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Efficiency Measurement of NDEA with Interval Data

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Abstract. Data envelopment analysis (DEA) is a non-parametric technique for evaluation of relative efficiency of decision making units described by multiple inputs and outputs. It is based on solving linear programming problems. Since 1978 when basic DEA model was introduced many its modifications were formulated. Among them are two or multi-stage models with serial or parallel structure often called network DEA models that are widely discussed in professional community in the last years. The exact known inputs and outputs are required in these DEA models. However, in the real world, the concern is systems with interval (bounded) data. When we incorporate such interval data into multi-stage DEA models, the resulting DEA model becomes a non-linear programming problem. In this study, we suggest an approach to measure the efficiency of series and parallel systems with interval data that preserves the linearity of DEA model. Also, the interval DEA models are proposed to measure the lower and upper bounds of the efficiency of each DMU with interval data.

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Keywords: DEA, Network DEA, Interval Data, Divisional efficiency, Overall efficiency.

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1. Introduction

Data envelopment analysis is a Non parametric method for measuring relative efficiency of decision making units based on multiple inputs and outputs that was invented by Fare and universalized by Charnes et al. Efficiency measurement is an important task in management, to better understand the past accomplishments of a unit and planning for its future development. Since the seminal work of Charnes, Cooper, and Rhodes (1978), data envelopment analysis (DEA) has been widely

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recognized as an effective technique for measuring the relative efficiency of a set of decision making units (DMUs) that apply multiple inputs to produce multiple outputs, with many theoretical developments and practical applications being reported (see, for example, the review of Cook & Seiford, 2009; Emrouznejad, Parker, & Tavares, 2008; Liu, Lu, Lu, & Lin, 2013a, 2013b; Seiford, 1996; Zhou, Ang, & Poh, 2008).

One drawback of the standard DEA models is ignoring internal operations within a DMU. The DEA model considers each DMU as a black box and uses only initial inputs and final outputs of the DMU to evaluate the efficiency. As a result, knowledge of the internal operations of a DMU is not taken into account in the analysis, and insights about how to improve the performance of the DMU from within become largely unclear. However, DMUs might include internal or network structures, which consist of several interactive processes. There are two basic structures for network systems, one is series and the other is parallel, where all processes operate independently as shown in Figs 1and2. Structurally, the parallel system is the same as the multi-period system. They differ only in factor definitions, in that the multi-period system requires the inputs and outputs of every period to be the same. Fure, Grabowski, Grosskopf, and Kraft (1997) are one of the earliest works on parallel systems, and their model aims to maximize the output system distance function. In their study of 57 Southern Illinois grain farms, the land was shared for crops of corn, soybeans, wheat, and double crop soybeans. Kao and Lin (2011) measured efficiency when the factors were qualitative, while Kao and Lin (2012) measured it when the observations were fuzzy numbers, with both works using the data in Beasley (1995) for illustration purposes. Rogge and Jaeger (2012) analyzed cost efficiency in the treatment of solid waste in 293 municipalities in Flanders, Belgium, using a ratio-form system efficiency model. There were six types of solid waste, residential, other municipal, packaging, other EPR, green, and bulky, with a shared input of handling costs. Da Cruz, Carvalho, and Marques (2013) applied the same model to measure the efficiency of the drinking water and wastewater services of 45 water utilities in Portugal with shared resources.

The series structure refers to a number of processes connected in sequence, where each process consumes the exogenous inputs and intermediate products produced by the preceding process, and produces exogenous outputs and intermediate products for the succeeding one to use. Although a series system can have as many processes as desired, except for theoretical studies, the largest system that has appeared in the literature has only five processes. Matthews (2013) studied the risk management and managerial efficiencies of 15 banks in China with an SBM model. The system was divided into three processes, where non-performing loans were inputs for the third one. Tsutsui and Goto (2009) used a weighted SBM model to study the performance of 90 vertically integrated electric power companies in the US. In this paper, in section 2, by considering weighted arithmetic mean DMSU, presented overall efficiency model for basic network system. Section3 is part that we presented the model network DEA with interval data. Section4 contains a numerical example that Meanwhile, the obtained results are compared with A. Ashrafi et al. [1] model and conclusions are given in section5.

