*International Journal of Mathematical Modelling & Computations*
*Vol. 14, No. 01, 2024, 21- 38*
DOI: 10.71932/ijm.2024.1128741

# Evaluation of Decision-Making Units with Interval Grey Data in DEA

Jafar Pourmahmoud [a,*], Mahdi Eini[b], Davood Darvishi Salokolaei[c] and Saeid Mehrabian[d]

[a] *Department of Applied Mathematics, Azarbaijan Shahid Madani University, Tabriz, Iran,*
[b] *Department of Applied Mathematics, Azarbaijan Shahid Madani University, Tabriz, Iran,*
[c] *Department of Mathematics, Payame Noor University, Tehran, Iran,*
[d] *Department of Computer Science, Faculty of Mathematics Sciences and Computer, Kharazmi University, Karaj, Iran.*

**Abstract.** When evaluating the efficiency of decision-making units, paying attention to the amount of indicators and their conditions is of particular importance. In classical data envelopment analysis models, inputs and outputs are deterministic. However, if the data is grey, to obtain reliable results, it is better to use the theory of grey systems in data envelopment analysis. In this paper, a new method for using interval grey data in CCR model is proposed. The proposed method on a practical example is used to check the condition of cerebral hemorrhage in stroke patients after the injection of Tissue Plasminogen Activator. The results obtained from the proposed method on this example show that the examination of cerebral hemorrhage status of stroke patients is more accurately calculated and these results are more reliable for decision makers.

Received: 11 January 2024 ; Revised: 20 March 2024; Accepted: 30 March 2024.

**Keywords**: Efficiency; Stroke; Interval grey number; Data envelopment analysis.

**Index to information contained in this paper**



## 1. Introduction

In recent years, the evaluation method of decision making units (DMUs) using data envelopment analysis (DEA) has been widely increased. This method was first introduced via CCR model by Cooper et al [2] with deterministic data. This model has been used to evaluate the airport [41], agricultural system [20], social networks [46] etc.

Since one of the most important factors in the progress of any country is the health its people, Evaluation of health system is very critical. Nunamaker first calculated the daily efficiency of nursing services using DEA method [53]. Kleinsorge and Karney also evaluated the performance of nursing homes using DEA [24].

Technical efficiency of Turkish hospitals was measured by DEA approach by Ersoy et al [31]. Zavras et al used DEA to calculate the efficiency of the national primary health

Corresponding author. Email: pourmahmoud@azaruniv.ac.ir


https://sanad.iau.ir/journal/ijm

care network of Greece [5]. Basson and Butler's method can be used to evaluate the efficiency of the operating rooms in the disabled veterans' health management system [36]. Marschall and Flessa proposed a two-stage DEA method to calculate efficiency [42]. Popescu et al evaluated the Romanian health system using the DEA method [16]. Kohl et al examined the health care situation in hospitals using the DEA method [49]. Gavurova et al calculated the efficiency of the health system in OECD countries with the dynamic network DEA approach [8]. Omir et al proposed a three-stage DEA model to evaluate the financial efficiency of the health care system [7].

Sometimes in decision-making problems, the data have non-deterministic conditions. In this case, if classical DEA models are used to calculate the efficiency of DMUs, the results may not be reliable. As a result, non-deterministic DEA models should be used. Data in non-deterministic DEA models are fuzzy, random, imprecise, uncertain, and grey. In Fuzzy DEA models, data are fuzzy [22, 41, 35]. In Stochastic DEA models, the data of DMUs are random [1, 4, 21]. Whenever the data is ordinal and interval, it is better to use Imprecise DEA models to obtain more accurate results [9, 19, 39]. For the evaluation of DMUs that do not have enough historical data, the relevant experts are used for the data, in this case Uncertain DEA models are used to evaluate the DMUs [10, 27, 44]. Sometimes, in some real problems, there are data that do not include any of the above. These data are called grey numbers which were first proposed by Deng in the grey systems theory [26]. This theory is an effective method for solving uncertainty problems with discrete data and incomplete information [55]. Grey is the concept of information poverty, lack of information, and uncertainty, in which black indicates completely unknown information, and white indicates completely clear information, and other information that is somewhat clear and somewhat vague is grey [37]. This theory has been used in the field of engineering [23], management [6], energy consumption prediction [40], optimization [17].

In grey systems theory, grey numbers are considered as a tool to represent the data of DMUs. Grey numbers are uncertain numbers that take their possible value from a set of numbers or an interval [50]. One type of these numbers is interval grey numbers where the exact value of the number is not known, but the interval of numbers is known. Each interval grey number has a grey kernel value and a grey degree. Hence, Guo et al presented an interval grey number ranking method based on kernel and grey degree [48]. Li and Wei's method can be used to evaluate decision making units with grey data [43]. Chen et al proposed grey classification evaluation based on AHP and interval grey number [30]. Ranking of interval grey numbers is of particular importance. Li and Li ranked interval grey numbers based on generalized grey [56].

In some decision-making problems, such as health systems, the data are grey. Li et al evaluated the resource configuration and service capability in a hospital based on the public-private partnership model with grey systems theory [57]. In a hospital, it is very important to identify the main risk factors. Delcea and Ioana-Alexandra ranked them with gray systems theory for better management [12]. There is inequity in terms of health financing among the countries of the Eastern Mediterranean Region, and Pourmohammadi et al evaluated the financing of the health system in these countries through gray systems theory and Shannon's entropy [32]. Javed et al., using grey decision analysis approaches, investigated patient satisfaction and quality of public and private health care services in Pakistan [47]. Peng et al suggested ranking the factors of healthcare resources using grey systems theory [58]. Through this theory, Çetin and Özen compared and ranked the countries' COVID-19 mobile applications to fight the corona virus pandemic [18].

Sometimes, in decision-making problems, the data are grey interval and to calculate the efficiency of DMUs with the DEA method, the grey DEA method can be used as in this case the results are more reliable for managers to make decisions. This method was first proposed by Yang [59]. Wang et al calculated production and marketing efficiency with grey DEA method [45]. Wang and Liu proposed the grey DEA method to calculate the efficiency of DMUs with interval grey data [29]. Wang et al conducted an experimental study of grey DEA in performance analysis [34]. Wang et al evaluated the performance of major Asian airlines with DEA window model and grey systems theory [13]. Wang et al used it to evaluate efficiency in e-commerce markets [14]. Tran, and Nguyen applied the gray DEA model to evaluate the productivity of Vietnamese tourism [54]. Pourmahmoud et al proposed a method to calculate the efficiency of DMUs with undesirable factors by using the principles of strong and weak disposability in DEA [28].

In grey DEA models, the methods available in the DEA approach have been used to evaluate DMUs with interval grey data. In this case, the grey condition of the data may not be maintained and the data may be considered deterministic. Therefore, the results may not be reliable for decision-making managers. To solve this problem, it is better to use grey concepts. In this paper, the interval grey CCR fractional model based on the division of interval grey numbers is proposed to calculate the efficiency of interval grey DMUs with DEA method. Finally, the condition of cerebral hemorrhage in stroke patients after the injection of Tissue Plasminogen Activator (TPA) drug is investigated by the proposed method.

The article is organized as follows. In Section 2, the basic required concepts are given. In Section 3, a Grey DEA model is proposed to measure the efficiency of DMUs with interval grey data. In section 4, the indicators of DMUs for checking the condition of cerebral hemorrhage in stroke patients after drug injection (TPA) are introduced. In section 6, the interpretation of the results of the proposed model is stated. In section 7, the conclusions of this study are discussed.

