

## Study of Oil Spill on the Sea Surface in the Presence of Thermal and Concentration Buoyancy Effects

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**Abstract.** Pollution occurs when the concentration of various chemical or biological constituents exceed a level implying negative impact on amenities, the eco-system, resources and human health. Oil spills are the serious environmental hazards which often exhibit long-term impacts. The main objective of response to an oil spill is to reduce its impact on nature and human health. This paper allows us to get a comprehensive idea of the oil spill impact in the presence of thermal and concentration buoyancy effects. The governing equations and their associated boundary conditions are first cast into dimensionless form and the resulting equations are then solved analytically using perturbation technique. The effect of various physical parameters such as Grashof number, Prandtl number, Schmidt number and chemical reaction parameter on the velocity, temperature and concentration profile as well as surface skin friction, heat and coefficient of mass transfers are discussed in detail with the help of graphs.

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**Keywords:** Oil spill, Concentration of oil particle, Heat and mass transfer, Chemical reaction, Perturbation technique.

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## 1. Introduction

Oil spill on the sea has become very common due to the development of oil industries and oil transport, which has a serious impact on the ocean ecological environment. Oil spill continue to happen as long as society depends on petroleum and its products. This is due to the potential for human error and equipment failure inherent in producing, transporting and storing petroleum. In the past, when an oil spill occurred, the location and extent of the spill, the potential behavior of

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the oil, and its impact on the environment were not immediately known. Today, technology available provides most of these informations immediately [7]. To assist this, mathematical models are developed to predict the trajectory or pathway and fate of oil. The study in this area help us to learn how the oil affect the environment and what improvements have to be made in the cleanup techniques to identify the gaps in technology.

The transport and fate of spilled oil can be affected by the physical, chemical and biological processes. These processes are divided into three phases: oil on the surface layer, oil in the water column and oil in the bed layer. Aghajanloo et al.[1] simulated the oil slick transport on the sea surface by an advection-diffusion model. Chen Hai-Zhou et al.[4] studied the oil spill model behavior and fate on the sea using Monte Carlo method. Perianez and Pascual-Granged [11] defined a model to simulate the dispersion of chemical/radioactive and oil spills based on particle-tracking technique and also discussed the effects of contamination concentration and time evolution of pollutant concentration. Wang et al.[14,15] investigated the vertical dispersion/motion of the spilled oil slick in the surface layer using Lagrangian discrete particle algorithm in two-dimension and extended the study to three-dimension. The Eulerian-Lagrangian model was developed by Nagheeby and Kolahdoozan [9] to simulate concentration distribution of oil on the water surface in two-phase fluid flow. Gamzaev [5] studied the oil spill on the sea surface affected by the forces due to gravity and viscous friction. Perianez [10] used two types of dispersion models namely finite difference and particle-tracking method for radionuclides and heavy metal concentrations in bed sediments and water column. Chao at al. [3] developed a three dimensional oil fate model to simulate the distribution of oil particle concentration in the water column.

Among these oil spill models, many of the researchers focus on the surface movement of oil spills. In this study, we considered the surface movement of the oil spill in the presence of thermal and concentration buoyancy effects including coriolis force. Also we assume that there exists a homogeneous first order chemical reaction in the flow. The problem is considered to be a transient flow in the horizontal channel. All the thermo physical properties are assumed to be constant in the linear momentum equation. The governing equations are approximated according to Bousinessq approximation and solved by the perturbation technique [8] using Rossby number as the perturbation parameter. The skin friction, heat and mass transfer characteristics of the resultant flow are also studied due to its importance at the surface of a water body.

## 2. Mathematical Formulation

We consider an unsteady two dimensional, laminar, incompressible, viscous flow of oil spilled at an open sea in a rectangular domain of length L and height h.

The governing equations for this problem are based on the balance laws of mass, linear momentum in the presence of thermal and concentration buoyancy effects including coriolis force (due to earth rotation), the energy equation and the species equation with homogeneous first order chemical reaction. These can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + g\beta_T (T - T_1) + g\beta_C (C - C_1) + fv \tag{2}$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - fu \tag{3}$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \tag{4}$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) - k_1 (C - C_1) \tag{5}$$

where,  $u$  and  $v$  are the components of velocities along  $x$  and  $y$  directions, respectively,  $T$  is the temperature of the oil,  $C$  is the oil concentration,  $T_1$  and  $C_1$  are the temperature and concentration at the upper surface of oil, respectively.  $\rho$  is the density of oil,  $\nu$  is the kinematic viscosity,  $C_p$  is the specific heat at constant pressure,  $D$  is the mass diffusivity,  $g$  is the gravitational acceleration,  $f$  is the coriolis parameter,  $k$  is the thermal conductivity,  $k_1$  is the chemical reaction parameter,  $\beta_T$  and  $\beta_C$  are the thermal and concentration expansion coefficients, respectively.

