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Limited Growth Prey Model and Predator Model Using Harvesting

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Abstract. In this paper, we have proposed a study on controllability and optimal harvesting of a prey predator model and mathematical non linear formation of the equation equilibrium point of Routh harvest stability analysis. The problem of determining the optimal harvest policy is solved by invoking Pontryagin′s maximum principle dynamic optimization of the harvest policy is studied by taking the combined harvest effect as a dynamics variable.

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1. Introduction

The first continuous time predator prey model was the Lotka Volterra model [8], It was proposed in the 1920 by Volterra. Discussed in the model Rosenzweig MacArthur model using formulation of predator prey interaction. Using the introduction to mathematical modeling and chaotic Dynamics, by Ranjit kumar Upadhyay et. al., [12]. In many resilience and stability of harvested predator prey

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system using harvested effort model formulated by [6]. Their approach controllability and optimal harvesting of the prey predator model in the method of optimal harvesting policy for combined harvesting for both predator and prey models have also been discussed in[3, 15]. In many work have been Holling type II function formulated the equation [9, 16]. Theory of the model origins and evaluation of the predator prey by [1]. Some concepts has been studied a predator prey system with arranging the mathematical model based in [2, 7]. In the present paper, we consider a predator prey model, assume optimal harvesting problem using economic dynamics phase diagrams and their economic application by Ronald shone[13]. Using the book mathematical theory of optimal processes by Pontryagin .L.S et. al.,[10]. Pontryagin′s principle problem suggested that optimal harvesting effort problem formatted the maximum value of continuous and discrete type model. Using the optimal processes by Clark.C.W. The equilibrium point and stability analysis calculated by [4, 13]. A simple mathematical model of non linear dynamics [5]. The model assumption and functional responses will be calculated by [3, 4, 11, 14].

2. The Mathematical Model

- In the Rosenzweig MacArthur model using formulation of predator model interaction are as follows, the life histories of each population involve continuous growth and overlapping generations. The predator p dies out exponentially in the absence of its most favorite prey q, the predators feeding rate saturates at high prey densities the age saturates of both the populations are ignored. It is assumed that the per capita growth rate of prey in the absence of predation defined by the function f grow. Logistically with the intrinsic growth rate r per capita rate of self reproduction on for the prey.
- The limited growth model using populations cannot continue growing exponentially over time due to limited resources or competition for food with other species where $f(p) = (r - \xi p)$ by $p = k = \frac{r}{\xi}$ carrying capacity.
- The functional response of the prey is taken as $A_1(p) = \frac{upq}{(p+w)}$and Holling type functional response.
- The functional response of the predator is taken as $A_2(q) = \frac{vpq}{(p+w_1)}$
- The predator population using harvesting. Let E represent the effort used for harvesting. Assume that catch per unit effort is proportion to the density of the stock level q. Where γ is called the catch ability coefficient and $E\gamma$ is called the mortality rate.

$$\frac{dp}{dt} = (r - \xi p)p - \frac{upq}{(p+w)} \tag{1}$$

$$\frac{dq}{dt} = \frac{vpq}{(p+w_1)} - sq - \gamma Eq \tag{2}$$

p : The prey population at time t.
q : The predator population at time t.
$(r - \xi p)$: Limited growth rate model.
r : The maximum per capita growth rate of prey population.
ξ: Measures the intensity of competition among individuals of species p for

spaces,food and so on.
u : Maximum rate of per capita removal of prey species p due to predator by its predator q.
s : Measures how fast the predator q will die when there is no prey to capture, kill and eat.
v : The conversion coefficient form individuals of prey into individuals of predator.
γ : The constant predation rate by the predator population.
E : The constant harvesting effort
w : value of population density of p at which per capita removal rate is half of its u
w_1 : population density of the prey at which per capita gain per unit time in q is half of its maximum value of v

