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Adomian Decomposition Method and Padé Approximation to Determine fin Efficiency of Convective Straight Fins in Solar Air Collector

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Abstract. In this paper, the nonlinear differential equation for the convection of the temperature distribution of a straight fin with the thermal conductivity depends on the temperature is solved using Adomian Decomposition Method and Pad approximation (PADM) for boundary problems. Actual results are then compared with results obtained previously using digital solution by RungeKutta method and a differential transformation method (DTM) in order to verify the accuracy of the proposed method.

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1. Introduction

Fins are employed in solar air collector to enhance the heat transfer between the plate absorber. The nonlinear fin problem has received a significant attention in

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recent years in view of its practical applications in semiconductors, heat exchangers, solar thermal collector ,power generators and electronic components[4]. Several researchers have applied well-known numerical techniques to solve the nonlinear fin equation. Some of the pertinent research works include. A.Joneidi [1] used Differential Transformation Method to determine fin efficiency of convective straight fins with temperature dependent thermal conductivity, and he compared to exact solution and fourth order Runge-Kutta numerical solution. and showed that the method provides high accuracy DTM to solve the problems of heat transfer in engineering. Jun-Sheng Duan [2] treated a temperature distribution parameterized of a convective straight fin with temperature-dependent thermal conductivity has been obtained by using a new modified decomposition method MDM.He Also we express the efficiency of the straight fin as a function of the two fin parameters. These results greatly facilitate the parameter analysis for the heat transfer model. As a comparison we also investigate the method of undetermined coefficients in the ADM, by which we find it is difficult to obtain the temperature distribution with any of the parameters β and Ψ . Safa Bozkurt Coskun [3] used variation iteration method VIM to analyze the efficiency of convective straight fins with temperature dependent thermal conductivity and he compared to Adomian decomposition method ADM and the results from finite element analysis, he concluded that, VIM is an advantageous method when compared to ADM in view of formulation and solution process and the results obtained from both methods and variation iteration method VIM gives good results with reasonable nonlinearity in the governing equation. However, with a highly nonlinear equation, a high-order expression may be needed, D.B. Kulkarni,[4] proposed a digital technique based on the minimization of residue to solve the same physical problem. The near-exact solution obtained thus is used to calculate the effectiveness of aileron. In the case of constant thermal conductivity, the results obtained are validated with analytical solutions, while in the case of variable thermal conductivity; obtained results are corroborated with those previously published in the literature. An excellent agreement in each case consolidates the proposed numerical technique.

2. Problem description

Consider a straight fin with a temperature-dependent thermal conductivity k (T), arbitrary constant cross-sectional area S, perimeter P and length L. The fin is attached to a base surface of uniform temperature Tb and its tip is insulated. Under steady-state conditions, the face of fin is exposed to a convective environment, where the temperature Ta and the heat transfer coefficient h are assumed to be uniform. That we shows in Figure 1 .The one-dimensional energy balance equation is given [1,2]

$$s\frac{d}{dx}\left[K\left(T\right)\frac{dt}{dx}\right]-Ph\left(T-T_a\right)=0 \tag{1}$$

Where T is the temperature distribution on the fin and the thermal conductivity of the fin material is assumed to be a linear function of temperature according to

$$K\left(T\right)=K_a\left[1+\lambda(T-T_a)\right] \tag{2}$$

And where k_a is the thermal conductivity at the ambient temperature and λ is

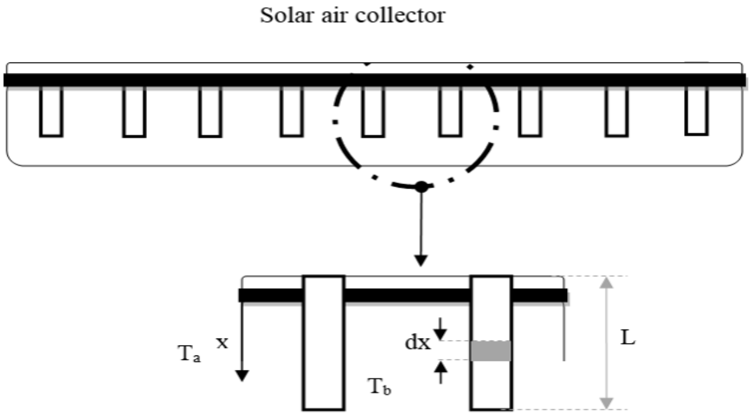


Figure 1. Geometry of a straight fin in solar air collector.