## 2. Basic concepts

In this section, the basic concepts used in the paper are given. The first DEA model or the fractional CCR model was introduced with deterministic data to evaluate the efficiency of units. Then, the way it is displayed with algebraic operators, calculating the grey kernel and normalizing interval grey numbers are described in more details in the following:

### 2.1 *Model CCR fractional*

Suppose $DMU_j, (j = 1,..,n)$ are decision units with deterministic variables that consume a vector of inputs$X_j \in R_+^m, (j = 1,..,n)$ to produce a vector of outputs $Y_j \in R_+^s, (j = 1,..,n)$ Cooper et al [2] presented the following model CCR fractional to check the efficiency of the Z unit under evaluation:

$$
\begin{aligned}
& max \frac{UY_z}{VX_z} \\
& s.t. \\
& \quad \frac{UY_j}{VX_j} \leq 1, j = 1,..,n \\
& \quad V = (v_1, \dots, v_m) \geq \varepsilon \mathbf{1}_m, U = (u_1, \dots, u_s) \geq \varepsilon \mathbf{1}_s \,.
\end{aligned}
\tag{1}
$$

Where $\mathbf{1}_K$ is a vector with K unit components, V and U are the vectors of weights of inputs and outputs, respectively, and $\varepsilon$ is a positive non-Archimedean number.

### 2.2 *Basic concepts in grey systems theory*

Every new theory that enters the world of science has its own basic concepts. These concepts include relationships and operations that must be defined in that theory. In grey systems theory, algebraic operators and the kernel of interval grey numbers are defined as follows. Let that the grey number of interval is represented by the symbol

$\otimes a \in [\underline{a}, \overline{a}] = \{t \in a : \underline{a} \le t \le \overline{a}\}$ that $\underline{a}$ the lower limit is and $\overline{a}$ is the upper limit.

**Definition 2.1** Let $\otimes a \in [\underline{a}, \overline{a}]$ and $\otimes b \in [\underline{b}, \overline{b}]$ are two interval grey numbers.

(i) Algebraic operators of two interval grey numbers $\otimes a$ and $\otimes b$ are defined as follows [50]:

$$\otimes a + \otimes b \in [\underline{a} + \underline{b}, \overline{a} + \overline{b}],$$
$$\lambda \otimes a \in [\lambda \underline{a}, \lambda \overline{a}], \lambda > 0,$$
$$\frac{\otimes a}{\otimes b} \in \left[ min\left\{ \frac{\underline{a}}{\underline{b}}, \frac{\underline{a}}{\overline{b}}, \frac{\overline{a}}{\underline{b}}, \frac{\overline{a}}{\overline{b}} \right\}, max\left\{ \frac{\underline{a}}{\underline{b}}, \frac{\underline{a}}{\overline{b}}, \frac{\overline{a}}{\underline{b}}, \frac{\overline{a}}{\overline{b}} \right\} \right].$$

(ii) The grey number kernel of interval $\otimes a$ is defined as follows [50]:

$$\otimes \hat{a} = \frac{\underline{a} + \overline{a}}{2}$$

(iii) The width of the kernel from the lower limit and the upper limit $\otimes a$ is defined as follows[52]:

$$\otimes a_w = \frac{\underline{a} - \overline{a}}{2}$$

(iv) The comparison of two interval grey numbers $\otimes a$ and $\otimes b$ with Hu and Wong's method is as follows [11]:

$$\otimes a \le \otimes b \Leftrightarrow \begin{cases} \otimes \hat{a} \ne \otimes \hat{b} & \text{if } \otimes \hat{a} < \otimes \hat{b} \\ \otimes \hat{a} = \otimes \hat{b} & \text{if } \otimes a_w \ge \otimes b_w \end{cases}$$

### 2.3 *Interval grey data normalization method*

To obtain accurate results using the presented models, the data should be normalized, because the data of DMUs may have been measured with different scales. The grey interval inputs and outputs of DMUs are normalized as [25]:

(i) Normalization of DMUs inputs:

Let that the inputs of $i$-th of $DMU_j$ are $\otimes X_{ij} \in [\underline{x}_{ij}, \overline{x}_{ij}],\ i = 1, \dots, m,\ j = 1, \dots, n$; the following values are calculated for each $i$-th input.

$$m_i = \min_{1 \le j \le n} \underline{x}_{ij}, \quad M_i = \max_{1 \le j \le n} \overline{x}_{ij}, \quad i = 1, \dots, m.$$

The normalized values of the inputs are obtained from the following relations:

$$\underline{x}_{ij}^* = \frac{M_i - \overline{x}_{ij}}{M_i - m_i}, \quad \overline{x}_{ij}^* = \frac{M_i - \underline{x}_{ij}}{M_i - m_i}, \quad i = 1, \dots, m, j = 1, \dots, n.$$

(ii) Normalization of DMUs outputs:

Let that the outputs of $r$-th of $DMU_j$ are $\otimes Y_{rj} \in [\underline{y}_{rj}, \overline{y}_{rj}],\ r = 1, \dots, s,\ j = 1, \dots, n$; the following values are calculated for each $r$-th output.

$$m_r = \min_{1 \le j \le n} \underline{y}_{rj}, \quad M_r = \max_{1 \le j \le n} \overline{y}_{rj}, \quad i = 1, \dots, s.$$

The normalized values of the outputs are obtained from the following relations:

$$\underline{y}^*_{rj} = \frac{M_r - \overline{y}_{rj}}{M_r - m_r}, \qquad \overline{y}^*_{rj} = \frac{M_r - \underline{y}_{rj}}{M_r - m_r}, \qquad r = 1, \dots, s, j = 1, \dots, n.$$

## 3. Proposed grey DEA model with interval grey data

Suppose there are $n$ DMUs with interval grey inputs and outputs as:

$$\otimes X_{ij} \in [\underline{x}_{ij}, \overline{x}_{ij}], \qquad \otimes Y_{rj} \in [\underline{y}_{rj}, \overline{y}_{rj}], \ \ i = 1, \dots, m, r = 1, \dots, s, j = 1, \dots, n.$$

Interval grey model CCR fractional is proposed for unit evaluation under $z$-th evaluation as follows:

$$\begin{aligned} &\max \frac{\sum_{r=1}^{s} u_r [\underline{y}_{rz}, \overline{y}_{rz}]}{\sum_{i=1}^{m} v_i [\underline{x}_{iz}, \overline{x}_{iz}]} \\ &\quad s.t. \\ &\frac{\sum_{r=1}^{s} u_r [\underline{y}_{rj}, \overline{y}_{rj}]}{\sum_{i=1}^{m} v_i [\underline{x}_{ij}, \overline{x}_{ij}]} \le 1, j = 1, .., n \\ &v_i, u_r \ge \varepsilon, i = 1, \dots, m, r = 1, \dots, s \ . \end{aligned} \tag{2}$$

Model (2) can be written as follows according to the multiplication of the scalar number by interval grey number

$$\begin{aligned} &\max \frac{[\sum_{r=1}^{s} u_r \underline{y}_{rz}, \sum_{r=1}^{s} u_r \overline{y}_{rz}]}{[\sum_{i=1}^{m} v_i \underline{x}_{iz}, \sum_{i=1}^{m} v_i \overline{x}_{iz}]} \\ &\quad s.t. \\ &\frac{[\sum_{r=1}^{s} u_r \underline{y}_{rj}, \sum_{r=1}^{s} u_r \overline{y}_{rj}]}{[\sum_{i=1}^{m} v_i \underline{x}_{ij}, \sum_{i=1}^{m} v_i \overline{x}_{ij}]} \le 1, j = 1, .., n, \\ &v_i, u_r \ge \varepsilon, i = 1, \dots, m, r = 1, \dots, s \ . \end{aligned} \tag{3}$$

Model (3) is a fractional interval grey model and cannot be easily solved with the help of mathematical software. To solve it, you can use the interval grey division method in definition 1 of section (1). In this case, we have four states for the objective function. By choosing the minimum value and the maximum optimal value of the four modes, the efficiency of DMUs is obtained as an interval grey number. In the following, each of the four modes is explained separately.