2. Network DEA

In Network DEA (NDEA) structure we deal with n decision making units (DMU_j , $j = 1, ..., n$). Each DMU is divided to K divisions ($k = 1, ..., K$), which x_{ijk} and y_{rjk} are the i th input supplied from outside,$i \in I^k$, where I^k is the index set of the exogenous inputs used by process k, and rth final output of the sys-

tem, $r \in O^k$, where O^k is the index set of the final outputs produced by process$k$, $k = 1, ..., K$, respectively, of the jth DMU. Clearly, the sums of x_{ijk} and y_{rjk} for all p processes are the system input x_{ij} and system output y_{rj} . Further. Let z_{djk} denote the d th intermediate produced by process a, $d \in M^k$, where M^k is the index set of the intermediate products used by process k, and z_{fjk} denote the f th intermediate product to be used by process b, $f \in N^k$, where N^k is the same intermediate products produced by process k. Assuming the most general case where the technologies of all processes are allowed to be different, the production possibility set defined by the general network structure is:

$$T = \Big\{(x, y, z) \mid \sum_{j=1}^{n} \lambda_{jk} x_{ijk} \leqslant x_i, i \in I^k, \sum_{j=1}^{n} \lambda_{jk} y_{rjk} \geqslant y_r, r \in O^k, \sum_{j=1}^{n} \lambda_j z_{djk} \leqslant z_d,$$
$$d \in M^k, \sum_{j=1}^{n} \lambda_j z_{fjk} \geqslant z_f, f \in N^k, \sum_{j=1}^{n} \lambda_{jk} = 1, \lambda_{jk} \geqslant 0, k = 1, ..., k\Big\}$$

In normal state of DEA, to calculate the efficiency, we divide total weighted outputs to total weighted inputs of the desired DMU. Know that there are two basic structures for network systems, one is series and the other is parallel. Now that the internal structure DMU is so efficient, to calculate in terms of divisional efficiency & overall efficiency, we use the model of (Zhu et al. 2004) "overall efficiency calculation of decision making unit with network structure by the use of arithmetic mean of the divisional efficiency".

2.1 *Network DEA model for series systems:*

The series structure refers to a number of processes connected in sequence, where each process consumes the exogenous inputs and intermediate products produced by the preceding process, and produces exogenous outputs and intermediate products for the succeeding one to use. As shown in Figure. 1, the structure is a generalization of the general multi-stage one.

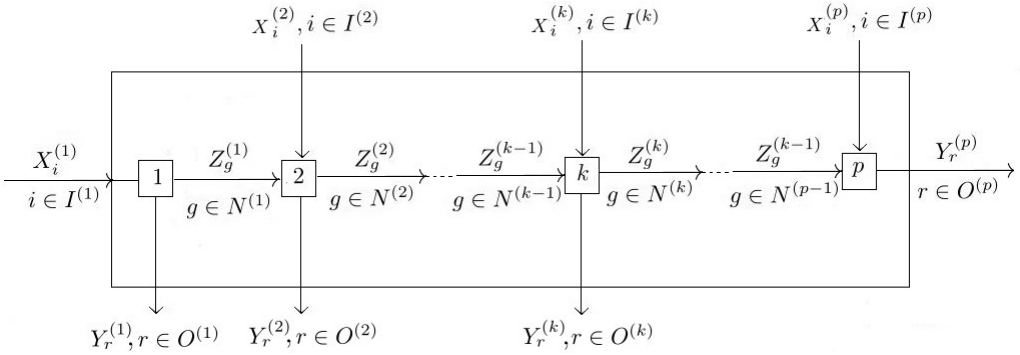

Figure 1. Series structures

In this part, by considering the inputs and outputs in one division of the desired DMU during a specific time process, we can evaluate the efficiency for that division in that process. Let DMU_j be the series system under evaluation. To measure the efficiency of the system is to find the multipliers u, v, w, and w' which produce the maximum efficiency under the constraint that the aggregation of the outputs is less than or equal to that of the inputs for all processes. Thus by using the definition

of relative efficiency, k division efficiency for the decision making units is defined as follows and will be represented by E_0^k.