the parameter describing the variation of the thermal conductivity. Employing the following dimensionless variables and parameters.

$$\begin{array}{l} \theta = \frac{T-T_a}{T_b-T_a}, \xi = \frac{X}{l}, \beta = \lambda\left(T_b - T_a\right) \\ \Psi^2 = \frac{hPL^2}{k_a S} \theta = \frac{T-T_a}{T_b-T_a} \\ \xi = \frac{X}{l}, = \left(T_b - T_a\right), \Psi^2 = \frac{hPL^2}{k_a S} \end{array} \tag{3}$$

Equation 4 is the governing nonlinear differential equation for the temperature distribution along the length of the fin and the nonlinearity is due to the term β .For β=0 (case of constant thermal conductivity), Equation 4 reduces to a linear differential equation, for which an analytical solution is available. With reference to the nonlinearity in the governing differential equation, it is difficult to arrive at an exact solution to the temperature distribution and researchers often resort to various numerical techniques in order to arrive at approximate solutions.

$$\frac{d^2\theta}{d\xi^2} + \beta\theta\frac{d^2\theta}{d\xi^2} + \beta\left(\frac{d\theta}{d\xi}\right)^2 - \Psi^2\theta \tag{4}$$

Subject to the boundary conditions

$$\frac{d\theta}{d\xi} = 0, xi = 0 \tag{5}$$

$$\theta = 1, xi = 1 \tag{6}$$

3. Fin Efficiency

The efficiency of convective fins is defined as the ratio of actual heat transfer to the maximum heat transfer which occurs if the base temperature Tb is maintained throughout the length of the fin. In such an ideal case, the maximum temperature difference can be realized along the entire length of the fin. A highly efficient fin would therefore be; smaller in length, has a high value of thermal conductivity and operates in a small value of convective heat transfer coefficient. All such Parameters help in maintaining the temperature along the fin as close to the base temperature as possible. Mathematically the efficiency is [1-4].

$$\eta = \frac{Q}{Q_{ideal}} = \frac{\int_0^l P(T-T_a)dx}{Pb(T_b-T_a)} = \int\limits_{\xi=0}^{1} \theta\left(\xi\right) d\xi \tag{7}$$

We consider the ranges of the two dimensionless fin parameters:

$$0 < \Psi \leq 1.5 and 0 \leq \beta \leq 1$$

4. Fundamentals of Adomian Decomposition Method

Consider the differential equation

$$Lu + Ru + Nu = g\left(t\right) \tag{8}$$

Where: N is a nonlinear operator, L is the highest ordered derivative and R represents the remainder of linear operator L.
By considering as an n-fold integration for an nth order of L, the principles of method consists on applying the operator L^{-1} to the expression (6). Indeed, we

$$L^{-1}Lu = L^{-1}g - L^{-1}Ru - L^{-1}Nu \tag{9}$$

The solution of Eq.(9) is given

$$u = \varphi + L^{-1}g - L^{-1}Ru - L^{1}Nu \tag{10}$$

Where φ is determined from the boundary or initial conditions. For the standard Adomian decomposition method, the solution u can be determined as the infinite series with the components given by:

$$u = \sum_{n=0}^{\infty} u_n \tag{11}$$

And the nonlinear term Nu is given as following:

$$Nu = \sum_{n=0}^{+\infty} A_n\left(u_0, u_1, \ldots, u_n\right) \tag{12}$$

Where $A_n's$, called Adomian polynomials has been introduced by George

$$A_n(u_0, u_1, ..., u_n) = 1/n! \left[d^n/(d\lambda^n)\left[N\left(\sum_{n=0}^{\infty}\lambda^i u_i\right)\right]\right]_{(\lambda=0)} , n = 0, 1, 2, ..., n \tag{13}$$

By substituting the given series (11), (12) into both sides of (13), we obtain the following expressions:

$$\sum_{n=0}^{\infty} u_n = \varphi + L^{-1}g - L^{-1}R\sum_{n=0}^{\infty} u_n - L^{-1}\sum_{n=0}^{+\infty} A_n \tag{14}$$