***The first case***

$$\max \frac{\sum_{r=1}^{s} u_r\, \underline{y}_{rz}}{\sum_{i=1}^{m} v_i\, \underline{x}_{iz}}$$
$$s.t.$$
$$\frac{\sum_{r=1}^{s} u_r\, \underline{y}_{rj}}{\sum_{i=1}^{m} v_i\, \underline{x}_{ij}} \le 1,\;\; j=1,\dots,n \qquad (a)$$
$$\frac{\sum_{r=1}^{s} u_r\, \underline{y}_{rj}}{\sum_{i=1}^{m} v_i\, \overline{x}_{ij}} \le 1,\;\; j=1,\dots,n \qquad (b) \qquad\qquad (4)$$
$$\frac{\sum_{r=1}^{s} u_r\, \overline{y}_{rj}}{\sum_{i=1}^{m} v_i\, \underline{x}_{ij}} \le 1,\;\; j=1,\dots,n \qquad (c)$$
$$\frac{\sum_{r=1}^{s} u_r\, \overline{y}_{rj}}{\sum_{i=1}^{m} v_i\, \overline{x}_{ij}} \le 1\,,\;\; j=1,\dots,n \qquad (d)$$
$$v_i, u_r \ge \varepsilon, i=1,\dots,m\,, r=1,\dots,s\;. \qquad (e)$$

Based on the assumption $\overline{x}_{ij} \le \overline{x}_{ij}, \underline{y}_{rj} \le \overline{y}_{rj}; i=1,\dots,m\,, r=1,\dots,s,\;\; j=1,\dots,n$ and $v_i, u_r \ge 0, i=1,\dots,m\,, r=1,\dots,s$ the following relationship can be written:

$$0 < \sum_{r=1}^{s} u_r\, \underline{y}_{rj} \le \sum_{r=1}^{s} u_r\, \overline{y}_{rj} \le \sum_{i=1}^{m} v_i\, \underline{x}_{ij} \le \sum_{i=1}^{m} v_i\, \overline{x}_{ij}\,, i=1,\dots,m\,, r=1,\dots,s. \qquad (5)$$

By changing Charens-Cooper variable [3], fractional programming models (4) can be converted into the following linear model:

$$E_z^{*1} = \max \sum_{r=1}^{s} \mu_r\, \underline{y}_{rz}$$
$$s.t. \qquad\qquad (6)$$
$$\sum_{i=1}^{m} w_i\, \underline{x}_{iz} = 1$$
$$\sum_{r=1}^{s} \mu_r\, \underline{y}_{rj} - \sum_{i=1}^{m} w_i\, \underline{x}_{ij} \le 0,\;\; j=1,..,n \qquad (a')$$
$$\sum_{r=1}^{s} \mu_r\, \underline{y}_{rj} - \sum_{i=1}^{m} w_i\, \overline{x}_{ij} \le 0,\;\; j=1,..,n \qquad (b')$$
$$\sum_{r=1}^{s} \mu_r\, \overline{y}_{rj} - \sum_{i=1}^{m} w_i\, \underline{x}_{ij} \le 0,\;\; j=1,..,n \qquad (c')$$
$$\sum_{r=1}^{s} \mu_r\, \overline{y}_{rj} - \sum_{i=1}^{m} w_i\, \overline{x}_{ij} \le 0,\;\; j=1,..,n \qquad (d')$$
$$w_i, \mu_r \ge \varepsilon, i=1,\dots,m\,, r=1,\dots,s\;. \qquad (e')$$

**Theorem 3.1** Models (4) is feasible.

***Proof.*** Suppose, according to the definition of inputs and outputs of DMUs, we have:

$$x_{iM} = min\{\underline{x}_{ij}, \overline{x}_{ij}\}, i = 1, \dots, m \quad j = 1, .., n$$
$$y_{rM} = max\{\underline{y}_{rj}, \overline{y}_{rj}\}, r = 1, \dots, s \quad j = 1, .., n.$$

Suppose, the number of elements of set $\{i: x_{iM} = 0\}$ is equal to α. Assuming

$$\delta = min\left\{\frac{1}{(m-\alpha)x_{iM}}, \frac{1}{sy_{rM}}\right\}, x_{iM} \neq 0, i = 1, \dots, m, r = 1, \dots, s,$$

consider the vector $v_i$ and $u_r$ as follows:

$$v_i = \begin{cases} \delta, & if \ x_{iM} = 0 \\ \frac{1}{(m-\alpha)x_{iM}}, & if \ x_{iM} \neq 0 \end{cases}, i = 1, \dots, m, u_r = \frac{1}{sy_{rM}} \ r = 1, \dots, s.$$

Now we prove that the vectors $v_i, i = 1, \dots, m$ and $u_r \ r = 1, \dots, s$ as defined above apply to the constraints of model (4).

According to the definition of $v_i, , i = 1, \dots, m$ and $u_r \ r = 1, \dots, s$, we can conclude that we have $v_i, u_r \geq \delta, i = 1, \dots, m, r = 1, \dots, s$ and

$$\sum_{r=1}^{s} u_r \overline{y}_{rj} = \sum_{r=1}^{s} \frac{1}{sy_{rM}} \overline{y}_{rj} \leq 1, j = 1, \dots, n. \quad (7)$$

According to the assumption of $\underline{y}_{rj} \leq \overline{y}_{rj}, j = 1, \dots, n, r = 1, \dots, s$and$u_r \geq \delta, \ r = 1, \dots, s$, according to $\sum_{r=1}^{s} u_r \underline{y}_{rj} \leq \sum_{r=1}^{s} u_r \overline{y}_{rj}$and relation (7), we have

$\sum_{r=1}^{s} u_r \underline{y}_{rj} \leq 1, j = 1, \dots, n$. On the other hand,

$$\sum_{i=1}^{m} v_i \underline{x}_{ij} = \sum_{i=1}^{m} \frac{1}{(m-\alpha)x_{iM}} \underline{x}_{ij} \geq \frac{(m-\alpha)}{(m-\alpha)} = 1, for \ x_{iM} \neq 0 \quad (8)$$

Based on the assumption $\underline{x}_{ij} \leq \overline{x}_{ij}, i = 1, \dots, m, \ j = 1, \dots, n$ and $v_i \geq \delta, \ i = 1, \dots, m$, according to $\sum_{i=1}^{m} v_i \underline{x}_{ij} \leq \sum_{i=1}^{m} v_i \overline{x}_{ij}, j = 1, \dots, n$and relation (8), we have

$\sum_{i=1}^{m} v_i \overline{x}_{ij} \geq 1, j = 1, \dots, n$. Therefore, the defined $v_i$ and $u_r$ is a feasible solution to model (4). ■

**Theorem 3.2** Models (6) is feasible.

***Proof.*** Suppose, according to the definition of inputs and outputs DMUs, we have:

$$x_{iz} = min\{\underline{x}_{ij}, \overline{x}_{ij}\}, \underline{x}_{ij}, \overline{x}_{ij} \neq 0, i = 1, \dots, m, \quad j = 1, .., n$$
$$y_{rM} = max\{\underline{y}_{rj}, \overline{y}_{rj}\}, r = 1, \dots, s. \quad j = 1, .., n$$

