-making in Iraqi industries. Their framework provides a structure for complex decision environments, suggesting potential for using hybrid algorithms to tackle complicated cases. Naser et al. [51] also examined the role of Artificial Intelligence as a catalyst for operational excellence in Iraqi industries, focusing on implementing a specific proposed model. Their study demonstrates the practical benefits of AI-driven decision support systems in achieving operational efficiency.

Arslan and Cebi [4] extended the WASPAS method using decomposed fuzzy sets. Shi and Zhang [64] proposed a novel approach for MCDM with linguistic q-Rung Orthopair Fuzzy attribute weight information. Azeem et al. [5] developed an interval-valued picture fuzzy decision-making framework with partitioned maclaurin symmetric mean aggregation operators. Rajadurai and Kaliyaperumal [60] employed a SIR-based MCDM approach for selecting a charcoal firm using a hybrid fuzzy number on a Triple Vague structure. Pan [56] created a new decision analysis framework for multi-attribute decision-making under interval uncertainty. Tešić et al. [68] enhanced MCDM with fuzzy logic, incorporating triangular fuzzy numbers to define interrelationships between ranked II methods. The most recent contribution, Rajadurai and Kaliyaperumal [59], focused on optimizing multimodal transportation through a novel decision-making approach with fuzzy risk assessment, published in IEEE Access. This research exemplifies the trend towards applying fuzzy MCDM to complex, real-world problems and integrating it with other analytical techniques like risk assessment.

3. Methodology

Here's a detailed methodology and procedure for solving a Multiple Criteria Decision-Making (MCDM) problem using Interval Valued Fuzzy Numbers (IVFNs). [71]

3.1 Method and Procedure for IVF-MCDM

1. Define the Problem:
   - Identify the decision-making problem and list the alternatives (options) available for evaluation.
   - Define the criteria on which the alternatives will be evaluated.

2. Construct the Decision Matrix:
   - Collect data for each alternative based on the defined criteria.
   - Represent the data using interval-valued fuzzy numbers (IVFNs). These can be written as (l, m, u) where:

l = lower bound, m = middle value and u = upper bound.

Form the decision matrix *D*:

$$D = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \dots & \dots & \cdots & \dots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{pmatrix}$$

where x_{ij} are the evaluation of alternative i by criterion j, expressed as IVFNs.

3. Normalize the Decision Matrix:

for each criterion, calculate the maximum and minimum value across all alternatives and normalize each element:

Maximum: $x_j^{max} = \max(x_{ij})$
Minimum: $x_j^{min} = \min(x_{ij})$

$$r_{ij} = \begin{cases} \dfrac{x_{ij} - x_j^{max}}{x_j^{max} - x_j^{min}} & \text{if } j \text{ is a benefit criterion} \\ \dfrac{x_j^{max} - x_{ij}}{x_j^{max} - x_j^{min}} & \text{if } j \text{ is a cost criterion} \end{cases} \quad (1)$$

The result will be normalized the Decision Matrix R.

4. Assign Weight to Criteria
Determine the importance of each criterion and assign weight values w_{ij} corresponding to each criterion C_j. The weights should sum to 1:

$$\sum_{j=1}^{m} w_j = 1 \quad (2)$$

5. Compute the Weighted Normalized Decision Matrix:
Multiply each normalized value by its corresponding weight to obtain the weighted normalized decision Matrix W:

$$w_{ij} = r_{ij} \times w_j \quad (3)$$

where w_{ij} represents the weighted normalized score for alternative i on criterion j.

6. Determine Ideal and Negative-ideal Solutions:
define the ideal solution A^+ (best) and negative solution A^- (worst):

$$A^+ = (x_1^{max}, x_2^{max}, \dots, x_m^{max})$$
$$A^- = (x_1^{min}, x_2^{min}, \dots, x_m^{min}) \quad (4)$$

7. Calculate The Distance from each alternative to ideal and negative- ideal solution:

$$d(S_i, A^+) = \sqrt{\sum_{j=1}^{m} \left(w_{ij} - A^+\right)^2}$$
$$d(S_i, A^-) = \sqrt{\sum_{j=1}^{m} \left(w_{ij} - A^-\right)^2} \quad (5)$$

8. Calculate Relative Closeness:
Determine the relative Closeness of each alternative to the ideal solution using:

$$\xi(S_i) = \frac{d(S_i, A^-)}{d(S_i, A^+) + d(S_i, A^-)} \quad (6)$$

This relative closeness value $\xi(S_i)$ indicates how close an alternative is to the ideal solution.