$$_{series}E_0^k = max \frac{\sum\limits_{r\in O^k} u_{rk}y_{rok} + \sum\limits_{f\in N^k} w'_{fk}z_{fok} + u_{0k}}{\sum\limits_{i\in I^k} v_{ik}x_{iok} + \sum\limits_{d\in M^k} w_{dk}z_{dok}}$$

$s.t.$

$$\frac{\sum\limits_{r\in O^k} u_{rk}y_{rjk} + \sum\limits_{f\in N^k} w'_{fk}z_{fjk} + u_{0k}}{\sum\limits_{i\in I^k} v_{ik}x_{ijk} + \sum\limits_{d\in M^k} w_{dk}z_{djk}} \leqslant 1 \qquad j = 1, ..., n\ ,\ k = 1, ..., K \qquad (1)$$

$$u_r \geqslant 0\ ,\ r = 1, ..., s_k \qquad v_i \geqslant 0\ ,\ i = 1, ..., m_k$$

$$w_d \geqslant 0\ ,\ d = 1, ..., D \qquad w'_f \geqslant 0\ ,\ f = 1, ..., D-1$$

The fractional form of the objective function can be linearized by equating the denominator to 1, as a constraint, and using the numerator as the objective function.

$$_{series}E_0^k = max \sum_{r\in O^k} u_{rk}y_{rok} + \sum_{f\in N^k} w'_{fk}z_{fok} + u_{0k}$$

$s.t.$

$$\sum_{i\in I^k} v_{ik}x_{iok} + \sum_{d\in M^k} w_{dk}z_{dok} = 1 \qquad (2)$$

$$\sum_{r\in O^k} u_{rk}y_{rjk} + \sum_{f\in N^k} w'_{fk}z_{fjk} + u_{0k} - \sum_{i\in I^k} v_{ik}x_{ijk} + \sum_{d\in M^k} w_{dk}z_{djk} \leqslant 0$$

$$j = 1, ..., n\ ,\ k = 1, ..., K$$

$$u_r \geqslant 0\ ,\ r = 1, ..., s_k \qquad v_i \geqslant 0\ ,\ i = 1, ..., m_k$$

$$w_d \geqslant 0\ ,\ d = 1, ..., D \qquad w'_f \geqslant 0\ ,\ f = 1, ..., D-1$$

$$u_{0k} : free$$

By the use of (2) model, the overall performance of decision making unit can be written as convex linear combination of parts efficiency. Calculation of this efficiency is actually the calculation of the desired DMU considering the efficiency of all their divisions. We display it by E_0 .

This efficiency can be evaluated by the weighted mean of divisional efficiency (E_0^k). Which is defined as follows:$E_0 = \sum\limits_{k=1}^{K} w^k E_0^k$ Notice that w^k weights shows the share of k division in the efficiency of the desired period for the unit under evaluation. Due to this definition w^k, $\sum\limits_{k=1}^{K} w^k = 1$. We are overall efficiency model as follows:

$$_{series}E_0^k = max \frac{1}{K} \sum_{k=1}^{K} \Big(\sum_{r \in O^k} u_{rk} y_{rok} + \sum_{f \in N^k} w'_{fk} z_{fok} + u_{0k} \Big)$$

s.t.

$$\frac{1}{K} \sum_{k=1}^{K} \Big(\sum_{i \in I^k} v_{ik} x_{iok} + \sum_{d \in M^k} w_{dk} z_{dok} \Big) = 1 \qquad (3)$$

$$\frac{1}{K} \sum_{k=1}^{K} \Big(\sum_{r \in O^k} u_{rk} y_{rjk} + \sum_{f \in N^k} w'_{fk} z_{fjk} + u_{0k} - \sum_{i \in I^k} v_{ik} x_{ijk} + \sum_{d \in M^k} w_{dk} z_{djk} \Big) \leqslant 0$$

$j = 1, ..., n$

$u_r \geqslant 0\,,\ r = 1, ..., s_k \qquad v_i \geqslant 0\,,\ i = 1, ..., m_k$

$w_d \geqslant 0\,,\ d = 1, ..., D \qquad w'_f \geqslant 0\,,\ f = 1, ..., D-1$

$u_{0k} : free$

2.2 *Network DEA model for parallel systems:*

There are two basic structures for network systems, one is series and the other is parallel, where all processes operate independently as shown in Figure. 2. Structurally, the parallel system is the same as the multi-period system. They differ only in factor definitions, in that the multi-period system requires the inputs and outputs of every period to be the same.