Suppose that $e_i, i = 1, .., m$ is standard vector where i-th component is 1 elsewhere 0, $t_i, i = 1, .., m$ is vector where i-th component is $\frac{1}{x_{iz}}$ elsewhere 0 and$\mu_r = \frac{1}{sy_{rM}}, r =$

$1,\dots,s$.Also Suppose that$w_i = e_i . t_i = \frac{1}{x_{iz}}, i =, \dots, m$. According to the definition of vector $w_i, i =, \dots, m$, the first constraint is established. Also, according to the defined vectors $w_i, i =, \dots, m,\ \mu_r, r = 1, \dots, s$ and the proof of theorem 3.1, the constraints $(a')$ to $(e')$ are established.
Suppose $\theta = min\left\{\frac{1}{x_{iz}}, \frac{1}{sy_{rM}}\right\}, i = 1, \dots, m,\ r = 1, \dots, s$, so it's conclude $w_i, \mu_r \geq \theta$, $i = 1, \dots, m,\ r = 1, \dots, s$. ■

**Theorem 3.3** The optimal value of model (6) is in the interval $(0,1]$, in other words $0 < E_z^{*1} \leq 1$.

***Proof***. In model (6) according to the first, second and fourth constraints, the second and fourth constraints can be written for the index$j = z$, so $\sum_{r=1}^{s} \mu_r \underline{y}_{rj} \leq 1, \sum_{r=1}^{s} \mu_r \overline{y}_{rj} \leq 1$. On the other hand, according to the first, third, and fifth constraints and considering relation (5), we have$\sum_{r=1}^{s} \mu_r \underline{y}_{rj} \leq 1, \sum_{r=1}^{s} \mu_r \overline{y}_{rj} \leq 1$, so the maximum optimal value of model (6) is less than and equal to one. The Proposition for the lower limit is also established according to the positive assumption of the data of the DMU $w_i, \mu_r \geq \varepsilon, i = 1, \dots, m, r = 1, \dots, s$. ■

**Theorem 3.4**

(i) For each the feasible solution of model (4), there is the feasible solution corresponding to model (6) and vice versa.
(ii) For each optimal solution of model (4), there is a corresponding optimal solution for model (6) so that the optimal values of model (4) and (6) are equal.

***Proof.*** Refer to [38]. ■

For model (6) to be feasible, an acceptable value for the non-Archimedean number $\varepsilon$ should be considered. To find this value, the trust-interval for $\varepsilon$ can be determined using Mehrabian et al's method [51]. To do so, it is necessary to calculate $\varepsilon_j^*, j = 1, \dots, n$ for all decision making units which are obtained by solving the following linear programming problem:

$$
\begin{aligned}
&\varepsilon^{*1} = max\ \varepsilon \\
&s.t. \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (9)\\
&\qquad \sum_{i=1}^{m} w_i\, \underline{x}_{iz} = 1 \\
&\qquad set\ constraints\ (a')\ \text{to}\ (e').
\end{aligned}
$$

By solving the above model, the optimal answer$\varepsilon^*$is obtained for all DMUs. If $\varepsilon^* = \min\{\varepsilon_1^*, \dots, \varepsilon_n^*\}$is assumed, then the trust-interval for $\varepsilon$ is $(0, \varepsilon^*]$. By choosing any value from this interval, an acceptable value for the non-Archimedean number $\varepsilon$ can be chosen and model (8) can be solved.

**Theorem 3.5** Model (9) is feasible and has a finite optimal solution.

***Proof.*** Suppose $\underline{x}_{iz} = \min_{j=1,\dots,n}\{\underline{x}_{ij}, \overline{x}_{ij}\}, i = 1, \dots, m$ and the number of elements of set $\{i: x_{iz} \neq 0\}$ is equal to β. Also, Assume vector V with components

$$v_i = \begin{cases} 0, & \underline{x}_{iz} = 0 \\ \dfrac{1}{\beta \underline{x}_{iz}}, & \underline{x}_{iz} \neq 0 \end{cases}, i = 1, \dots, m, \qquad and\ u_r = 0, r = 1, \dots, s, \varepsilon = 0.$$

The vector$(v_i, u_r, \varepsilon), i = 1, \dots, m, r = 1, \dots, s,$ defined above applies to all model constraints (9). So a solution is feasible, as a result, model (9) is feasible. By multiplying the sides of limit $v_i \geq \varepsilon$ in $\sum_{i=1}^{m} \underline{x}_{iz}$, we will have: $\sum_{i=1}^{m} v_i \underline{x}_{iz} \geq \varepsilon \sum_{i=1}^{m} \underline{x}_{iz}$. in this case, according to the first limitation of model (9), the relation $\varepsilon \leq 1/\sum_{i=1}^{m} \underline{x}_{iz}$ is obtained. So model (9) has a finite optimal solution.

***The second case*** The ratio of the weighted sum of the lower limit of the outputs to the weighted sum of the upper limit of the inputs. In this case, the objective function should be considered based on the lower limit of the outputs and the upper limit of the inputs with the same constraints as model (4). Therefore, the corresponding fractional programming model is as follows:

$$\begin{aligned} & max \ \frac{\sum_{r=1}^{s} u_r \underline{y}_{rz}}{\sum_{i=1}^{m} v_i \overline{x}_{iz}} \\ & \quad s.t. \\ & \quad\quad set\ constraints\ (a) \text{ to } (e). \end{aligned} \tag{10}$$

The feasibility of model (10) is proved similar to theorem 3.1. Similarly, model (10) is linearized to model (11).

$$\begin{aligned} & E_z^{*2} = max \sum_{r=1}^{s} u_r \underline{y}_{rz} \\ & \quad s.t. \\ & \quad\quad \sum_{i=1}^{m} w_i \overline{x}_{iz} = 1 \\ & \quad\quad set\ constraints\ (a') \text{ to } (e'). \end{aligned} \tag{11}$$

The feasibility of the above model is similar to the proof of Theorem 3.2. The optimal value of model (11) is in interval $(0, 1]$ which can be proved similar to theorem 3.3 As mentioned about how to obtain the trust-interval in the first case, to obtain the trust-interval in the second case in model (11) is used to solve linear programming problem the following:

$$\begin{aligned} & \varepsilon^{*2} = max\ \varepsilon \\ & s.t. \\ & \quad \sum_{i=1}^{m} w_i \overline{x}_{iz} = 1 \\ & \quad set\ constraints\ (a') \text{ to } (e'). \end{aligned} \tag{12}$$

Model (12) is feasible and has a finite optimal solution which can be proved similarly to Theorem 3.5

***The third case*** The ratio of the weighted sum of the upper limit of the outputs to the weighted sum of the lower limit of the inputs. In this case, the objective function should be considered based on the upper limit of the outputs and the lower limit of the inputs with the same constraints as model (4). Therefore, the corresponding fractional programming model is as follows:

$$\max \frac{\sum_{r=1}^{s} u_r \overline{y}_{rz}}{\sum_{i=1}^{m} v_i \overline{x}_{iz}} \quad s.t. \quad set\ constraints\ (a)\ \text{to}\ (e). \tag{13}$$

The feasibility of model (13) is proved similar to theorem 3.1. Similarly, model (13) is linearized to model (14).

$$E_z^{*3} = \max \sum_{r=1}^{s} \mu_r \overline{y}_{rz} \quad s.t. \quad \sum_{i=1}^{m} w_i \underline{x}_{iz} = 1, \quad set\ constraints\ (a')\ \text{to}\ (e'). \tag{14}$$

The feasibility of the model (14) is similar to the proof of Theorem 3.2. The optimal value of model (14) is in interval $(0,1]$ which can be proved similar to theorem 3.3. Considering that the limitations of model (14) are the same as the limitations of model (9), then the trust-interval of model (14) can be considered according to model (9).