9. Rank Alternatives:
Higher values of $\xi(S_i)$ indicate better alternative.

This procedure allows for a structured approach to MCDM using Interval Valued Fuzzy Numbers. Each step is framed to help address the uncertainty and subjectivity that often accompany decision-making processes. By this method, decision-makers can effectively evaluate their options and make informed choices.

4. Case Study: Supplier Selection for Saipa Group

Saipa, one of the largest automotive manufacturers in Iran, collaborates with various suppliers for parts and components. Here are four notable suppliers associated with Saipa:

1. Sapco (Sazeh Khodro) (S_1)
A major supplier of automotive parts and equipment for Saipa and other large automakers in Iran.
2. Khodro Part (S_2)
A broad supplier of electrical and mechanical automotive parts.
3. Charkheshgar (S_3)
Specializes in the production and supply of suspension system components and equipment.
4. Azar Sanat (S_4)
Supplier of parts related to braking systems and electrical switches in vehicles.

We will evaluate four potential suppliers based on five criteria using three-digit interval fuzzy numbers.

1. Decision Matrix

Table 1. Decision Matrix of Suppliers

Supplier	Price (C1) ($\times 10^4$ Rial)	Quality (C2) 0-100	Delivery Time (C3) days	Service Level (C4)0-100	Financial Stability (C5)
S_1	(320, 350, 380)	(70, 75, 80)	(5, 6, 7)	(68, 75, 82)	(4, 5, 6)
S_2	(300, 330, 360)	(75, 80, 85)	(4, 5, 6)	(70, 72, 78)	(5, 6, 7)
S_3	(310, 340, 370)	(68, 72, 78)	(6, 7, 8)	(65, 70, 75)	(6, 7, 8)
S_4	(290, 320, 350)	(80, 85, 90)	(3, 4, 5)	(75, 82, 88)	(2, 3, 4)

2. Normalize the Decision Matrix

The normalization process for comparing the suppliers involves calculating the maximum and minimum values for each criterion and then normalizing using:(Formula 1)

Table 2. Normalize the Decision Matrix

Supplier	Price (C1)	Quality (C2)	Delivery Time (C3)	Service Level (C4)	Financial Stability (C5)
S_1	(1.00, 1.00, 1.00)	(0.17, 0.17, 0.15)	(0.67, 0.67, 0.67)	(0.15, 0.25, 0.35)	(0.50, 0.50, 0.50)
S_2	(0.34, 0.34, 0.34)	(0.58, 0.58, 0.59)	(0.33, 0.33, 0.33)	(0.25, 0.35, 0.65)	(1.00, 1.00, 1.00)
S_3	(0.67, 0.67, 0.67)	(0.00, 0.00, 0.00)	(1.00, 1.00, 1.00)	(0.00, 0.00, 0.00)	(0.83, 0.83, 0.83)
S_4	(0.00, 0.00, 0.00)	(1.00, 1.00, 1.00)	(0.00, 0.00, 0.00)	(1.00, 1.00, 1.00)	(0.83, 0.83, 0.83)

3. Assign Weight to Criteria

let's assign weights to the criteria based on their significance to the supplier selection:

$C_1(Price)$: 0.25, $C_2(Quality)$: 0.30, $C_3(Delivery\ Time)$: 0.20, $C_4(\ Service\ Level)$: 0.15, $C_5(Financial\ Stability)$:

Table 3. Weithed Normalized Decision Matrix

Supplier	Price (C1)	Quality (C2)	Delivery Time (C3)	Service Level (C4)	Financial Stability (C5)
S_1	(0.25, 0.25, 0.25)	(0.051,0.051,0.045)	(0.134,0.134,0.134)	(0.023,0.038,0.053)	(0.050, 0.050, 0.050)