According to Eq. (14), the recursive expression which defines the ADM components u_n is given as:

$$u_0 = \varphi + L^{-1}g, u_{n+1} = -L^{-1}\left(Ru_n + A_n\right) \tag{15}$$

Finally after some iterations, the solution of the studied equation can be given s an infinite series by :

$$u = u_0 + u_1 + u_2 + u_3 + \ldots + u_n \tag{16}$$

5. Application of ADM-Pad to the Nonlinear Problem

Considering the Eq. (9), Eq. (4) can be written as:

$$L\theta = -\beta\theta\theta'' - \beta\theta'^2 + \varphi^2\theta \tag{17}$$

Where the differential operator L given by $L = \frac{d^2}{d\eta^2}$ The inverse of operator L is expressed by L^{-1} and can be represented as:

$$L^{-1} = \int_0^{\eta}\int_0^{\eta} \bullet d\eta d\eta \tag{18}$$

The application of Eq. (10) on Eq. (4) and considering the boundary conditions (5,6), we obtain:

$$\theta(\eta) = \theta(0) + \theta^{'}(0)\eta + L^{-1}(Nu) \tag{19}$$

Where:

$$Nu = -\beta\theta\theta'' - \beta\theta'^2 + \varphi^2\theta \tag{20}$$

The values of(0) and $\theta'(0)$ depend on boundary condition. The boundary conditions are expressed as following:

$$\theta^{'}(0) = 0 \tag{21}$$

$$\theta\left(0\right) = c \tag{22}$$

By applying the boundary conditions (5) and considering$\theta\left(0\right)=c$, we obtain:

$$\theta(\eta)=\sum_{n=0}^{\infty}\theta_0=\theta_0+L^{-1}\left(Nu\right) \tag{23}$$

Where:$\theta_0=c$ The terms of Adomian polynomials, are defined by applying Adomian decomposition method as following:

$$A_0=c\phi^2 \tag{24}$$

$$A_1=-\beta c^2\phi^2+\frac{1}{2}cx^2\phi^4-\frac{3}{2}\beta c^2x^2\phi^4 \tag{25}$$

$$\begin{aligned}A_2&=\beta^2c^3\phi^2-\tfrac{1}{2}\beta c^2\phi^2\eta^2\phi^2-\tfrac{1}{2}\beta c^2\eta^2\phi^4\\&+\tfrac{9}{2}\beta^2c^3\eta^2\phi^4\\&-\tfrac{3}{2}\beta^3c^4\eta^2\phi^4+\tfrac{1}{24}c\phi^2\eta^4\phi^4-\tfrac{1}{8}\beta c^2\phi^2\eta^4\phi^4\\&-\tfrac{5}{8}\beta c^2\eta^4\phi^6\\&+\tfrac{5}{2}B^2c^3\eta^4\phi^6-\tfrac{15}{8}\beta^3c^4\eta^4\phi^6-\tfrac{7}{144}\beta c^2\eta^6\phi^8\\&+\tfrac{7}{24}\beta^2c^3\eta^6\phi^8\\&-\tfrac{7}{16}\beta^3c^4\eta^6\phi^8\end{aligned} \tag{26}$$

By using algorithm (15), the first few components of the solution are:

$$\theta_1=\frac{1}{2}c\eta^2\phi^2 \tag{27}$$

$$\theta_2\ =-\frac{1}{2}\beta c^2\eta^2\phi^2+\frac{1}{24}c\eta^4\phi^4-\frac{1}{8}\beta c^2\eta^4\phi^4 \tag{28}$$

$$\begin{aligned}\theta_3\ &=\tfrac{1}{2}\beta^2c^3\eta^2\phi^2-\tfrac{1}{24}\beta c^2\phi^2\eta^4\phi^2-\tfrac{1}{24}\beta c^2\eta^4\phi^4+\tfrac{3}{8}\beta^2c^3\eta^4\phi^4-\\&\tfrac{1}{8}\beta^3c^4\eta^4\phi^4+\tfrac{1}{720}c\phi^2\eta^6\phi^4-\tfrac{1}{240}\beta c^2\phi^2\eta^6\phi^4-\tfrac{1}{48}\beta c^2\eta^6\phi^6+\tfrac{1}{12}\beta^2c^3\eta^6\phi^6\\&-\tfrac{1}{16}\beta^3c^4\eta^6\phi^6-\tfrac{\beta c^2x^8\phi^8}{1152}+\tfrac{1}{192}\beta^2c^3\eta^8\phi^8-\tfrac{1}{128}\beta^3c^4\eta^8\phi^8\end{aligned} \tag{29}$$

Finally, the solution for convergent channel is given by Adomian decomposition method as:

$$\theta\left(\eta\right)=\theta_0+\theta_1+\theta_2+....+\theta_n=c+\frac{1}{2}c\eta^2\phi^2-\frac{1}{2}\beta c^2\eta^2\phi^2+\frac{1}{24}c\eta^4\phi^4-\frac{1}{8}\beta c^2\eta^4\phi^4\ \ +\ldots \tag{30}$$

The value of constant c is obtained by solving Eq. (30) using Eq. (7).