***The fourth case*** The ratio of the weighted sum of the upper limit of the outputs to the weighted sum of the upper limit of the inputs. In this case, the objective function should be considered based on the upper limit of the outputs and the upper limit of the inputs with the same constraints as model (4). Therefore, the corresponding deficit planning model is as follows:

$$\max \frac{\sum_{r=1}^{s} u_r \overline{y}_{rz}}{\sum_{i=1}^{m} v_i \overline{x}_{iz}} \quad s.t. \quad set\ constraints\ (a)\ \text{to}\ (e). \tag{15}$$

The feasibility of model (15) is proved similar to theorem 3.1. Similarly, model (15) is linearized to model (16).

$$E_z^{*4} = \max \sum_{r=1}^{s} \mu_r \overline{y}_{rz} \quad s.t. \quad \sum_{i=1}^{m} w_i \overline{x}_{iz} = 1, \quad set\ constraints\ (a')\ \text{to}\ (e'). \tag{16}$$

The feasibility of the model (16) is similar to the proof of Theorem 3.2. The optimal value of model (16) is in interval $(0,1]$which can be proved similar to theorem 3.3. Considering that the limitations of model (16) are the same as the limitations of model (11), then the trust-interval of model (16) can be considered according to model (11).

After obtaining the optimal values of models (8), (11), (14) and (16), the parameter values of the lower and upper limits of efficiency for$DMU_z$ are calculated as follows:

$$\underline{E}_z^* = min\{E_z^{*1}, E_z^{*2}, E_z^{*3}, E_z^{*4}\}, \overline{E}_z^* = max\{E_z^{*1}, E_z^{*2}, E_z^{*3}, E_z^{*4}\} \quad (17)$$

The efficiency of $DMU_j, j = 1, \dots, n$ with interval grey number data can be shown as $\otimes E_j^* \in \left[\underline{E}_j^*, \overline{E}_j^*\right], j = 1, \dots, n$ and defined as follows:

**Definition 3.1** In evaluating the efficiency of $DMU_j, j = 1, \dots, n$ interval grey inputs and outputs, the following cases occur. Whenever,

(1) $\underline{E}_j^* = \overline{E}_j^*, j = 1, \dots, n$, then $DMU_j, j = 1, \dots, n$ is first level efficient.

(2) $\underline{E}_j^* < \overline{E}_j^*, j = 1, \dots, n$, then $DMU_j, j = 1, \dots, n$ is the efficient of the second level.

(3) $\underline{E}_j^* < \overline{E}_j^* < 1, j = 1, \dots, n$, then $DMU_j, j = 1, \dots, n$ is inefficient.

**Lemma 3.1** For the efficiency values of DMUs, $\underline{E}_j^* \leq \overline{E}_j^*, j = 1, \dots, n$.

***Proof.*** According to the data of $DMU_j, j = 1, \dots, n$ and $w_i, \mu_r \geq 0, i = 1, .., m, r = 1, \dots, s$, we have the following relations:

$$\sum_{i=1}^{m} w_i \underline{x}_{ij} \leq \sum_{i=1}^{m} w_i \overline{x}_{ij}, \sum_{r=1}^{s} \mu_r \underline{y}_{rj} \leq \sum_{r=1}^{s} \mu_r \overline{y}_{rj}.$$

Considering relation (7), we will have

$$0 < \sum_{r=1}^{s} \mu_r \underline{y}_{rj} \leq \sum_{r=1}^{s} \mu_r \overline{y}_{rj} \leq \sum_{i=1}^{m} w_i \underline{x}_{ij} \leq \sum_{i=1}^{m} w_i \overline{x}_{ij}.$$

The ruling is established by considering relation (17) and the above relation. ■

## 4. Applications

Nowadays, professionals in the health system are faced with various diseases. One of these diseases is stroke. In a stroke, blood is not supplied to an area of the brain, and as a result, part of the body's neurological function is disturbed. This disease is one of the most important causes of death and disability [33]. Medicines have been used to treat this disease. One of these medicines is TPA. When injecting this medicine, neurologists pay attention to clinical and laboratory indicators. In this study, the indicators that are checked by neurologists are divided into two categories, before and after the administration of TPA to the stroke patient. The relevant indicators before injecting the drug are: Age, TPA Dosage, Weight, Blood pressure, Glucose, Door Needle Time (DNT), National Institutes of Health Stroke Scale (NIHSS), and Platelet Count (PLT). The indicator after drug injection is Hemorrhagic transformation infarct (HTI). In the following, each of the indicators mentioned above is briefly explained.

***Age*** The probability of cerebral hemorrhage caused by TPA injection is higher in elderly patients. Therefore, the lower the value of this indicator, the more confidently the neurologist can inject TPA to the patient.

***TPA Dosage*** Considering the patient's clinical conditions and blood test results, the dosage of TPA drug is injected after the patient visits the hospital under the supervision of a neurologist. The best and most effective time of TPA drug injection is from the moment of stroke to 270 minutes later.
Weight: The stroke patient's weight is an indicator considered when injecting the TPA drug to the patient. In this study, each patient's weight is calculated based on kilograms. The injection of TPA drug dose is based on this indicator.

***DNT*** Neurologists, when examining the risk of cerebral hemorrhage caused by TPA injection, attach special importance to the DNT indicator. This indicator is considered from the time the patient enters the hospital until the moment of TPA injection. The lower the DNT value, the lower the risk of bleeding from TPA injection.

***NIHSS*** After a stroke patient is admitted to the hospital, questions such as how conscious is the patient? Is the patient able to communicate properly with others? And other things that the relevant expert asks to the patient and the patient's companion in the form of a questionnaire. By completing this questionnaire by the relevant expert, the NIHSS indicator is obtained, representing the general history of the stroke patient during hospitalization. This indicator with a qualitative scale needs to be converted into a quantitative scale. After conversion, the minimum value of the NIHSS index is zero, and its maximum value is 42. According to a neurologist, the closer the NIHSS indicator value is to zero, the more brain bleeding caused by TPA injection may not happen to the patient.

***Blood pressure*** Neurologists believe that lack of blood pressure control is one of the most important factors that play a significant role in causing a stroke. If the blood pressure is higher than 18 after the stroke patient is admitted to the hospital when TPA is injected into the patient. In that case, the risk of cerebral hemorrhage increases, and it is dangerous for the patient. For this reason, if the patient has this condition, the blood pressure should be reduced to below 18 with the relevant medicine, and then TPA injection should be done.

***Glucose*** Another stroke indicator is glucose, which the hospital's blood laboratory determines at the time of admission. The high level of this factor increases the risk of bleeding in stroke patients treated with TPA. If the patient's glucose level is more than 400 when injecting the drug, the glucose level is reduced to less than 400 with insulin, and then the TPA drug is injected.

***PLT*** The number of PLT is determined by the blood laboratory at the time of hospitalization. According to a neurologist, if the number is between 100,000 and 400,000, the risk of bleeding due to TPA injection to the patient is low.