Using Boussinesq approximation, the flow variables become functions of  $y$  and time  $t$  only. The Equations (1) to (5) that describe the physical situation are given by

$$\frac{\partial v}{\partial y} = 0 \tag{6}$$

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta_T (T - T_1) + g\beta_C (C - C_1) + fv \tag{7}$$

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} \tag{8}$$

$$\frac{\partial C}{\partial t} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_1 (C - C_1) \tag{9}$$

Under these assumptions, the appropriate boundary conditions for the velocity, temperature and concentration fields are:

$$\left. \begin{aligned} u = 0, T = T_0 + \epsilon e^{nt} (T_0 - T_1), C = C_0 + \epsilon e^{nt} (C_0 - C_1) \text{ at } y = 0 \\ u = u_0, T = T_1, C = C_1 \text{ at } y = L \end{aligned} \right\} \tag{10}$$

where,  $T_0$  and  $C_0$  are the temperature and concentration at the lower surface of oil, respectively,  $u_0$  and  $n$  are constants and  $\epsilon$  is the perturbation parameter.

We now introduce the following non-dimensional quantities:

$$u^* = \frac{u}{v_0}, \quad t^* = \frac{tv_0^2}{\nu}, \quad y^* = \frac{yv_0}{\nu}, \quad \theta = \frac{T - T_1}{T_0 - T_1}, \quad \phi = \frac{C - C_1}{C_0 - C_1}$$

where,  $\theta$  and  $\phi$  are dimensionless temperature and concentration respectively.

Making use of the non-dimensional variables in Equations (6) to (9), neglecting the (\*) symbol gives

$$\frac{\partial v}{\partial y} = 0 \quad (11)$$

which implies  $v = v_0$ , where  $v_0$  is a real positive constant, also considered as the characteristic velocity and

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gc\phi + \frac{1}{Ro} \quad (12)$$

$$\frac{\partial \theta}{\partial t} + \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} \quad (13)$$

$$\frac{\partial \phi}{\partial t} + \frac{\partial \phi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - K\phi \quad (14)$$

where,  $Gr = \frac{\nu g \beta_T (T_0 - T_1)}{v_0^3}$  is the Grashof number,  $Gc = \frac{\nu g \beta_C (C_0 - C_1)}{v_0^3}$  is the modified Grashof number,  $Ro = \frac{v_0}{fL}$  is the Rossby number,  $Pr = \frac{\rho C_p \nu}{k}$  is the Prandtl number,  $Sc = \frac{\nu}{D}$  is the Schmidt number and  $K = \frac{k_1 \nu}{v_0^2}$  is the chemical reaction parameter.

The boundary conditions (10) in non-dimensional form are:

$$\left. \begin{aligned} u = 0, \theta = 1 + \epsilon e^{nt}, \phi = 1 + \epsilon e^{nt} \text{ at } y = 0 \\ u = 1, \theta = 0, \phi = 0 \text{ at } y = 1 \end{aligned} \right\} \quad (15)$$

### 3. Method of Solution

Equations (12) to (14) represent a set of partial differential equations whose solutions are difficult to solve in a closed-form. However, it can be reduced to a set of ordinary differential equations that can be solved analytically, by employing perturbation technique [6, 12, 13]. This can be done by representing the velocity, temperature and the concentration as:

$$\left. \begin{aligned} u(y, t) &= u_0(y) + \epsilon e^{nt} u_1(y) + O(\epsilon^2) \\ \theta(y, t) &= \theta_0(y) + \epsilon e^{nt} \theta_1(y) + O(\epsilon^2) \\ \phi(y, t) &= \phi_0(y) + \epsilon e^{nt} \phi_1(y) + O(\epsilon^2) \end{aligned} \right\} \quad (16)$$

Substituting Equations (16) into Equations (12) to (14), neglecting the higher order of ( $\epsilon^2$ ) and simplifying we obtain the following set of differential equations

for  $u_0, \theta_0, \phi_0$  and  $u_1, \theta_1, \phi_1$ .