We apply Laplace transformation to $\theta\left(\eta\right)=\theta_0+\theta_1+\theta_2+\theta_3$, which yields

$$-\frac{35\beta c^2\phi^8}{s^9}+\frac{210\beta^2c^3\phi^8}{s^9}-\frac{315\beta^3c^4\phi^8}{s^9}+\frac{c\phi^6}{s^7}-\frac{18\beta c^2\phi^6}{s^7}+\frac{60\beta^2c^3\phi^6}{s^7}-\frac{45\beta^3c^4\phi^6}{s^7}+$$
$$\frac{c\phi^4}{s^5}-\frac{5\beta c^2\phi^4}{s^5}+\frac{9\beta^2c^3\phi^4}{s^5}-\frac{3\beta^3c^4\phi^4}{s^5}+\frac{c\phi^2}{s^3}-\frac{\beta c^2\phi^2}{s^3}+\frac{\beta^2c^3\phi^2}{s^3}+\frac{c}{s}$$

For the sake of simplicity, let$s=\frac{1}{\eta}$, then

$$-35\beta c^2\phi^8\eta^9+210\beta^2c^3\phi^8\eta^9-315\beta^3c^4\phi^8\eta^9+c\phi^6\eta^7-18\beta c^2\phi^6\eta^7+$$
$$60\beta^2c^3\phi^6\eta^7-45\beta^3c^4\phi^6\eta^7+c\phi^4\eta^5-5\beta c^2\phi^4\eta^5+9\beta^2c^3\phi^4\eta^5-$$
$$3\beta^3c^4\phi^4\eta^5+c\phi^2\eta^3-\beta c^2\phi^2\eta^3+\beta^2c^3\phi^2\eta^3+\mathrm{c}\ \eta$$

Notice that, we can get to the value of c by using the shooting method to solve the boundary value problem (4) together the boundary conditions (5),(c =0.9213147484091304‘, $\beta=0.5$ and$\phi=0.5$).Then [5/5] Padé approximate of $L[\theta(\eta)]$ yields:

$$\frac{5}{5}=\frac{0.00250221-2.50891\,\eta+2.01442\,\eta^2+0.164675\,\eta^3+0.571892\eta^4-0.20996\eta^5}{1-1.20\,(-0.92+\eta)+0.79\,(-0.9+\eta)^2-0.14\,(-0.92+\eta)^3-0.16\,(-0.92+\eta)^4+0.136476\,(-0.92+\eta)^5}$$

Recalling$=\frac{1}{s}$, we obtain [5/5] in terms of s

$$\frac{5}{5}=\frac{0.00250221-\frac{2.50891}{s}+\frac{2.01442}{s^2}+\frac{0.164675}{s^3}+\frac{0.571892}{s^4}-\frac{0.20996}{s^5}}{1-1.20\left(-0.92+\frac{1}{s}\right)+0.79\left(-0.9+\frac{1}{s}\right)^2-0.14\left(-0.92+\frac{1}{s}\right)^3-0.16\left(-0.92+\frac{1}{s}\right)^4+0.13\left(-0.92+\frac{1}{s}\right)^5}$$

By using inverse Laplace transformation to[5/5], we obtain the solution $\dot{\theta}\,(\eta)$by using the after treatment techniquewhich improves the accuracy of the ADM

$$\dot{\theta}\,(\eta)=(-0.000707271+0.\ \mathrm{i})+0.274563\ \mathrm{e}^{-0.867664\,\eta}-(0.0721158$$
$$+0.0803442\ \mathrm{ie}^{(0.393851\ -0.382903\ \mathrm{i})\eta}-(0.0721158\ -0.0803442\ \mathrm{i})\,\mathrm{e}^{(0.393851\ +0.382903\ \mathrm{i})\eta}$$
$$+(0.400408\ +0.0736334\ \mathrm{i})\,\mathrm{e}^{(0.420784\ +0.128103\ \mathrm{i})\eta}$$
$$+(0.400408\ -0.0736334\ \mathrm{i})\ \mathrm{e}^{(0.420784\ -0.128103\mathrm{i})\eta}$$