***HTI*** This indicator shows the bleeding status of the patient's brain. After the injection of TPA, images are taken of the patient's brain with a CT scan machine. Then the images are checked by a neurologist.
The patient's brain bleeding status has three states. In this study, grey systems theory with interval data is used to quantify each of the three cases. The grey numbers of the interval corresponding to the three cases are listed in Table (1).

Table 1. Clinical description of HTI of stroke patients.

| HTI Score | Clinical Description |
|---|---|
| [90,100] | There is no bleeding in the brain |
| [40,90] | Bleeding occurred in the brain, but it did not cause any problems for the patient |
| [0,40] | Bleeding has occurred in the brain and it is dangerous for the patient |

In this study, in 2020, the data of 30 stroke patients were collected from seven registry data centers. Due to the confidentiality of the information, it is not allowed to publish all laboratory and clinical information of all patients. For this reason, in table (2) only information about one patient is provided as an example. The purpose of investigating the condition of cerebral hemorrhage in stroke patients after TPA drug injection is based on clinical and laboratory indicators in which each patient is considered as a DMU. The considered clinical and laboratory indicators are listed in table (3) and also the indicators are specified based on the type of input and output.

Table 2. The information of the one patient.

| Age | TPA Dosage | Weight | DNT | NIHSS | Blood pressure | Glucose | PLT | HTI |
|---|---|---|---|---|---|---|---|---|
| 57 | [50, 81] | [85, 95] | [44, 61] | [11, 15] | [70, 135] | [224, 304] | [284, 324] | [90, 100] |

Table 3. The description of inputs and outputs in the treatment of stroke patients.

| Descriptions | Inputs/Output |
|---|---|
| Blood pressure | Inputs |
| Glucose | |
| TPA Dosage | |
| Weight | |
| DNT | |
| Age | |
| NIHSS | |
| PLT | |
| HTI | Output |

## 5. Result and discussion

In this study, the aim is to investigate the condition of cerebral hemorrhage in stroke patients after TPA injection. to this end, based on the type of patient data, the models proposed in section 3 are used. On the other hand, according to the type of data related to patient inputs and outputs, it is clear that the data do not have the same unit of measurement. Therefore, it is necessary to normalize the data to use it for the proposed models. Therefore, we normalize the data using the relations in section 3.2. An example of normalized data is given in the table below.

Table 4. Normalized data for one patient in Table 3

| Age | TPA Dosage | Weight | DNT | NIHSS |
|---|---|---|---|---|
| 0.5714 | [0, 0.6078] | [0, 0.2128] | [0.6763, 0.7746] | [0.5455, 0.6667] |
| Blood pressure | Glucose | PLT | HTI | |
| [0.5592, 0.9868] | [0.3955, 0.7591] | [0.1888, 0.3605] | [1 ,1] | |

In order to obtain the cerebral hemorrhage status of stroke patients after TPA drug injection, it is necessary to use the results of models (8), (11), (14) and (16) in relationships (17). After implementing these items, the results are summarized and listed in Table 5. In this table, the second column shows the efficiency of each DMU as an interval grey number.

Table 5. Efficiency of DMUs based on relation (17).

| DMU | Efficiency |
|---|---|
| DMU01 | [0.8331,1] |
| DMU02 | [0.8054,1] |
| DMU03 | [0, 0] |
| DMU04 | [0.5187, 0.7151] |
| DMU05 | [0.6702, 1] |
| DMU06 | [0.6996, 1] |
| DMU07 | [0.6056, 1] |
| DMU08 | [0.9477, 1] |
| DMU09 | [0.7487, 1] |
| DMU10 | [0, 0] |
| DMU11 | [0.6431, 0.8241] |
| DMU12 | [0.6976, 0.8130] |
| DMU13 | [0.9043, 1] |
| DMU14 | [0.6464, 0.9589] |
| DMU15 | [0.6697, 0.8484] |
| DMU16 | [0.7004, 1] |
| DMU17 | [0.3738, 0.8741] |
| DMU18 | [0.3646, 1] |
| DMU19 | [0.6135, 0.7673] |
| DMU20 | [0.3349, 0.7613] |
| DMU21 | [0.4301, 1] |
| DMU22 | [0.7918, 1] |
| DMU23 | [0.3064, 0.7760] |
| DMU24 | [0, 0] |
| DMU25 | [0.3583, 1] |
| DMU26 | [0.6479, 1] |
| DMU27 | [0.6368, 1] |
| DMU28 | [0.8881, 1] |
| DMU29 | [0.7663, 1] |
| DMU30 | [0, 0] |

According to definition 2, the following results are obtained from Table (5):

(i) Patients 1,2,5,6,7,8,9,13,16,18,21,22,25,26,27,28 and 29 have second-level efficient.

(ii) Patients 3,4,10,11,12,14,15,17,19,20,23,24,30 are inefficient.

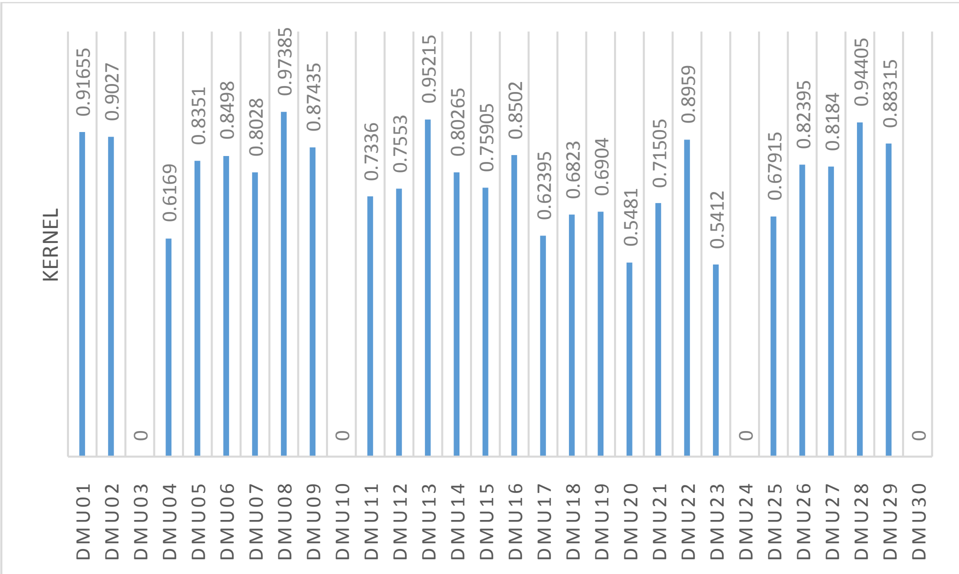
(iii) In this study, there is no first-level efficient.

According to the results, after TPA drug injection, among 30 patients, 20 patients did not have brain bleeding. 6 patients had mild cerebral bleeding, but it was not dangerous for the patient. 4 patients had severe cerebral hemorrhage and died after drug injection.

To check the condition of cerebral hemorrhage in stroke patients using the proposed method, a few patients are randomly selected. Among all the patients, the interval grey efficiency of patient No. 8 is higher than the rest of the patients. The efficiency of this patient is of the second level efficiency. The lower bound efficiency of this patient is higher than the lower bound efficiency of all patients. With the approval of neurologists, this patient had the best conditions in terms of minimal clinical and laboratory indicators during TPA injection. By comparing patient number 8 with patient number 29, it can be observed that patient number 8 at the age of 85 has better clinical conditions than patient number 29 at the age of 67. Comparison of the interval grey efficiency kernel of these two patients reveals that after the injection of TPA drug the complications caused by stroke in patient 8 were less compared to patient 29.