Zeroth order equations:

$$u_{0yy} - u_{0y} + Gr\theta_0 + Gc\phi_0 + \frac{1}{Ro} = 0 \tag{17}$$

$$\theta_{0yy} - Pr\theta_{0y} = 0 \tag{18}$$

$$\phi_{0yy} - Sc\phi_{0y} - ScK\phi_0 = 0 \tag{19}$$

First order equations:

$$u_{1yy} - u_{1y} - nu_1 + Gr\theta_1 + Gc\phi_1 = 0 \tag{20}$$

$$\theta_{1yy} - Pr\theta_{1y} - Prn\theta_1 = 0 \tag{21}$$

$$\phi_{1yy} - Sc\phi_{1y} - Sc(n + K)\phi_1 = 0 \tag{22}$$

subject to the boundary conditions,

$$\left. \begin{aligned} u_0 = 0, \theta_0 = 1, \phi_0 = 1 \text{ at } y = 0 \\ u_0 = 1, \theta_0 = 0, \phi_0 = 0 \text{ at } y = 1 \end{aligned} \right\} \tag{23}$$

$$\left. \begin{aligned} u_1 = 0, \theta_1 = 1, \phi_1 = 1 \text{ at } y = 0 \\ u_1 = 0, \theta_1 = 0, \phi_1 = 0 \text{ at } y = 1 \end{aligned} \right\} \tag{24}$$

The solutions to Equations (17)-(19) and (20)-(22) using the boundary conditions (23) and (24) respectively, gives the velocity, temperature and concentration distributions as

$$\begin{aligned} u(y, t) = & \left\{ \frac{e^y - 1}{1 - e} \left[ \frac{Gr}{1 - e^{pr}} \left( \frac{1 - e^{pr}}{pr(pr - 1)} - e^{pr} \right) + \frac{Gc}{e^{m_1} - e^{m_2}} \right. \right. \\ & \left. \left( \frac{e^{m_2}(e^{m_1} - 1)}{m_1(m_1 - 1)} + \frac{e^{m_1}(1 - e^{m_2})}{m_2(m_2 - 1)} \right) + \frac{1}{Ro} - 1 \right] + \frac{Gr}{1 - e^{pr}} \left( \frac{1 - e^{pry}}{pr(pr - 1)} - ye^{pr} \right) \right. \\ & \left. + \frac{Gc}{e^{m_1} - e^{m_2}} \left( \frac{e^{m_2}(e^{m_1y} - 1)}{m_1(m_1 - 1)} + \frac{e^{m_1}(1 - e^{m_2y})}{m_2(m_2 - 1)} \right) + \frac{y}{Ro} \right\} \\ & + \epsilon e^{nt} \left\{ \frac{Gr}{e^{m_3} - e^{m_4}} \left[ \frac{e^{m_4}(e^{m_3y} - e^{m_7y})}{m_3^2 - m_3 - n} + \frac{e^{m_3}(e^{m_7y} - e^{m_4y})}{m_4^2 - m_4 - n} \right. \right. \\ & \left. \left. + \frac{e^{m_7y} - e^{m_8y}}{e^{m_7} - e^{m_8}} \left( \frac{e^{m_4}(e^{m_7} - e^{m_3})}{m_3^2 - m_3 - n} + \frac{e^{m_3}(e^{m_4} - e^{m_7})}{m_4^2 - m_4 - n} \right) \right] \right. \\ & \left. + \frac{Gc}{e^{m_5} - e^{m_6}} \left[ \frac{e^{m_6}(e^{m_5y} - e^{m_7y})}{m_5^2 - m_5 - n} + \frac{e^{m_5}(e^{m_7y} - e^{m_6y})}{m_6^2 - m_6 - n} \right. \right. \\ & \left. \left. + \frac{e^{m_7y} - e^{m_8y}}{e^{m_7} - e^{m_8}} \left( \frac{e^{m_6}(e^{m_7} - e^{m_5})}{m_5^2 - m_5 - n} + \frac{e^{m_5}(e^{m_6} - e^{m_7})}{m_6^2 - m_6 - n} \right) \right] \right\} \end{aligned} \tag{25}$$

$$\theta(y, t) = \frac{1}{1 - e^{pr}} (e^{pry} - e^{pr}) + \epsilon e^{nt} \frac{1}{e^{m_3} - e^{m_4}} (e^{m_3} e^{m_4 y} - e^{m_4} e^{m_3 y}) \quad (26)$$