3. Boundedness of the Model

All solutions of model equation(1,2) are uniformly bounded.
Proof:
Let $p(t), q(t)$ be any solution of model(1,2)
Now,consider the equation $A = vp + uq$,we get differential with respect t,

$$\frac{dA}{dt} = v\frac{dp}{dt} + u\frac{dq}{dt}$$

$$\frac{dA}{dt} = v(r - \xi p)p - \frac{upq}{(p+w)} + u\frac{vpq}{(p+w_1)} - sq - \gamma EqP$$

$$\frac{dA}{dt} = vr - \xi p^2 - \frac{vupq}{(p+w)} + \frac{uvpq}{(p+w_1)} - usq - u\gamma EqP \text{and Let} w = w_1$$

$$\frac{dA}{dt} \leqslant vr - \xi p^2 - 0 - usq - u\gamma Eq$$

$$\leqslant vp(r - \xi p) - qu(s + \gamma E) + 0, \text{where } \rho = min\{r - \xi p, s + \gamma E\}$$

$$\leqslant 0 + \rho A,$$

$$\frac{dA}{dt} + \rho A \leqslant 0$$

Applying a theorem on differential inequalities,we obtain
$0 \leqslant A(p, q) \leqslant 0 + A(p(0), q(0))e^{-t\rho}$
and for $t \to \infty$
$0 \leqslant A(p, q) \leqslant \epsilon$
Thus all solutions of (1,2)enter into the region
$B = \{(p, q) : 0 \leqslant A \leqslant \epsilon, \text{for any } \epsilon\}$

4. Equilibrium Point

The first critical point (0,0)
The second critical point $\left\{p = \frac{r}{\xi}, q = 0\right\}$
The third critical point$\{p^* = -\frac{w_1\ (E\gamma + s)}{E\gamma + s - v}, q^* = \frac{1}{u}[(r - \xi p^*)(p^* + w)]\}$
The jacobian matrix

$$\begin{bmatrix} -p\xi + r - \dfrac{uq}{p+w} + p\left(-\xi + \dfrac{uq}{(p+w)^2}\right) & -\dfrac{pu}{p+w} \\ q\left(\dfrac{v}{p+w_1} - \dfrac{vp}{(p+w_1)^2}\right) & \dfrac{vp}{p+w_1} - E\gamma - s \end{bmatrix}$$

5. Nature of the Equilibrium and Stability Analysis

In this section, we shall discuss the stability properties of the equilibrium J_0, J_1, J_2. The Jacobian of the system about the equilibrium point $J_0(0,0)$ is given by

$$\begin{bmatrix} r & 0 \\ 0 & -E\gamma - s \end{bmatrix}$$

Hence the eigenvalues of this system are $\lambda_1 = r$ and $\lambda_2 = (-E\gamma - s) < 0$
The equilibrium point $J_0(0,0)$ is a unstable point.
Since $Re(\lambda) \neq 0$for both eigenvalues.The fixed point is hyperbolic.
since the eigenvalues are real and are of opposite signs.
Hence $J_0(0,0)$ is a hyperbolic saddle point.
i.e $J_0(0,0)$ is a saddle point.
The axial equilibrium point $\left\{p = \frac{r}{\xi}, q = 0\right\}$exists.
Then $\left\{p = \frac{r}{\xi}, q = 0\right\}$ is asymptotically stable(or) unstable (or)hyperbolic saddle point according as $\frac{vr}{r+\xi w_1} - \gamma E - s + r \gtrless 0$.

Proof:
The Jocobian matrix for $J_1\left\{p = \frac{r}{\xi}, q = 0\right\}$ is given by

$$\begin{bmatrix} -r & -\dfrac{ru}{r+\xi w} \\ 0 & \dfrac{rv}{r+\xi w_1} - \gamma E - s \end{bmatrix}$$

The eigenvalues are$\lambda_1 = -r$ and $\lambda_2 = \frac{vr}{(r + w_1\xi)} - \gamma E - s$
If $\lambda < 0$, That is $\frac{vr}{(r+w_1\xi)} < \gamma E + s$, Then the equilibrium point $J_1\left(\frac{r}{\xi}, 0\right)$ is asymptotically stable.
when $E > \frac{1}{\gamma}\left\{\frac{rv}{r+\xi w_1} - s\right\}$ [or] $(E\gamma + s) > \frac{rv}{r+\xi w_1}$
Otherwise $J_1\left\{p = \frac{r}{\xi}, q = 0\right\}$ is unstable.
If depends on the value of the parameters $v, r, w_1, s, E, \xi, \gamma$.If$\lambda_2 > 0$the eigenvalues are real and are of opposite sings and the fixed point $\left\{p = \frac{r}{\xi}, q = 0\right\}$ is a hyperbolic saddle point if$(E\gamma + s) > \frac{rv}{r+\xi w_1}$
The conversion rate of the predator is sufficient to overcome losses due to death and harvesting.