The grey efficiency value of patients 3, 10, 24 and 30 is equal to zero. In other words, the brain bleeding condition of these patients is severe after TPA injection. These patients did not recover and died. Our proposed method has diagnosed these patients as inefficient. These patients are ranked in the last categories. According to the clinical and laboratory indicators, these patients had the worst conditions compared to the patients who did not have cerebral hemorrhage. In Figure 1, patients are ranked according to the state of cerebral hemorrhage after TPA injection.

Using the method presented in this study, Neurologists will have fewer medical errors and patients recover faster and resume their normal life sooner. Regarding the clinical and clinical factors of heart attack patients, cardiologists can also benefit from the presented method.




Figur 1. Rank the DMUs.

## 6. Conclusion

Classical DEA models are used to evaluate the efficiency of DMUs with deterministic data. If the data is grey, these DEA models alone cannot be used anymore as the results obtained may not be reliable. Therefore, it is better for decision-making managers to employ the theory of grey systems. In this study, based on the division of interval gray numbers, a new method was proposed to calculate the efficiency of DMUs with interval grey data.

In any country, the health of every single person is crucial, so the evaluation and improvement of the health system should always be the top priority for authorities. One of the most important parts of the health system is the one related to the treatment of stroke patients. The amount of indicators that are effective in increasing the risk of bleeding due to the use of TPA drugs cannot be measured accurately, but they can be specified as grey numbers. For this reason, the value of these indicators was considered as an interval grey number. In this paper, the condition of cerebral hemorrhage in stroke patients after TPA drug injection was investigated using our proposed model. According to the obtained results, most of the patients who had interval grey efficiency of the second type had desirable clinical and laboratory conditions and did not have brain bleeding when TPA was injected. The patients who were inefficient had little or severe brain bleeding after TPA injection. Neuroscientists confirmed that this proposed model worked correctly. For further research, the proposed method can be developed for other systems such as meteorology, agriculture, etc.

## Acknowledgment

The authors would like to thank Dr. Leila Afshar Hezarkhani, Assistant Professor Neuroscience Research Center, Health Technology Institute, Kermanshah University of Medical Sciences, Kermanshah, Iran, for his efforts in the data gathering and the help during this research. It should be noted that Dr. Leila Afshar Hezarkhani confirmed the study results.

## References

[1] A. Amirteimoori, B. K. Sahoo, V. Charles, S. Mehdizadeh, A. Amirteimoori, B. K. Sahoo, V. Charles and S. Mehdizadeh, Stochastic Data Envelopment Analysis, Stochastic Benchmarking: Theory and Applications, (2022) 55-76.

[2] Charnes, W. W. Cooper and E. Rhodes, Measuring the efficiency of decision-making units., European Journal of Operational Research **3 (4)** (1979) 339.

[3] A. Charnes, W. W.Cooper, Programming with linear fractional functionals, Naval Research logistics quarterly, **9**(**3-4**) (1962) 181-186.

[4] A. Hadi-Vencheh, M. Khodadadipour, Y.Tan, H. Arman and D. Roubaud, Cross-efficiency analysis of energy sector using stochastic DEA: Considering pollutant emissions, Journal of Environmental Management, **364** (2024) 121319.

[5] A. I. Zavras, G. Tsakos, C. Economou and J. Kyriopoulos, Using DEA to evaluate efficiency and formulate policy within a Greek national primary health care network, Journal of Medical Systems **26** (2002) 285-292.

[6] A. Mahmoudi and S.A. Javed, Grey multiple criteria decision making methods: a literature review, ICSES Transaction on Neural and Fuzzy Computing, **2** (**2**) (2019) 1-13.

[7] A. S. Omir, A. Z. Panzabekova and A. A, Satybaldin, Evaluating the Financial Efficiency of the Healthcare System: A Three-Stage DEA Model Analysis. R-Economy. ***4*9**(**4**) (2023) 353-365.

[8] B. Gavurova, K. Kocisova and J. Sopko, Health system efficiency in OECD countries: dynamic network DEA approach, Health Economics Review, **11** (2021) 1-25.

[9] B. Jiang, C. Yang, Q. Dong and J. Li, Ecological efficiency evaluation of China's port industries with imprecise data, Evolutionary Intelligence, (2024) 1-12.

[10] B. Jiang, E. Chi and J. Li, Uncertain Data Envelopment Analysis for Cross Efficiency Evaluation with Imprecise Data, Mathematics, **10**(**13**) (2022) 2161.

[11] B.Q. Hu, S. Wang, A novel approach in uncertain programming part I: new arithmetic and order relation for interval numbers, Journal of Industrial and Management Optimization, **2**(**4**) (2006) 351-371.

[12] C. Delcea and B. Ioana-Alexandra, Fostering risk management in healthcare units using grey systems theory, Grey Systems: Theory and Application, **6**(**2**) (2016) 216-232.

[13] C. N. Wang, T. T. Tsai, H. P. Hsu, L. H. Nguyen, Performance evaluation of major Asian airline companies using DEA window model and grey theory. Sustainability, **11**(**9**) (2019) 2701.

[14] C. N. Wang, T. T. Dang, N. A. T. Nguyen, T. T. H. Le, Supporting better decision-making: A combined grey model and data envelopment analysis for efficiency evaluation in e-commerce marketplaces. Sustainability, **12**(**24**) (2020) 10385.

[15] C. N. Wang, K. N. C. Imperial, C. C. Huang and T. T. Dang, Output targeting and runway utilization of major international airports: A comparative analysis using DEA, Mathematics, **10**(**4**) (2022) 551.

[16] C. Popescu, L. Asandului and P. Fatulescu, A data envelopment analysis for evaluating Romania's health system, Procedia-Social and Behavioral Sciences, **109** (2014) 1185-1189.

[17] Tiwari, A. F. Sherwani, M. Asjad and A. Arora, Grey relational analysis coupled with principal component analysis for optimization of the cyclic parameters of a solar-driven organic Rankine cycle, Grey Systems: Theory and Application, **7**(**2** ) (2017) 218-235.

[18] E. Çetin and H. Özen, Ranking eHealth Efforts of Countries to Fight Coronavirus Pandemic via Grey Systems Theory: Evidence from National COVID-19 Mobile Apps. In Integrity, Transparency and Corruption in Healthcare & Research on Health, Singapore: Springer Nature Singapore, **2** (2023) 143-178.

[19] E. Vaezi, S. E. Najafi, S. M. Haji Molana, F. Hosseinzadeh Lotfi and M. Ahadzadeh Namin, Measuring performance of a hybrid system based on imprecise data: Modeling and solution approaches. Journal of Industrial Engineering and Management Studies, **6**(**2**) (2019) 214-241.

[20] H. M. A. Ilyas, M. Safa, A. Bailey, S. Rauf and A. Khan, Energy efficiency outlook of New Zealand dairy farming systems: An application of data envelopment analysis (DEA) approach, Energies, **13**(**1**) (2020) 251.

[21] H. Omrani, M. Shamsi, A. Emrouznejad and T.Teplova, A robust DEA model under discrete scenarios for assessing bank branches, Expert Systems with Applications, **219** (2023) 119694.

[22] H. Pourbabagol, M. Amiri, M. T. Taghavifard and P. Hanafizadeh, A new fuzzy DEA network based on possibility and necessity measures for agile supply chain performance evaluation: A case study, Expert Systems with Applications, **220** (2023) 119552.

[23] H.Yan and L. Zhang, The research of the grey system theory applied on buildings deformation monitoring, Journal of Chemical and Pharmaceutical Research, **6**(**7**) (2014) 2627-2629.