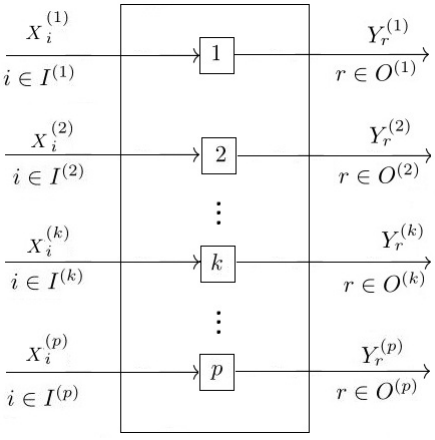

Figure 2. Parallel structure

If let DMU_j be the parallel system under evaluation than the model for measuring the system overall efficiency can be formulated as:

$$_{parallel}E_0^k = max \sum_{r\in O^k} u_{rk}y_{rok} + u_0$$

$s.t.$

$$\sum_{i\in I^k} v_{ik}x_{iok} = 1 \qquad (4)$$

$$\sum_{r\in O^k} u_{rk}y_{rok} + u_0 - \sum_{i\in I^k} v_{ik}x_{iok} \leqslant 0 \qquad j = 1, ..., n\,,\ k = 1, ..., K$$

$$u_r \geqslant 0\,,\ r = 1, ..., s_k \qquad v_i \geqslant 0\,,\ i = 1, ..., m_k$$

$$u_{0k} : free$$

Overall efficiency of the weighted sum of the efficiency the units is calculated as follows:

$$_{parallel}E_0^k = max \frac{1}{K}\sum_{k=1}^{K}\big(\sum_{r\in O^k} u_{rk}y_{rok} + u_0\big)$$

$s.t.$

$$\frac{1}{K}\sum_{k=1}^{K}\big(\sum_{i\in I^k} v_{ik}x_{iok}\big) = 1 \qquad (5)$$

$$\sum_{k=1}^{K}\big(\sum_{r\in O^k} u_{rk}y_{rjk} + u_0 - \sum_{i\in I^k} v_{ik}x_{iok}\big) \leqslant 0 \qquad j = 1, ..., n\,,\ k = 1, ..., K$$

$$u_r \geqslant 0\,,\ r = 1, ..., s_k \qquad v_i \geqslant 0\,,\ i = 1, ..., m_k$$

$$u_{0k} : free$$

3. Network DEA with Interval Data

Interval inputs and outputs are one of sorts' non-precision data which are placed in range of upper and lower bound that are defined by spans. Assuming that the levels of inputs, output and intermediate product are not exactly know, the true data for $DMU_j, (j = 1, , n)$ are known to lie within bounded intervals.

Unlike the original DEA models, we assume further that the levels of inputs and outputs are not known exactly, the true input and output data known to lie within bounded intervals, i.e., $x_{ijk} \in [\underline{x}_{ijk}, \overline{x}_{ijk}], y_{rjk} \in [\underline{y}_{rjk}, \overline{y}_{rjk}]$, $z_{djk} \in [\underline{z}_{djk}, \overline{z}_{djk}]$ and $z_{fjk} \in [\underline{z}_{fjk}, \overline{z}_{fjk}]$ with upper and lower bounds of the intervals given as constants and assumed strictly positive. In this case, the NDEA-efficiency can be an interval.

Now consider the following network DEA models with imprecise data:

$$_{series}E_0 = max \frac{1}{K}\sum_{k=1}^{K}\big(\sum_{r\in O^k} u_{rk}y_{rok} + \sum_{f\in N^k} w'_{fk}z_{fok} + u_{0k}\big)$$

$s.t.$

$$\frac{1}{K}\sum_{k=1}^{K}\big(\sum_{i\in I^k} v_{ik}x_{iok} + \sum_{d\in M^k} w_{dk}z_{dok}\big) = 1 \qquad (6)$$

$$\sum_{k=1}^{K}\big(\sum_{r\in O^k} u_{rk}y_{rjk} + \sum_{f\in N^k} w'_{fk}z_{fjk} + u_{0k} - \sum_{i\in I^k} v_{ik}x_{ijk} - \sum_{d\in M^k} w_{dk}z_{djk}\big) \leqslant 0$$