6. Results

The results of comparison between Adomian Decomposition Method and Pad approximation PADM solutions with a numerical results is presented in table 1 the case thermal conductivity is constant ($\beta=0$ and $\Psi=0.55$) the results of this analysis are gathered against the obtained by the numerical solution by Runge-Kutta fourth order method . a very good agreement between the results is also observed, which confirms the excellent validity of the PADM.
To better visualize the effectiveness of the analytical technique used, we present In the table 2 the comparison Adomian Decomposition Method and Pad approximation PADM solutions with a numerical solution by Runge-Kutta fourth order method and also comparing the Differential Transformation Method DTM with a

Table 1. Comparison between PADM and Numerical Results for temperature Distribution when (β=0 and Ψ = 0.55)

	Numerical(Nu)	PADM	ERROR
0	0.8657248454	0.865724851	5.6×10^{-9}
0.05	0.8660522223	0.8660522241	1.8×10^{-9}
0.1	0.8670345910	0.8670345902	8.0×10^{-9}
0.15	0.8686726946	0.8686726926	2.0×10^{-9}
0.2	0.8709677708	0.8709677700	8.0×10^{-9}
0.25	0.8739215557	0.8739215584	2.7×10^{-9}
0.3	0.8775362886	0.8775362916	3.0×10^{-9}
0.35	0.8818147002	0.8818147034	3.2×10^{-9}
0.40	0.886760026	0.8867600296	3.6×10^{-9}
0.45	0.892376007	0.8923760104	3.4×10^{-9}
0.50	0.8986668901	0.898666893	2.9×10^{-9}
0.55	0.9056374321	0.9056374354	3.3×10^{-9}
0.60	0.9132929055	0.9132929092	3.7×10^{-9}
0.65	0.9216391005	0.921639104	3.5×10^{-9}
0.70	0.9306823288	0.9306823328	4.0×10^{-9}
0.75	0.940429430	0.940429434	4.0×10^{-9}
0.80	0.9508877761	0.9508877803	4.2×10^{-9}
0.85	0.9620652764	0.9620652806	4.2×10^{-9}
0.90	0.9739703841	0.9739703885	4.4×10^{-9}
0.95	0.9866121031	0.9866121080	4.9×10^{-9}
1.00	0.9999999948	0.9999999999	5.1×10^{-9}

Table 2. Values of temperature Distribution by DTM and PADM Method (ψ=0.5 ,β=0.2)

$\beta = 0.2$ and $\Psi = 0.5$				
Numerical (NU)	**DTM**	**(PADM)**	**DTM-NU**	**PADM-NU**
0.9034471757	0.9034471796	0.9034471644	3.9×10^{-9}	1.1×10^{-8}
0.9036863048	0.9036863060	0.9036862908	1.2×10^{-9}	1.4×10^{-8}
0.9044037548	0.9044037536	0.9044037386	1.2×10^{-9}	1.6×10^{-9}
0.9055997311	0.9055997276	0.9055997129	3.5×10^{-9}	1.8×10^{-8}
0.9072745754	0.9072745703	0.9072745559	5.1×10^{-9}	1.9×10^{-8}
0.9094287648	0.9094287601	0.9094287460	4.7×10^{-9}	1.8×10^{-8}
0.9120629141	0.9120629117	0.9120628982	2.4×10^{-9}	1.5×10^{-8}
0.9151777745	0.9151777756	0.9151777626	1.1×10^{-9}	1.1×10^{-8}
0.9187742359	0.9187742374	0.9187742251	1.5×10^{-9}	1.0×10^{-8}
0.9228533156	0.9228533176	0.9228533059	2.0×10^{-9}	9.0×10^{-9}
0.9274161677	0.9274161700	0.9274161593	2.3×10^{-9}	8.4×10^{-9}
0.9324640796	0.9324640824	0.9324640726	2.8×10^{-9}	7.0×10^{-9}
0.9379984709	0.9379984741	0.9379984652	3.2×10^{-9}	5.7×10^{-9}
0.9440208923	0.9440208963	0.9440208882	4.0×10^{-9}	4.1×10^{-9}
0.9505330252	0.9505330293	0.9505330226	4.1×10^{-9}	2.6×10^{-9}
0.9575366796	0.9575366840	0.9575366783	4.4×10^{-9}	1.3×10^{-9}
0.9650337932	0.9650337981	0.9650337933	4.9×10^{-9}	1.0×10^{-10}
0.9730264303	0.9730264355	0.9730264320	5.2×10^{-9}	1.7×10^{-9}
0.9815167805	0.9815167860	0.9815167838	5.5×10^{-9}	3.3×10^{-9}
0.9905071567	0.9905071626	0.9905071614	5.9×10^{-9}	4.7×10^{-9}
0.9999999937	0.9999999996	1.0000000000	5.9×10^{-9}	6.3×10^{-9}