[24] I. K. Kleinsorge and D. F. Karney. Management of nursing homes using data envelopment analysis, Socio-Economic Planning Sciences **26**(**1**) (1992) 57-71.

[25] J .H. Wu, C.B. Chen, An alternative form for grey relational grades, the journal of Grey System, **11** (**1**) (1999) 7-12.

[26] J.L. Deng, The control problems of grey systems", Systems and Control Letters, **15** (1982) 288-294.

[27] J. Pourmahmoud and N. Bagheri, Providing an uncertain model for evaluating the performance of a basic two-stage system, Soft Computing, **25** (2021) 4739-4748.

[28] J. Pourmahmoud, M. Eini, D. Darvishi Salokolaei, S. Mehrabian, Hybrid model for evaluation of DMUs with principles of strong and weak disposability in the presence of grey undesirable factors, Iranian Journal Of Operations Research, **140**(**2**) (2021) 32-46.

[29] J. Wang and S. Liu, Efficiency measures in DEA with grey interval data under the hypotheses of data consistency, Grey Systems: Theory and Application, **2**(**1**) (2012) 63-69.

[30] K. Chen, P. Chen, L. Yang and L. Jin, Grey clustering evaluation based on AHP and interval grey number, International Journal of Intelligent Computing and Cybernetics, **12**(**1**) (2019) 127-137.

[31] K. Ersoy, S. Kavuncubasi, Y. A. Ozcan, II and J. M Harris, Technical efficiencies of Turkish hospitals: DEA approach, Journal of Medical Systems **21** (1997) 67-74.

[32] K. Pourmohammadi, P. Shojaei, H. Rahimi and P. Bastani, Evaluating the health system financing of the Eastern Mediterranean Region (EMR) countries using Grey Relation Analysis and Shannon Entropy, Cost Effectiveness and Resource Allocation, **16** (2018) 1-9.

[33] L.B. Goldstein, D.B. Matchar, Clinical assessment of stroke, Amer. Med. Assoc, **271** (1994) 1114–1120.

[34] L. W. Wang, T. T. Tran and N. T. Nguyen, An empirical study of hybrid DEA and grey system theory on analyzing performance: a case from Indian mining industry, Journal of Applied Mathematics, **2015**(**1**) (2015) 395360.

[35] M.Bagheri, A., Ebrahimnejad, S. Razavyan, F. Hosseinzadeh Lotfi and N. Malekmohammadi, Fuzzy arithmetic DEA approach for fuzzy multi-objective transportation problem, Operational Research, (2022) 1-31.

[36] M. D. Basson and, T. Butler, Evaluation of operating room suite efficiency in the Veterans Health Administration system by using data-envelopment analysis, The American journal of surgery, **192**(**5**) (2006) 649-656.

[37] M.H. Kamfiroozi, A. Aliahmadi and M. Jafari Eskandari, Application of Three Parameter Interval Grey Numbers in Enterprise Resource Planning Selection, International Journal of Information Security System Management. (2012) 72-77.

[38] M. S. Bazaraa, H. D. Sherali, C. M. Shetty, Nonlinear programming: theory and algorithms, John Wiley & sons, (2013).

[39] M., Toloo, E. Keshavarz and A. Hatami-Marbini, An interval efficiency analysis with dual-role factors, OR Spectrum, **43** (2021) 255-287.

[40] M. Zhang, H. Guo, M. Sun, S. Liu and J. Forrest, A novel flexible grey multivariable model and its application in forecasting energy consumption in China, Energy, **239** (2022) 122441.

[41] M. Zadmirzaei, F. Hasanzadeh, A. Susaeta and E. Gutiérrez, A novel integrated fuzzy DEA–artificial intelligence approach for assessing environmental efficiency and predicting CO2 emissions. Soft Computing, **28**(**1**) (2024) 565-591.

[42] P. Marschall and S. Flessa, Efficiency of primary care in rural Burkina Faso. A two-stage DEA analysis, Health economics review, **1** (2011) 1-15.

[43] P. Li and C. Wei, A new two-stage grey evaluation decision-making method for interval grey numbers, Kybernetes, **47**(**4**) (2018) 801-815.

[44] R. K. Shiraz, H., Fukuyama and J. Vakili, A note on, some new ranking criteria in data envelopment analysis under uncertain environment, Computers & Industrial Engineering, **131** (2019) 259-262.

[45] R. T. Wang, C. T. B. Ho and K. Oh, Measuring production and marketing efficiency using grey relation analysis and data envelopment analysis, International Journal of Production Research, **48**(**1**) (2010) 183-199.

[46] S. Ang, R., Zheng, F. Wei and F. Yang, A modified DEA-based approach for selecting preferred benchmarks in social networks, Journal of the Operational Research Society, **72**(**2**) (2021) 342-353.

[47] S. A. Javed, S. Liu, A. Mahmoudi and M. Nawaz, Patients' satisfaction and public and private sectors' health care service quality in Pakistan: application of grey decision analysis approaches, the International Journal of Health Planning and Management, **34**(**1**) (2019) 168-182.

[48] S. D. Guo, S. Liu and Z. Fang, Algorithm rules of interval grey numbers based on different "kernel" and the degree of greyness of grey numbers, Grey Systems: Theory and Application, **7**(**2**) (2017) 168-178.

[49] S. Kohl, J., Schoenfelder, A. Fügener and J. O. Brunner, The use of Data Envelopment Analysis (DEA) in healthcare with a focus on hospitals, Health care management science, **22** (2019) 245-286.

[50] S. Liu and Y. Lin, Grey information: theory and practical applications, Springer Science & Business Media, (2006).

[51] S. Mehrabian, G. R. Jahanshahloo, M. R. Alirezaee, G. R. Amin, An assurance interval for the non-Archimedean epsilon in DEA models, Operations Research, **48**(**2**) (2000) 344-347.

[52] S.H. Nasseri, A. Yazdani, D. Darvishi salikolaei, A primal simplex algorithm for solving linear programming problem with grey cost coefficients, Journal of New Researches in Mathematics, **1**(**4**) (2016) 115-135.

[53] T. R. Nunamaker, Measuring routine nursing service efficiency: a comparison of cost per patient day and data envelopment analysis models, Health services research **18**(**2**) Pt 1 (1983) 183.

[54] T. T. Tran, N. T. Nguyen, Applying Data Envelopment Analysis and Grey Model for the Productivity Evaluation of Vietnamese Tourism. Industrial Engineering & Management Systems, **22**(**3**) (2023) 298-312.

[55] U. R. Tuzkaya and E. Yolver, R&D Project selection by integrated gray analytic network process and gray relational analysis: An implementation for home appliances company, Aeronaut Space Technol. **8**(**2**) (2015) 35-41.

[56] X. Li and L. Li, One ranking method of interval grey number based on generalized greyness, Grey Systems: Theory and Application, **13**(**2**) (2023) 340-356.

[57] X. Li, X. Liao, X. Tan and H. Wang, Using grey relational analysis to evaluate resource configuration and service ability for hospital on public private partnership model in China, Grey Systems: Theory and Application, **49**(**2**) (2014) 260-272.

[58] X. Peng, X. Tang, Y. Chen and J. Zhang, Ranking the healthcare resource factors for public satisfaction with health system in China-based on the grey relational analysis models, International Journal of Environmental Research and Public Health, **18**(**3**) (2021) 995.

[59] Y.S. Yang, Data envelopment analysis (DEA) model with interval gray numbers, International journal of information and management sciences, **9**(**4**) (1998)11-23.