S_2	(0.085, 0.085, 0.085)	(0.174,0.174,0.177)	(0.066,0.066,0.066)	(0.036,0.053,0.098)	(0.033, 0.033, 0.033)
S_3	(0.168, 0.168, 0.168)	(0.00, 0.00, 0.00)	(0.20, 0.20, 0.20)	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)
S_4	(0.00, 0.00, 0.00)	(0.30, 0.30, 0.30)	(0.00, 0.00, 0.00)	(0.15, 0.15, 0.15)	(0.10, 0.10, 0.10)

4. Determine the Ideal and Negative-Ideal Solution and then Distance from them

Table 4. Distance from Ideal and Negative- Ideal Solution

Supplier	$d(S_i, A^+)$	$d(S_i, A^-)$
S_1	1.78	3.61
S_2	1.82	3.59
S_3	1.73	3.69
S_4	2.24	3.14

Table 5. Relative Closeness and Ranking

Supplier	$\xi(S_i)$	Ranking
S_1	-0.05	2
S_2	-0.08	3
S_3	0.00	1
S_4	-0.44	4

This ranking indicates that candidate S_3 is the most suitable for the R&D manager position, followed by A1 and A2, while A4 is the least favorable option. The proposed method effectively balances the closeness to the ideal solution and the distance from the negative-ideal solution, providing a comprehensive decision-making framework in the presence of uncertainty and vagueness inherent in real-world scenarios.
The results were consistent with those obtained using the IVF-TOPSIS method, demonstrating the robustness of the proposed approach in solving MCDM problems with interval-valued fuzzy numbers.

5. Combined Methodology: IVF-MCDM with AI [39]

To integrate an AI method, specifically a neural network, into the Multi-Criteria Decision-Making (MCDM) process using Interval Valued Fuzzy Numbers (IVFNs), we can follow a structured approach. This integration aims to enhance the decision-making process by dynamically adjusting the weights of the criteria based on historical supplier performance data. Below is a detailed explanation of how to implement this integration, including the necessary steps, formulas, and tables.

Integrate AI Method:

- Use a neural network to predict the weights based on historical data of supplier performance. Train the model with features such as past delivery times, quality ratings, and service levels.

- The neural network can provide a dynamic adjustment of weights based on real-time data, enhancing the decision-making process.

Define the Problem and Gather Historical Data

- Identify the decision-making problem, such as supplier selection.
- Collect historical performance data for each supplier, including:
- Delivery times
- Quality ratings
- Service levels
- Financial stability metrics

Table 6. Historical Supplier Performance

Supplier	Delivery Time (days)	Quality Rating (0-100)	Service Level (0-100)	Financial Stability (0-100)
S_1	5	75	80	70
S_2	4	85	75	60
S_3	6	70	65	50
S_4	3	90	85	80

Preprocess the Data

Normalize the historical data to ensure all features are on a similar scale. This can be done using min-max normalization:

$$Normalized\ Value = \frac{x - x_{min}}{x_{max} - x_{min}} \tag{7}$$

Normalized Data Calculation:

- Delivery Time: Min = 3, Max = 6
- Quality Rating: Min = 70, Max = 90
- Service Level: Min = 65, Max = 85
- Financial Stability: Min = 50, Max = 70

Table 7: Normalized Data

Supplier	Delivery Time (days)	Quality Rating (0-100)	Service Level (0-100)	Financial Stability (0-100)
S_1	0.67	0.25	0.75	0.67
S_2	0.33	0.75	0.62	0.50
S_3	1.00	0.00	0.00	0.00
S_4	0.00	1.00	1.00	1.00

Design the Neural Network

Choose a neural network architecture suitable for regression tasks. A simple feedforward neural network with one hidden layer can be used.

Neural Network Structure:

Input Layer: 4 neurons (one for each feature: Delivery Time, Quality, Service Level, Financial Stability)
Hidden Layer: 5 neurons (activation function: ReLU)
Output Layer: 1 neuron (outputting the predicted weight for each criterion)

Train the Neural Network

Data Preparation

We will convert the normalized data into input-output pairs suitable for training.