numerical solution; in this case the thermal conductivity ($\beta = 0.2$ and $\Psi = 0.5$), It should be noted that the final results from PADM and DTM are in good agreement with the Runge-Kutta-Fehlberg method which is a well-tested numerical solution.

Figure 2 Illustrate the effect of parameters thermo-geometrics of fin for the temperature variation when the thermal conductivity is constant ($\beta = 0$).This figure display that increasing in the values of thermo-geometric fin parameter produce decrease in values of dimensionless temperature.

Table 3. Constant temperature $\theta(0)$ for Different Values of Ψ when(β=0 and β=0.5) (Analytical and numerical results)

	$\beta = 0$		$\beta = 0.5$	
Ψ	$\theta_{\mathrm{Numerical}}(0)$	$\theta_{\mathrm{PADM}}(0)$	$\theta_{\mathrm{Numerical}}(0)$	$\theta_{\mathrm{PADM}}(0)$
0	1.0000000000	1.000000000000	1.000000000000	1.000000000000
1	0.6480542598	0.648054273663	0.729675735735	0.729675885432
2	0.2658022228	0.265802228834	0.338845479529	0.338845387545
3	0.0993279206	0.099327927433	0.132792821432	0.132790991254
4	0.0366189869	0.036618994101	0.049747705386	0.049747683898
5	0.0134752797	0.013475289742	0.018408630316	0.018407079100
6	0.0049574730	0.004957513903	0.006785909916	0.006786372098

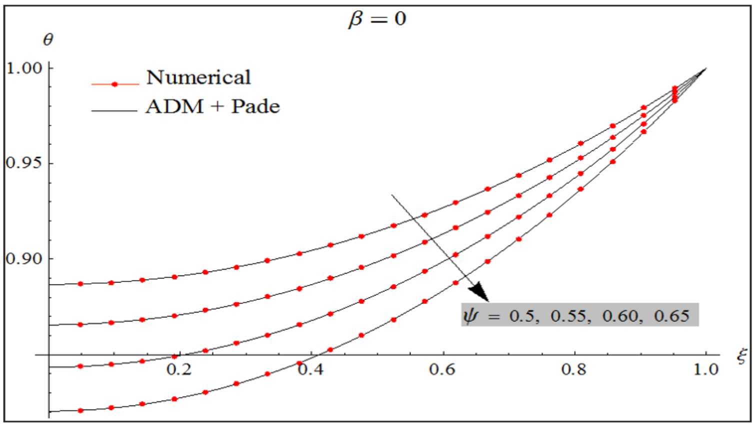
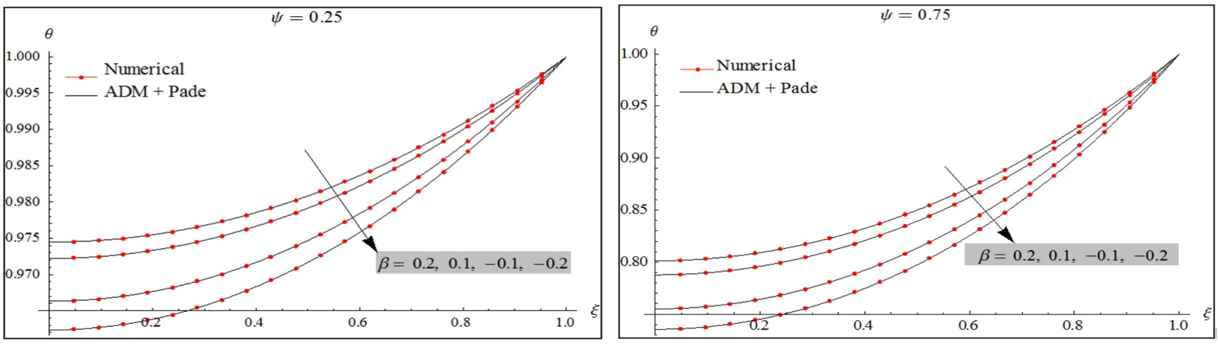