6. The Interior Equilibrium Point: Stability Analysis

The interior equilibrium point (p^*, q^*) exists.Then(p^*, q^*) is locally asymptotically stable or unstable according as

(1) $(E\xi + s) > \dfrac{\xi w(E\gamma - v + s) + rv}{(2\xi w_1 + r)}$

(2) $(E\xi + s) > \dfrac{rv}{\xi w_1 + r}$

Proof:
The eigenvalues of $J_2(p^*, q^*)$ are the root of $\Lambda(\lambda) = c_2\lambda^2 + c_1\lambda + c_0 = 0$
by the Routh-Hurwitz theorem, the necessary and sufficient conditions for local stability are given by $c_2 > 0, c_1 > 0, c_0 > 0$.
Now,$c_1 > 0$and

$$\frac{w_1\,(E\gamma + s)\,(-E\gamma\, w\xi + 2\,E\gamma\, w_1\,\xi + E\gamma\, r - sw\xi + 2\,sw_1\,\xi + vw\xi + rs - rv)}{(E\gamma\, w - E\gamma\, w_1 + sw - sw_1 - vw)\,(E\gamma + s - v)} > 0$$

gives

$$(E\xi + s)(2\xi w_1 + r) > \xi w(E\gamma - v + s) + rv$$

[or]

$$(E\xi + s) > \frac{\xi w(E\gamma - v + s) + rv}{(2\xi w_1 + r)}$$

Therefore sufficient condition is locally stability.
Now $c_0 > 0$ and $c_0 = -\dfrac{(E\gamma + s)\,(E\gamma\, w_1\,\xi + E\gamma\, r + sw_1\,\xi + rs - rv)}{v} > 0$,
gives $E\xi + s > \dfrac{rv}{\xi w_1 + r}$ is always satisfied.
Otherwise, The equilibrium point (p^*, q^*) is asymptotically stable.
Its clear that (p^*, q^*) is locally asymptotically stable or unstable according as $c_1 > or < 0$.

7. Optimal Harvesting Policy

The optimal plan for harvesting a prey and predator equation.This approach is based on the work of Pontryagian et al(1962),The present valueJ_{max} of a continuous time stream of revenues is given by $J = \int_0^\infty e^{-\delta t}\pi(p, q, E, t)dt$
where $\pi(p, q, E, t) = (p_1\gamma q - C)E$
C: The fixed constant cost per unit of harvesting intensity.
p_1 : There is a fixed constant price per unit biomass of the harvested.
δ : The instantaneous annual rate of discount.
γ : The catch ability coefficient value,$\gamma > 0$.
The control variable E(t) is subject to the constrains $0 \leqslant E(t) \leqslant E_{max}$ where E_{max} stands for a feasible upper limit on the harvesting effort.
The Hamiltonian for the problem is given by

$$H = e^{-t\delta}\,(p_1\gamma q - C)\,E + \left[-p^2\xi + pr - \frac{up}{p + w}\right]\lambda_1 + \left[\frac{qvp}{p + w_1} - E\gamma\, q - sq\right]\lambda_2 \quad (3)$$

where λ_1 andλ_2 are adjoint variables.
The adjoint equations are

$$\frac{d\lambda_1}{dt} = -\frac{\partial H}{\partial p} = -\left[-2\,p\xi + r - \frac{uq}{p+w} + \frac{uqp}{(p+w)^2}\right]\lambda_1 - \left[\frac{qv}{p+w_1} - \frac{qvp}{(p+w_1)^2}\right]\lambda_2$$

$$\frac{d\lambda_1}{dt} = \left[2p\xi - r + \frac{uqw}{(p+w)^2}\right]\lambda_1 - \left[\frac{qvw}{(p+w_1)^2}\right]\lambda_2 \tag{4}$$

$$\frac{d\lambda_2}{dt} = -\frac{\partial H}{\partial q} = -e^{-t\delta}p_1\gamma E - \left[-\frac{up}{(p+w)}\right]\lambda_1 - \left[\frac{vp}{(p+w_1)} - \gamma E - s\right]\lambda_1$$

$$\frac{d\lambda_2}{dt} = -e^{-t\delta}p_1\gamma E + \left[\frac{up}{(p+w)}\right]\lambda_1 + \left[-\frac{vp}{(p+w_1)} + \gamma E + s\right]\lambda_1 \tag{5}$$