$$\phi(y, t) = \frac{1}{e^{m_1} - e^{m_2}} (e^{m_1} e^{m_2 y} - e^{m_2} e^{m_1 y}) + \epsilon e^{nt} \frac{1}{e^{m_5} - e^{m_6}} (e^{m_5} e^{m_6 y} - e^{m_6} e^{m_5 y}) \quad (27)$$

where,

$$m_1 = \frac{Sc + \sqrt{Sc^2 + 4KSc}}{2}; \quad m_2 = \frac{Sc - \sqrt{Sc^2 + 4KSc}}{2};$$

$$m_3 = \frac{Pr + \sqrt{Pr^2 + 4nPr}}{2}; \quad m_4 = \frac{Pr - \sqrt{Pr^2 + 4nPr}}{2};$$

$$m_5 = \frac{Sc + \sqrt{Sc^2 + 4(n+K)Sc}}{2}; \quad m_6 = \frac{Sc - \sqrt{Sc^2 + 4(n+K)Sc}}{2};$$

$$m_7 = \frac{1 + \sqrt{1 + 4n}}{2}; \quad m_8 = \frac{1 - \sqrt{1 + 4n}}{2}$$

From the point of view of applications in technology, it is of interest to know the wall shear stress, surface heat and mass transfer rate at both the walls.

In non-dimensional form, the coefficient of skin friction  $C_f$  is defined as,

$$C_f = \frac{\tau_w}{\rho v_0^2} = \frac{\partial u}{\partial y}$$

Also the dimensionless form of coefficients of heat and mass transfer, characterized by Nusselt number (Nu) and Sherwood number (Sh) that describe the behavior of convective flow and diffusion rate are defined as,

$$\frac{Nu}{Re_x} = -\frac{\partial \theta}{\partial y}$$

and

$$\frac{Sh}{Re_x} = -\frac{\partial \phi}{\partial y}$$

where,  $Re_x = \frac{L v_0}{\nu}$  is the local Reynolds number.

#### 4. Results and Discussions

Numerical evaluation of the analytical results for the velocity, temperature and concentration distributions as well as coefficients of skin friction, heat and mass transfer have been computed using MATHEMATICA 8.0. The results obtained are discussed for various values of chemical reaction parameter, Grashof number,

Prandtl number and Schmidt number. The values of other physical parameters are fixed as real constants.

Figures 1 to 4 reveal that the velocity profile of the oil slick are found to be parabolic in nature, they also represent the effects of Grashof number, Prandtl number, Schmidt number and chemical reaction parameter. Figures 1 and 2 depict that an increase in chemical reaction parameter and Schmidt number results with a decrease in velocity. In Figure 3, the increase in Grashof number results in increasing velocity. This is because increasing the buoyancy ratio tends to accelerate the fluid flow. Increase in Prandtl number also shows an increase in velocity from Figure 4.

The behavior of temperature distribution for different values of Prandtl number is illustrated through Figure 5. It is observed that the temperature increases with increase in Prandtl number. The oil viscosity greater than the thermal diffusivity results in high Prandtl number ( $Pr > 1$ ) and hence the momentum diffusion governs the flow, not the thermal diffusion. Figures 6 and 7 show the effects of chemical reaction parameter and Schmidt number on concentration distribution. These figures indicate that increase in chemical reaction parameter and Schmidt number results in decreasing concentration distribution.

The coefficient of skin friction ( $C_f$ ), the non-dimensional parameter that characterizes the viscous friction forces of the flow over a solid surface is analyzed at the lower surface  $y=0$  and upper surface  $y=1$ , from the Figures 8 and 9 for different values of chemical reaction parameter and Schmidt number. Both the figures reveal that the coefficient of skin friction at the bottom surface  $y=0$  decreases with increase in chemical reaction parameter and Schmidt number. For the case at the upper surface  $y=1$ , a reverse trend happens. The negative values show that flow reversal arises within the boundary layer [2]. It can be seen that  $C_f$  value increases with increase in Grashof number at the lower surface, but decreases with increase in Grashof number at the upper surface.

Heat transfer is the thermal energy in transient due to temperature difference. It is one of the most observed phenomena in nature. Nusselt number, the dimensionless coefficient of heat transfer is obtained at both the lower and upper surfaces and depicted through Figure 10. As the Prandtl number varies, the rate of heat transfer remains the same at the lower surface and there is a rapid increase in the rate of heat transfer at the upper surface.