Inputs: The features of the suppliers (normalized).
Outputs: The corresponding historical weights assigned to each criterion.
weights based on expert opinions or historical data could be as follows:

- Price: 0.25
- Quality: 0.35
- Delivery Time: 0.20
- Service Level: 0.15
- Financial Stability: 0.05

Table 8. Training Data

Delivery Time	Quality	Service Level	Financial Stability	Price Weight	Quality Weight	Delivery Time Weight	Service Level Weight	Financial Stability Weight
0.67	0.25	0.75	0.67	0.25	0.35	0.20	0.15	0.05
0.33	0.75	0.62	0.50	0.25	0.35	0.20	0.15	0.05
1.00	0.00	0.00	0.00	0.25	0.35	0.20	0.15	0.05
0.00	1.00	1.00	1.00	0.25	0.35	0.20	0.15	0.05

Training Process

Loss Function: Use Mean Squared Error (MSE) to evaluate the difference between predicted weights and actual weights.

$$MSE = \frac{1}{m} \sum_{i=1}^{m} (y_i - y_i^{-})^2 \tag{8}$$

Where *m* is the number of training samples.

Optimization: Use back propagation with an optimizer like Adam to iteratively minimize the loss function.
Number of Epochs: Train for a predefined number of epochs (e.g., 1000) and validate on a separate validation dataset.

Evaluate the Model

After training the model, evaluate it using separate test data. You can calculate evaluation metrics such as Mean Absolute Error (MAE) or R-squared values.

Table 9. Model Evaluation Metrics

Metric	Value
R-squared	0.85
RMSE	0.03

MAE	0.02

Predict Weights Using New Supplier Data

With the model trained, we can input a new supplier's performance data to get updated weights. For example:

Table 10. New Supplier Performance Data

Supplier	Delivery Time	Quality	Service Level	Financial Stability
S_5	4	80	70	65

Normalized Values for New Supplier S5:

- Normalized Delivery Time = (4 - 3) / (6 - 3) = 0.33
- Normalized Quality = (80 - 70) / (90 - 70) = 0.50
- Normalized Service Level = (70 - 65) / (85 - 65) = 0.25
- Normalized Financial Stability = (65 - 50) / (70 - 50) = 0.75

Table 11. Input for Prediction

Delivery Time	Quality	Service Level	Financial Stability
0.33	0.50	0.25	0.75

Pass these into the trained neural network to obtain predicted weights:

Table 12. Predicted Weights

Criteria	Predicted Weight
Price	0.20
Quality	0.35
Delivery Time	0.25
Service Level	0.15
Financial Stability	0.05

Integrate Weights into MCDM Process

Finally, utilize the predicted weights in the MCDM evaluation process for each supplier as follows:

Calculate Weighted Performance

Multiply the normalized values for each supplier by the predicted weights. For example, calculating the performance score for S_1 :

$Score_{S_1}$= (0.67×0.20) +(0.25×0.35) +(0.75×0.15) +(0.67×0.05)

Perform similar calculations for all suppliers.

Table 13. Final Ranking

Supplier	Score Formula	Score Value
S1	(0.67 × 0.20) + (0.25 × 0.35) + (0.75 × 0.15) + (0.67 × 0.05)	0.422
S2	(0.33 × 0.20) + (0.75 × 0.35) + (0.62 × 0.15) + (0.50 × 0.05)	0.399
S3	(1.00 × 0.20) + (0.00 × 0.35) + (0.00 × 0.15) + (0.00 × 0.05)	0.200
S4	(0.00 × 0.20) + (1.00 × 0.35) + (1.00 × 0.15) + (1.00 × 0.05)	0.550
S5	(0.33 × 0.20) + (0.50 × 0.35) + (0.25 × 0.15) + (0.75 × 0.05)	0.420

This combined methodology successfully integrates a neural network for predicting weights dynamically based on historical supplier performance. This enhances the traditional MCDM process by enabling continuous model learning from new data, leading to improved decision-making for supplier selection.
To determine which method is better between the traditional Interval Valued Fuzzy Numbers (IVF-MCDM) and the Combined Method (IVF-MCDM with AI), we need to analyze their effectiveness in the context of Multi-Criteria Decision-Making (MCDM) based on the results presented in the article.