From (1)and(2), We first consider a optimal equilibrium solution of the above problem so that we may take simplify the equation

$$p\left(r - \xi p - \frac{uq}{(p+w)}\right) = 0$$
$$r = \xi p + \frac{uq}{(p+w)}$$
$$q\left(\frac{vp}{(P+w_1)} - \gamma E - s\right) = 0$$
$$\gamma E + s = \frac{vp}{(p+w_1)}$$

(4)and (5) become

$$\frac{d\lambda_2}{dt} = -e^{-\delta t}p_1\gamma E + \left[\frac{up}{(p+w)}\right]\lambda_1 + 0$$
$$\frac{d\lambda_1}{dt} = \left[p\xi - \frac{upq}{(p+w)^2}\right]\lambda_1 - \left[\frac{qvw_1}{(p+w_1)^2}\right]\lambda_2$$

differentiate (4) with respect to t,we get

$$\frac{d^2(\lambda_1)}{dt^2} = \left[p\xi - \frac{upq}{(p+w)^2}\right]\frac{d\lambda_1}{dt} - \left[\frac{qvw_1}{(p+w_1)^2}\right]\frac{d\lambda_2}{dt}$$

Given the second order derivative

$$\frac{d^2\lambda_1}{dt} - \left[p\xi - \frac{upq}{(p+w)^2}\right]\frac{d\lambda_1}{dt} - \left[\frac{qvw_1}{(p+w_1)^2}\right]\left[\frac{up}{(p+w)}\right]\lambda_1 = -\frac{qvw_1}{(p+w_1)^2}e^{-\delta t}p_1\gamma E$$

$$\frac{d^2\lambda_1}{dt^2} + A_1\frac{d\lambda_1}{dt} + B_1\lambda_1 = M_1e^{-\delta t} \tag{6}$$

where$M_1 = -\dfrac{qvw_1}{(p+w_1)^2}p_1\gamma E$ and $A_1 = -\left[p\xi - \dfrac{upq}{(p+w)^2}\right] < 0$
$B_1 = -\left[\dfrac{qvw_1}{(p+w_1)^2}\right]\left[\dfrac{up}{(p+w)}\right] > 0$
where

$$N = \mu^2 + A_1\mu + B_1 = 0 \tag{7}$$

andμ_1, μ_2 are the root of the auxiliary equation. Therefore the two roots of equations (6) are either both real and positive or complex conjugates with positive real parts. The discriminant of (7) is$X = A_1^2 - 4B_1$

case:1

In this case, The roots of (7) are real and positive. The roots are given by
$\mu_1 = \frac{-A_1+\sqrt{X}}{2}and\mu_2 = \frac{-A_1-\sqrt{X}}{2}$
The general solution is
$\lambda_1(t) = \zeta_1 e^{\mu_1 t} + \zeta_2 e^{\mu_2 t} + \dfrac{M_1}{N}e^{-\delta t}$
Where $N = \mu^2 + A_1\mu + B_1 \neq 0$ and $\zeta_1, \zeta_2$are arbitrary constants

case:2

In this case, The roots of (7) are complex conjugates with positive real pares.
The root are given by $\mu_1 = \frac{-A_1+i\sqrt{X}}{2}and\mu_2 = \frac{-A_1-i\sqrt{X}}{2}$
The general solution is
$\lambda_1(t) = e^{-A_1 t}\zeta_3 cos(t\sqrt{-X}) + \zeta_4 sin(t\sqrt{-X}) + \dfrac{M_1}{N}e^{-\delta t}$
Where $\zeta_3, \zeta_4$are arbitrary constants
In case 1, The current shadow price $e^{t\delta}\lambda_1$ remains bounded as $t \to \infty$ if and only if $\zeta_1 = \zeta_2 = 0$and then $e^{t\delta}\lambda_1 = (\dfrac{M_1}{N}) =$ constant.
In case 2 , The current shadow price $e^{t\delta}\lambda_1$ remains bounded as $t \to \infty$ if and only if $\zeta_3 = \zeta_4 = 0$and then $e^{t\delta}\lambda_1 = (\dfrac{M_1}{N}) =$ constant.
Then in both the cases we have $e^{t\delta}\lambda_1 = (\dfrac{M_1}{N})$
and similar term yields. $e^{t\delta}\lambda_1 = (\dfrac{M_2}{N})$
Where$M_2 = \{\mu + [\dfrac{uqw}{(p+w)^2} - p\xi]\}p_1\gamma E$. Again the condition that the Hamiltotion H must be a maximum for $E \in V_E = [0, E_{max}]$ gives the condition