Mass transfer refers to relative motion of chemical species due to concentration gradients. Mass transfer is used in different scientific disciplines for different processes and mechanisms that involve diffusive and convective transport of chemical species within physical systems. Sherwood number, the dimensionless coefficient of mass transfer is discussed at both the surfaces through Figure 11 for different values of chemical reaction parameter. At the surface  $y=0$ , increase in chemical reaction parameter increases the Sherwood number and the increase in Schmidt number also results in increase of Sherwood number. At the other surface  $y=1$ , we find a decrease in Sherwood number when chemical reaction parameter increases and this decrease occurs for increase in values of Schmidt number also. Figures 12-14 show velocity, temperature and concentration variation for different time. From these figures, we see that the velocity, temperature and concentration of spilled oil increases with time.

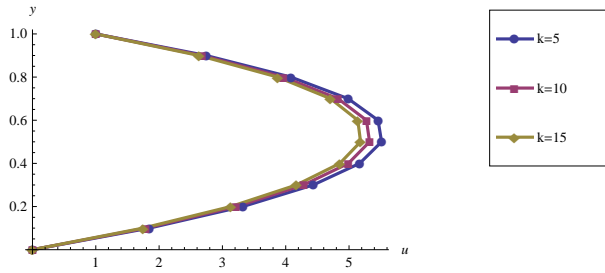


Figure 1. Effects of chemical reaction parameter on velocity profile

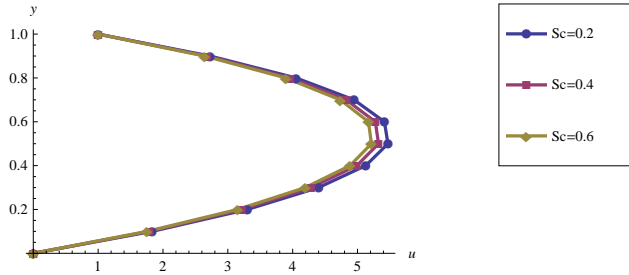


Figure 2. Effects of Schmidt number on velocity profile