$j = 1, ..., n$

$x_{ijk} \in [\underline{x}_{ijk}, \overline{x}_{ijk}], \qquad y_{rjk} \in [\underline{y}_{rjk}, \overline{y}_{rjk}]$

$z_{djk} \in [\underline{z}_{djk}, \overline{z}_{djk}], \qquad z_{fjk} \in [\underline{z}_{fjk}, \overline{z}_{fjk}]$

$u_r \geqslant 0\,,\ r = 1, ..., s_k \qquad v_i \geqslant 0\,,\ i = 1, ..., m_k$

$w_d \geqslant 0\,,\ d = 1, ..., D \qquad w'_f \geqslant 0\,,\ f = 1, ..., D-1$

$u_{0k} : free$

and

$$_{parallel}E_0 = max \frac{1}{K}\sum_{k=1}^{K}\big(\sum_{r\in O^k} u_{rk}y_{rok} + u_{0k}\big)$$

$s.t.$

$$\frac{1}{K}\sum_{k=1}^{K}\big(\sum_{i\in I^k} v_{ik}x_{iok}\big) = 1 \qquad (7)$$

$$\sum_{k=1}^{K}\big(\sum_{r\in O^k} u_{rk}y_{rjk} + \sum_{f\in N^k} w'_{fk}z_{fjk} + u_{0k} - \sum_{i\in I^k} v_{ik}x_{ijk}\big) \leqslant 0 \qquad j = 1, ..., n$$

$x_{ijk} \in [\underline{x}_{ijk}, \overline{x}_{ijk}], \qquad y_{rjk} \in [\underline{y}_{rjk}, \overline{y}_{rjk}]$

$z_{djk} \in [\underline{z}_{djk}, \overline{z}_{djk}], \qquad z_{fjk} \in [\underline{z}_{fjk}, \overline{z}_{fjk}$

$u_r \geqslant 0\,,\ r = 1, ..., s_k \qquad v_i \geqslant 0\,,\ i = 1, ..., m_k$

$u_{0k} : free$

The NDEA-efficiency score attained by DMUo in Models (6 & 7) is not worse (less) than any other NDEA-efficiency score that the DMU might attain, by adjusting the levels of the outputs, inputs and intermediate products within the limits of the bounded intervals.

3.1 *Upper and lower bounds of NDEA-efficiency*

The upper bound of interval NDEA-efficiency is obtained from the optimistic viewpoint and the lower bound is obtained from the pessimistic viewpoint. The following

models provide such an upper bound of interval NDEA-efficiency for DMUo:

$$_{series}E_0 = max \frac{1}{K}\sum_{k=1}^{K}\big(\sum_{r\in O^k} u_{rk}\overline{y}_{rok} + \sum_{f\in N^k} w'_{fk}\overline{z}_{fok} + u_{0k}\big)$$

$s.t.$

$$\frac{1}{K}\sum_{k=1}^{K}\big(\sum_{i\in I^k} v_{ik}\underline{x}_{iok} + \sum_{d\in M^k} w_{dk}\underline{z}_{dok}\big) = 1 \qquad (8)$$

$$\sum_{k=1}^{K}\big(\sum_{r\in O^k} u_{rk}\overline{y}_{rjk} + \sum_{f\in N^k} w'_{fk}\overline{z}_{fjk} + u_{0k} - \sum_{i\in I^k} v_{ik}\underline{x}_{ijk} - \sum_{d\in M^k} w_{dk}\underline{z}_{djk}\big) \leqslant 0$$

$$0 = j = 1, ..., n$$

$$\sum_{k=1}^{K}\big(\sum_{r\in O^k} u_{rk}\underline{y}_{rjk} + \sum_{f\in N^k} w'_{fk}\underline{z}_{fjk} + u_{0k} - \sum_{i\in I^k} v_{ik}\overline{x}_{ijk} - \sum_{d\in M^k} w_{dk}\overline{z}_{djk}\big) \leqslant 0$$