$$\frac{\partial H}{\partial E} = e^{-\delta t}(\gamma q p_1 - C)E - \lambda_2 q\gamma = 0 \tag{8}$$

$$\lambda_2 qx = e^{-\delta t}\frac{\partial \pi}{\partial E}$$

This indicates that the total user cost of harvest per unit effort must be equal to the discounted value of the future profit at the steady state effort level.
We must rewrite (8) in the form $\gamma q(p_1 - \dfrac{M_2}{N}) = C$
Given the optimal equilibrium populations $p = p_\delta, q = q_\delta$ and the optimal harvesting effort $E = E_\delta$when$\delta \to \infty$ equation(9) leads to the obvious result $(\gamma q_\infty p_1) = C$.
Therefore$\pi(p, q, E) = 0$. This shows that an infinite discount leads to complete dissipation of economic revenue.

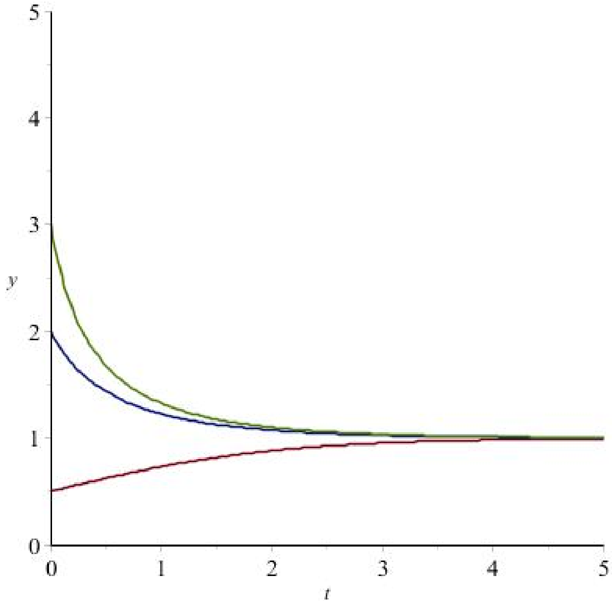

Figure 1. Limited growth rate,x axis is time series t and y axis is prey values let as consider y=p(prey).

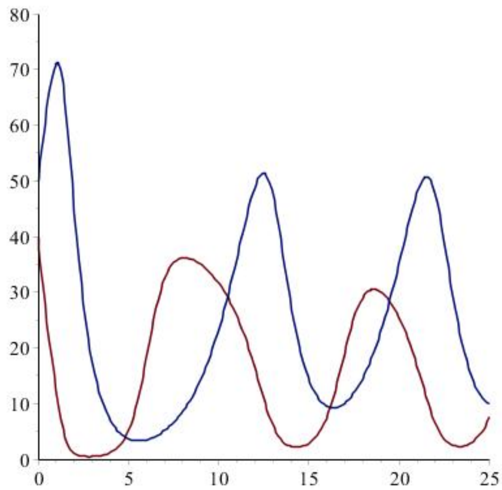

Figure 2. Prey predator interaction

8. Numerical Results

In our numerical experiments we use maple software and, In this section we illustrate some of the key findings of the system (1,2) numerically around the positive stability for a wide range of parameter values. First, we consider the limited growth range of parameter values r and ξ. Given we assume the values $f(p) = (r - \xi p)p$. where carrying capacity= $\frac{r}{\xi}, r = 1, \xi = 1$. In the initial value from

$p_1(0) = 0.5, p_2(0) = 2, p_3(0) = 3$. The growth rate value is decreasing figure1.