Figure 2. Temperature distribution in convective fins thermo-geometric fin parameter (ψ) when β=0 With PADM method and Ns



(a) (b)

Figure 3. Temperature distribution in convective fins with variable thermal conductivity with PADM method and Ns

Figure 3 shows the temperature distributions along the dimension of the surface coil with β ranging from 0.2 to -0.2 are shown in Figure 3(a),3(b) for different the values of parameters thermo-geometrics ψ =0.75 and ψ = 0.25, respectively. If the thermal conductivity of material increases with the fin temperature, the values of fin temperature in the axis (ξ =0) with parameters thermo-geometrics (ψ =0-6) and thermal conductivity β (0, 0.5) is shown in table 3.

Figure 4 shows the behavior of the fin efficiency relative to ψ and β. it's clearly that the fin efficiency increases as the thermal conductivity parameter,βincreases. To explain the effect of parameter β, we note that while the temperature increases as βincreases,the PADM results were checked against NS which were in excellent agreement with each other.

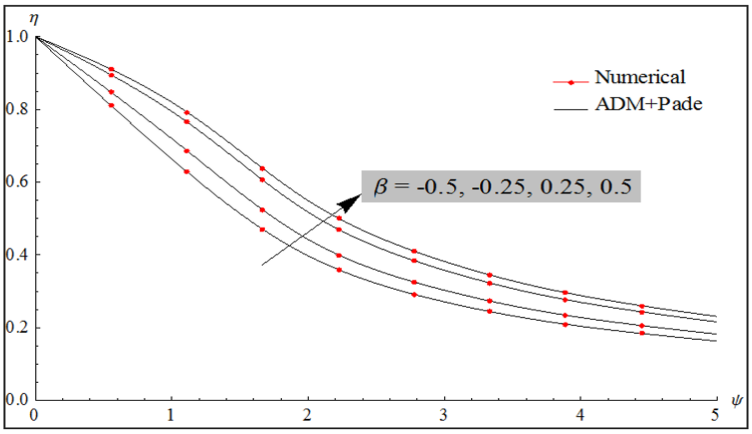


Figure 4. Variation of the fin efficiency with variable thermal conductivity with PADM method and Ns

The Results obtained shown in the figures above for the analytical solution proposed the method Adomian Decomposition and Pad approximation PADM compared with the numerical solution of Runge Kutta method , an excellent agreement in each case, in order to verify the accuracy of analytical technique adopted efficiency, a comparison with other studies is presented in Table 2 ,the results show that our results agree well with the results of the literature, which justifies the applicability, the efficiency and greater precision analytical technique PADM adopted other analytical methods used in articles published previously as the differential transformation method (DTM) [1],a good agreement with the results of our proposed methods(PADM).

7. Conclusion

In this study ,The PADM have been utilized to solve a nonlinear differential equation for the convection of the temperature distribution of a straight fin with the thermal conductivity depends on the temperature is solved for boundary problems. Actual results are then compared with results obtained previously using digital solution by RungeKutta method and articles published previously as the differential transformation method DTM in order to verify the accuracy of the proposed method. Figures and tables clearly show that these methods have good approximations to the solution of this nonlinear equation with high accuracy. After the audit, we analyze the effects of some physical parameters for this problem, such as thermo-setting end geometric and setting the thermal conductivity, the thermal conductivity of fin increase with increasing of average temperature rises.

8. Nomenclature

Cp	specific heat J $Kg^{-1}k^{-1}$
h	Convective heat transfer coefficient W. $M^{-2}\cdot k^{-1}$
K_a	Conductivity W. $M^{-1}\cdot k^{-1}$
l	Fin length M
q	heat flux W.M^{-2}
T	Temperature K
Ta	ambient temperature K
S	Surface M^2
η	Fin efficiency
ξ	dimensionless coordinate
Ψ	thermo-geometricfin parameter
β	dimensionless parameter describing variation of the thermal conductivity
θ	dimensionless temperature
b	base

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