$$0 \neq j = 1, ..., n$$

$$u_r \geqslant 0\,,\ r = 1, ..., s_k \qquad v_i \geqslant 0\,,\ i = 1, ..., m_k$$

$$w_d \geqslant 0\,,\ d = 1, ..., D \qquad w'_f \geqslant 0\,,\ f = 1, ..., D-1$$

$$u_{0k} : free$$

and

$$_{parallel}E_0 = max \frac{1}{K}\sum_{k=1}^{K}\big(\sum_{r\in O^k} u_{rk}\overline{y}_{rok} + u_{0k}\big)$$

$$s.t. \frac{1}{K}\sum_{k=1}^{K}\big(\sum_{i\in I^k} v_{ik}\underline{x}_{iok}\big) = 1 \qquad (9)$$

$$\sum_{k=1}^{K}\big(\sum_{r\in O^k} u_{rk}\overline{y}_{rjk} + u_{0k} - \sum_{i\in I^k} v_{ik}\underline{x}_{ijk}\big) \leqslant 0 \qquad j = 1, ..., n$$

$$\sum_{k=1}^{K}\big(\sum_{r\in O^k} u_{rk}\underline{y}_{rjk} + u_{0k} - \sum_{i\in I^k} v_{ik}\overline{x}_{ijk}\big) \leqslant 0 \qquad 0 \neq j = 1, ..., n$$

$$u_r \geqslant 0\,,\ r = 1, ..., s_k \qquad v_i \geqslant 0\,,\ i = 1, ..., m_k \qquad u_{0k} : free$$

Models (8 and 9) are Network DEA models with exact data, where the levels of inputs and outputs are adjusted in favors of DMUo and aggressively against the other DMUs. For DMUo, the inputs are adjusted at the lower bounds and the outputs at the upper bounds of the intervals. Unfavorably for the other DMUs, the inputs are contrarily adjusted at their upper bounds and the outputs at their lower bounds. The models below provide a lower bound of NDEA-efficiency score

for DMUo:

$$_{series}E_0 = max \frac{1}{K} \sum_{k=1}^{K} \big(\sum_{r \in O^k} u_{rk} \underline{y}_{rok} + \sum_{f \in N^k} w'_{fk} \underline{z}_{fok} + u_{0k} \big)$$

$s.t.$

$$\frac{1}{K} \sum_{k=1}^{K} \big(\sum_{i \in I^k} v_{ik} \overline{x}_{iok} + \sum_{d \in M^k} w_{dk} \overline{z}_{dok} \big) = 1 \qquad (10)$$

$$\sum_{k=1}^{K} \big(\sum_{r \in O^k} u_{rk} \underline{y}_{rjk} + \sum_{f \in N^k} w'_{fk} \underline{z}_{fjk} + u_{0k} - \sum_{i \in I^k} v_{ik} \overline{x}_{ijk} - \sum_{d \in M^k} w_{dk} \overline{z}_{djk} \big) \leqslant 0$$

$$0 = j = 1, ..., n$$

$$\sum_{k=1}^{K} \big(\sum_{r \in O^k} u_{rk} \overline{y}_{rjk} + \sum_{f \in N^k} w'_{fk} \overline{z}_{fjk} + u_{0k} - \sum_{i \in I^k} v_{ik} \underline{x}_{ijk} - \sum_{d \in M^k} w_{dk} \underline{z}_{djk} \big) \leqslant 0$$

$$0 \neq j = 1, ..., n$$

$$u_r \geqslant 0\,,\ r = 1, ..., s_k \qquad v_i \geqslant 0\,,\ i = 1, ..., m_k$$

$$w_d \geqslant 0\,,\ d = 1, ..., D \qquad w'_f \geqslant 0\,,\ f = 1, ..., D-1$$

$$u_{0k} : free$$

and

$$_{parallel}E_0 = max \frac{1}{K} \sum_{k=1}^{K} \big(\sum_{r \in O^k} u_{rk} \underline{y}_{rok} + u_{0k} \big)$$