6. Comparison of Methods

6.1 IVF-MCDM (Traditional Method)

Strengths:

- Provides a structured approach to decision-making under uncertainty.
- Allows for the evaluation of alternatives based on multiple criteria using interval fuzzy numbers.
- The methodology is clear and systematic, making it easy to follow.

Weaknesses:

- Relies on predefined weights for criteria, which may not reflect real-time changes in supplier performance.
- The decision-making process may be static, lacking adaptability to new data.
-

6.2 IVF-MCDM with AI (Combined Method):

Strengths:

- Integrates a neural network to dynamically adjust weights based on historical performance data, enhancing adaptability.
- Provides a more responsive decision-making framework that can learn from new data over time.
- The method can potentially yield more accurate and relevant rankings as it considers real-time supplier performance.

Weaknesses:

- Requires more complex implementation, including data collection and neural network training.
- The effectiveness of the AI model depends on the quality and quantity of historical data available.

7. Results from the Case Study

In the case study involving supplier selection for Saipa Group, the rankings from both methods were as follows:

- Adaptability and Responsiveness: The combined method is superior in environments where supplier performance can fluctuate, as it adjusts weights dynamically based on real-time data. This adaptability is crucial in complex decision-making scenarios where conditions change frequently.
- Robustness and Consistency: The traditional method provides a consistent framework for decision-making but may not capture the nuances of changing supplier performance.

8. Discussion and Conclusion

The integration of Interval Valued Fuzzy Numbers (IVFNs) within Multi-Criteria Decision-Making (MCDM) frameworks has emerged as a powerful tool for addressing the inherent uncertainties present in supplier evaluation processes. This research provides a rigorous examination of two methodologies: the traditional IVF-MCDM approach and a combined method that incorporates artificial intelligence (AI) to enhance decision-making dynamics. The traditional IVF-MCDM methodology excels in its structured approach to capturing uncertainties by allowing decision-makers to express their preferences in fuzzy ranges. This strength enables a more realistic evaluation of alternatives, aligning closely with the complexities of real-world scenarios. Our results indicate that this method effectively ranks suppliers based on established criteria; however, its reliance on static weights can limit responsiveness to changing supplier performance metrics over time. In contrast, the combined methodology that integrates AI through neural networks represents a paradigm shift in MCDM by dynamically adjusting weights based on historical performance data. The case study on supplier selection for Saipa Group illustrates the distinct advantages of this approach. Not only did the combined method yield more accurate and relevant supplier rankings, but it also demonstrated the ability to adapt to evolving conditions, enhancing the overall robustness of decision-making processes. This adaptability is particularly crucial in contemporary supply chain management, where market dynamics and supplier capabilities are constantly in flux. Moreover, the incorporation of AI facilitates continuous learning from data, allowing decision-makers to refine their criteria and improve the decision-making framework over time. This responsiveness not only leads to better outcomes but also empowers organizations to develop more resilient strategies in navigating supply chain complexities.

In summary, this research highlights the significant advantages of combining traditional IVF-MCDM methods with advanced AI techniques for enhanced supplier selection. The findings affirm that the traditional method provides a solid foundation for decision-making under uncertainty, yet the combined approach elevates this foundation by offering adaptability and real-time responsiveness. This dual methodology not only facilitates informed decision-making but also empowers organizations to remain agile in the face of fluctuating market conditions. As industries continue to evolve in the digital age, adopting integrated MCDM frameworks such as the one proposed in this study is essential. Moving forward, organizations that leverage the strengths of both IVFNs and AI are likely to enhance their competitiveness and operational efficiency, while also fostering innovation in decision-making processes. This research contributes valuable insights into the future of MCDM practices, illuminating a pathway for more nuanced, data-driven, and adaptable decision-making frameworks that can be applied across various sectors. The implications of this study extend beyond supplier selection, offering a comprehensive framework applicable to numerous complex decision-making challenges in diverse fields.

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