(i)When$p < \frac{r}{\xi}, f(p) > 0$. This implies that the population is decreasing with

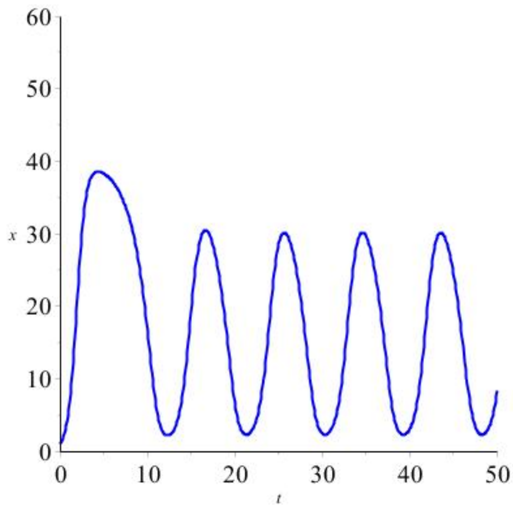

Figure 3. Time series plot,x axis is time series t and y axis is prey value let as consider x=p(prey)

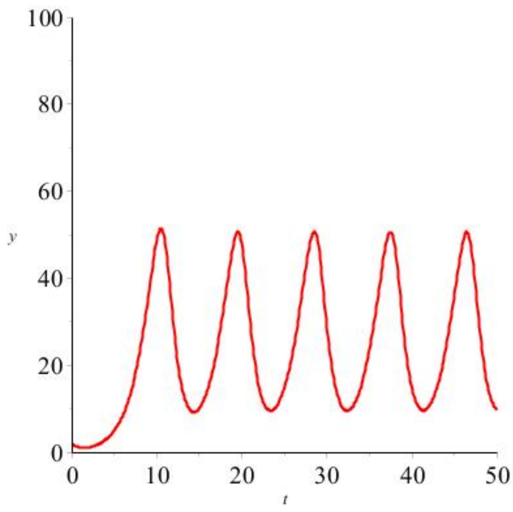

Figure 4. Time series plot,x axis is time series t and y axis is predator value let as consider y=q(predator)

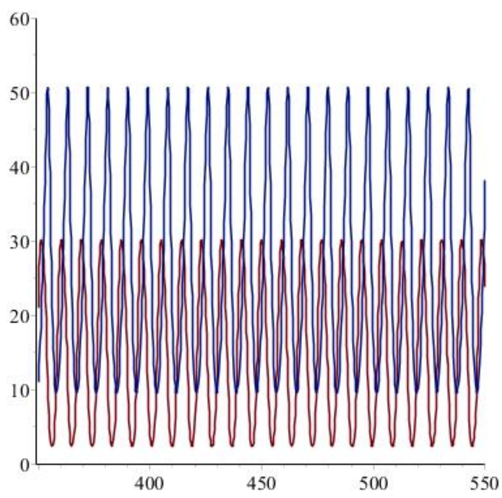

Figure 5. Prey predator interaction.

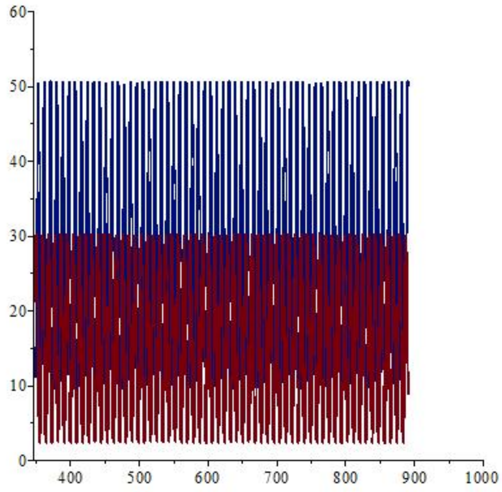

Figure 6. Prey predator interaction.

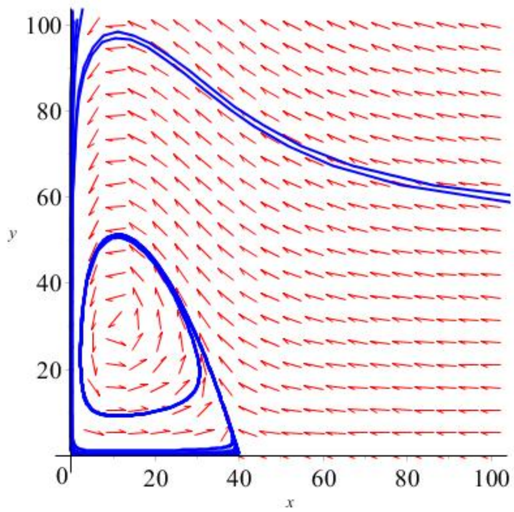

Figure 7. Competing species model,let us consider x axis prey values x=p(prey)and y axis predator values y=q(predator)