$s.t.$

$$\frac{1}{K} \sum_{k=1}^{K} \big(\sum_{i \in I^k} v_{ik} \overline{x}_{iok} \big) = 1 \qquad (11)$$

$$\sum_{k=1}^{K} \big(\sum_{r \in O^k} u_{rk} \underline{y}_{rjk} + u_{0k} - \sum_{i \in I^k} v_{ik} \overline{x}_{ijk} \big) \leqslant 0 \qquad j = 1, ..., n$$

$$\sum_{k=1}^{K} \big(\sum_{r \in O^k} u_{rk} \overline{y}_{rjk} + u_{0k} - \sum_{i \in I^k} v_{ik} \underline{x}_{ijk} \big) \leqslant 0 \qquad 0 \neq j = 1, ..., n$$

$$u_r \geqslant 0\,,\ r = 1, ..., s_k \qquad v_i \geqslant 0\,,\ i = 1, ..., m_k \qquad u_{0k} : free$$

Models (10 & 11) are also Network DEA models with exact data. For DMUo, the inputs are adjusted at their upper bounds and the outputs at their lower bounds and for the other DMUs, the inputs are adjusted at their lower bounds and the outputs at their upper bounds. Therefore, Models (8 & 9) and models (10 & 11) provide for each DMU a bounded interval$[\underline{E}^*_0, \overline{E}^*_0]$ in which its possible NDEA-efficiency scores lie, from the worst to the best case. Based on these mentioned models, the following theorem is proved easily.

THEOREM 3.1 $\underline{E}^*_0 \leqslant E^*_0 \leqslant \overline{E}^*_0$.

COROLLARY 3.2 *If* $\underline{E}^* = 1$ *, then* $\underline{E}_0^*, DMUo is always overall ef ficient.$

COROLLARY 3.3 *If* $\overline{E}^* < 1$ *, then DMUo is always inefficient.*

COROLLARY 3.4 *If* $\underline{E}^* \,\&\, \overline{E}^* < 1$ *, then DMUo will be overall efficient in some intervals and sometimes inefficient in the others.*

When DMUs are evaluated with interval data, therefore they can classify as follows:

$E^{++}$is as all the DMUs which are overall efficient with any combination of their inputs, outputs and intermediate products which are call as fully efficient. $E^{++} = \{DMU_j \, \underline{E}_j = \overline{E}_j = 1\}$.

$E^{+}$is the DMUs which are overall efficient in their maximal status but for some data levels they lose their efficiency. $E^{+} = \{DMU_j \, \underline{E}_j < 1 \,\&\, \overline{E}_j = 1\}$.

E^{-} is the DMUs that are overall inefficient in each case.$E^{-} = \{DMU_j \, \overline{E}_j < 1\}$. We investigate the state of the efficiency scores for the efficient units in E^{+} , and will determine part of intervals of data inputs and outputs in which DMU is fully efficient.

4. Numerical example

In this section we may be cited paper Ashrafi et al. [1]. they using the network structure model Kao and Hwang and they were able to calculate efficiency interval of the unit under evaluation. We applied the network DEA model for measuring the efficiency of those data. Then we compared the obtained results of the model above with the measured efficiency by Ashrafi et al.

The proposed model is applied to an example consisting of 15 two-stage processes with two interval inputs (x_1 and x_2), two interval intermediate products (z_1 and z_2) and two interval outputs (y_1 and y_2). The data are provided in Table 1.

Table 1. The interval data set of 15 DMUs.

DMU	X_1	X_2	Z_1	Z_2	Y_1	Y_2
1	[8,14]	[16,19]	[30,35]	[51,58]	[13,22]	[21,29]
2	[4,17]	[18,22]	[35,43]	[43,48]	[19,24]	[15,22]
3	[7,16]	[11,26]	[33,42]	[38,45]	[18,25]	[21,30]
4	[5,11]	[11,24]	[38,44]	[50,61]	[12,17]	[12,19]
5	[4,13]	[18,24]	[49,58]	[60,66]	[13,25]	[10,23]
6	[8,15]	[15,21]	[41,48]	[42,49]	[12,19]	[22,30]
7	[7,11]	[12,29]	[36,46]	[32,38]	[18,25]	[12,18]
8	[4,10]	[14,25]	[47,53]	[45,53]	[14,25]	[18,24]
9	[12,16]	[12,23]	[32,40]	[51,58]	[11,20]	[12,18]
10	[5,10]	[10,20]	[32,40]	[32,39]	[20,28]	[19,30]
11	[5,15]	[14,27]	[40,48]	[51,61]	[11,20]	[21,25]
12	[6,12]	[12,23]	[41,47]	[20,41]	[20,26]	[9,13]
13	[6,11]	[15,26]	[42,49]	[55,62]	[21,24]	[17,23]
14	[10,14]	[22,26]	[40,49]	[40,50]	[19,25]	[11,19]
15	[8,14]	[21,27]	[37,45]	[32,39]	[21,26]	[10,17]

Table 2 shows efficiency interval for divisional and overall efficiency.