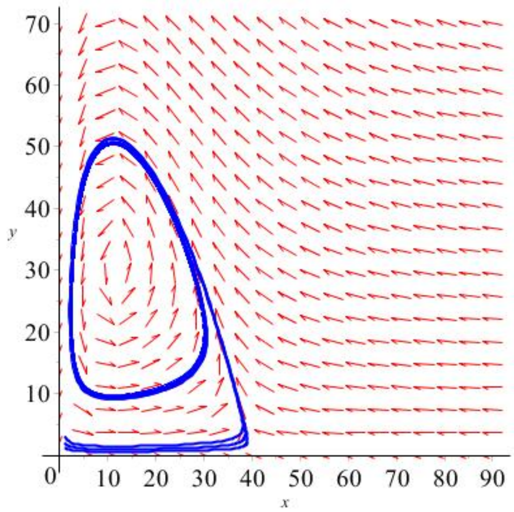

Figure 8. Prey predator model phase plot,let us consider x axis prey values x=p(prey)and y axis predator values y=q(predator

time and approaches $p = \dfrac{r}{\xi}$.
(ii)When $p > \dfrac{r}{\xi}, f(p) < 0$. This implies that the population is decreasing with time toward $p = \dfrac{r}{\xi}$.
In figure 3, The time series plot prey equation we have calculated by prey values is asymptotically stable and unstable values. In figure 4, The time series plot predator equation we have calculated by predator values is asymptotically stable and unstable values. The predator prey system of the equation (1,2) interaction value we can find out the parameter values $r = 2.0, \xi = 0.05, w = 10, w_1 = 10, u = 1.0, s = 1.0, v = 2.0, \gamma = 1, E = 0.05$using these sets of parametric values and initial values(p(0)=40,q(0)=50)the range of x,y direction(0,5...25 to 0,10,..,80) The interaction values of prey predator model is very clear model in the figure (2).Same as the parameter values change the initial value (p(0)=10,q(0)=10) the range of x,y direction (350...550 to 0,10,..,60) and (400,...,1000 to 0,...,60) the interaction values figure (5,6) given. In the initial value change of prey predator model interaction values late of the interaction we can find out the model equation.
The interior equilibrium point will be structurally unstable ($\Lambda(\lambda) > and \leqslant 0$). In the figure (7) the competing prey predator species will be existence limit cycle exists and bounders conditions exists. In figure (8) the prey predator model using Lotka Volterra model the species range is limit cycle exists and bounders condition exists.

9. Discussion and Result

Motivated by real world considerations we have proposed and studied a predator prey model in which the predator has a Holling type II functional response and limited prey growth. Given that the prey consumption provides the energy for predator activity and that the predator functional response represents the prey consumption rate per predator. We assumed that the per capita birth and death rates for the predator were respectively increasing and decreasing functions of the predator prey functional response. In this model very complicated limited growth prey value is very small and predator harvesting value is very small. The prey growth rate is high and competition food supply is very lowest value. The prey predator population is very lower value because the predator equation using harvesting effort consumption. So for as the dynamical behavior is concerned, it is observed that all the solutions of the system (1,2) are uniformly bounded which implies that the system is biologically well behaved. Nature of the equilibrium$(0, 0)$ and stability analysis discussed in the stability condition proved by the saddle point. The axial equilibrium point $(\frac{r}{\xi}, 0)$. The criterion given in the theorem 5.1, Provides the condition for stability or instability of the axial equilibrium point which is in turn, indicates ecological balance or imbalance. It is also observed that the existence of the positive equilibrium point(p^*, q^*). The criterion given in theorem 6.1, Provides the condition for stability or instability of the interior equilibrium point which, in turn, indicates ecological balance or imbalance. The optimal harvest policy of exploiting is given and its solution is derived by using Pontryagin$'$s maximum principle. It is shown that the total user cost of harvest per unit effort must be equal to the the discount value of the future profit at the steady state effort. Further it is noticed that an infinite discount rate leads to complete dissipation of economic revenue. It is also established that $\delta = 0$ leads to the maximization of the net economic revenue. Dynamic optimization of the harvest policy is carried out by taking

$E(t)$ as a dynamic variable. The harvesting policy problem is the prey population cost per unit effort time. In the mathematical modeling and diagrams using maple software version 18^{th} as used for numerical illustration.

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