Table 2. Tdivisional efficiency overall efficiency.

DMU	$[\underline{E}_0^k, \overline{E}_0^k]$	DMSU	$[\underline{E}_0, \overline{E}_0]$
1	[1.0000,1.0000]	1	[0.4840,0.6536]
		2	[0.6400,1.0000]
2	[0.6881,1.0000]	1	[0.3977,0.7413]
		2	[0.5049,0.7836]
3	[0.6572,1.0000]	1	[0.2844,0.9545]
		2	[0.5316,0.9632]
4	[0.7053,1.0000]	1	[0.4765,1.0000]
		2	[0.3116,0.5297]
5	[0.7095,1.0000]	1	[0.5104,1.0000]
		2	[0.2561,0.5830]
6	[0.6826,0.9304]	1	[0.4880,0.8000]
		2	[0.4884,0.7787]
7	[0.6442,1.0000]	1	[0.3179,0.9583]
		2	[0.4933,0.8440]
8	[0.7017,1.0000]	1	[0.4924,1.0000]
		2	[0.3622,0.6221]
9	[0.7324,1.0000]	1	[0.3998,0.8715]
		2	[0.3190,0.7142]
10	[0.6898,1.0000]	1	[0.4000,1.0000]
		2	[0.6898,1.0000]
11	[0.6897,1.0000]	1	[0.3703,0.9238]
		2	[0.4611,0.6586]
12	[0.6956,1.0000]	1	[0.4456,0.9791]
		2	[0.5574,1.0000]
13	[0.7200,0.9744]	1	[0.4079,0.8310]
		2	[0.4897,0.6530]
14	[0.6884,0.9825]	1	[0.3846,0.5568]
		2	[0.4431,0.7142]
15	[0.6541,0.9690]	1	[0.3425,0.5485]
		2	[0.5744,0.8659]

As a result, the upper efficiency scores for DMUs 2, 3, 4, 5, 7, 8, 9, 10, 11, 12 and 15 are in unity while the lower efficiency scores for all DMUs are less than one. According to argument in last section, the 15 DMUs are categorized into three sets in terms of their interval efficiency scores as follows:

$E^{++} = \{1\}$
$E^{+} = \{2, 3, 4, 5, 7, 8, 9, 10, 11, 12\}$
$E^{-} = \{6, 13, 14, 15\}$

We know that one of relevant problems to network DEA models are that these models are not always received efficient DMU. But additional constraints on model that do not affect the Measurements of efficiency directly, and usage twice inputs and outputs in manufacturing the space of possible are the drawbacks of the model used by Ashrafi et al. So we can say that network DEA with proposed network structure presents more precise answer compared to Ashrafi et al.

5. Conclusion

In this paper, we consider weighted arithmetic mean DMSU and we deal with series and parallel systems with interval data. A linear approach to deal with interval data was proposed for these types of systems. Interval DEA models for measuring the lower and upper bounds of the overall efficiency scores of series and parallel systems with interval data have been proposed as well. In this study, we proposed DEA models for measuring the efficiency of series and parallel systems with interval data. The efficiency measurement of series and parallel systems with ordinal data and ratio bounded data can also be considered in future studies. Also, since the series and parallel are two basic structures of a network system, a complicated system could be represented by an equivalent parallel system of series components or series system of parallel components. Based on the proposed DEA models for DMUs with series and parallel structure, a network DEA model for DMUs with general multistage or network structure needs to be developed.

So, it is supposed NDEA models are not complete. However, researchers always are trying developing a new model for those NDEA that both concludes variety of data envelopment analysis in this unit and presents same analysis for similar cases.

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