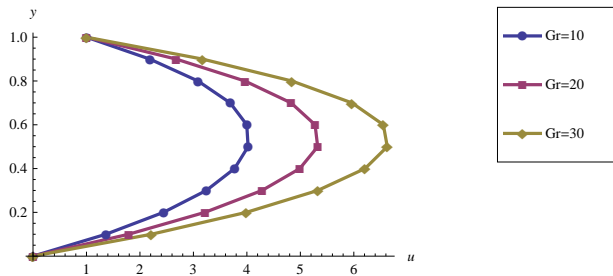


Figure 3. Effects of Grashof number on velocity profile

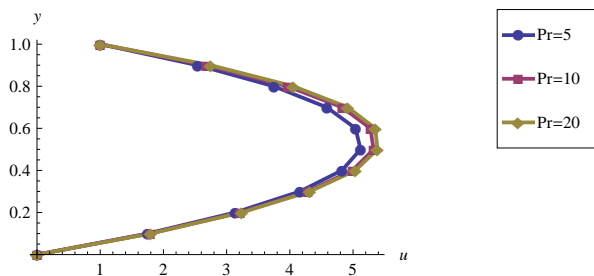


Figure 4. Effects of Prandtl number on velocity profile

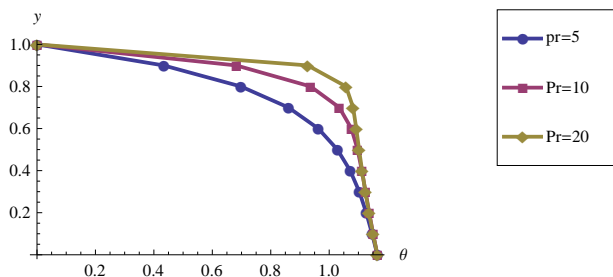


Figure 5. Effects of Prandtl number on temperature distribution



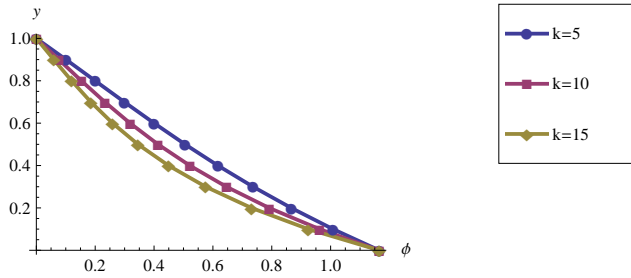


Figure 6. Effects of chemical reaction parameter on concentration distribution

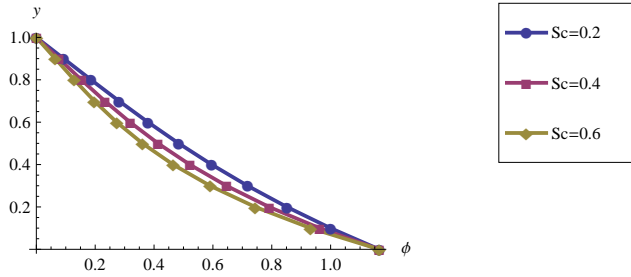


Figure 7. Effects of Schmidt number on concentration distribution

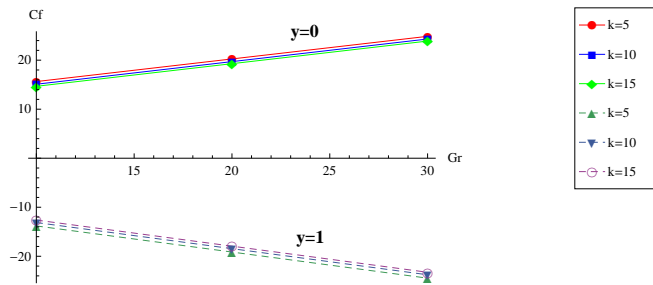


Figure 8. Effects of chemical reaction parameter on coefficient of skin friction

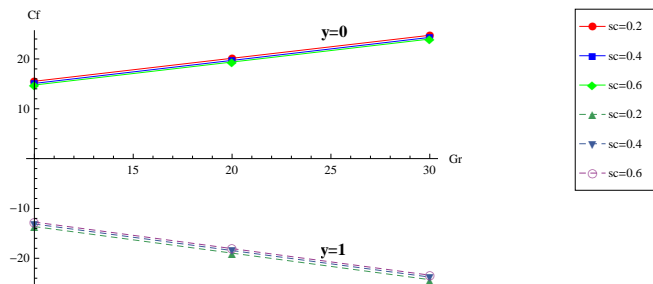


Figure 9. Effects of Schmidt number on coefficient of skin friction

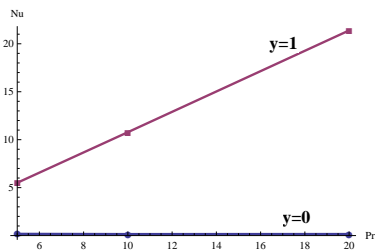


Figure 10. Prandtl number versus coefficient of heat transfer

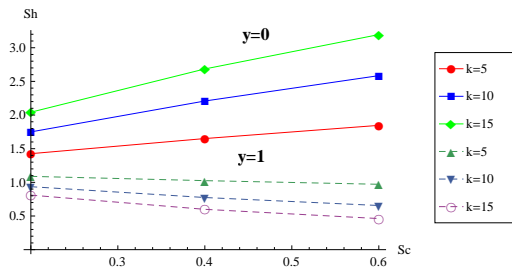


Figure 11. Effects of chemical reaction parameter on coefficient of mass transfer

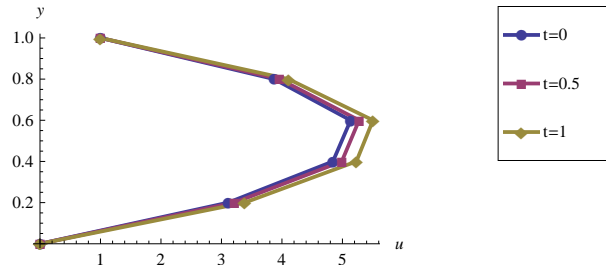


Figure 12. Time evolution of velocity profile

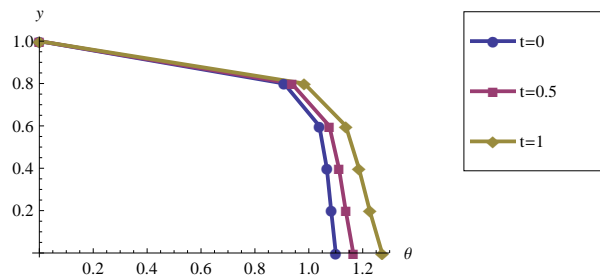


Figure 13. Time evolution of temperature distribution

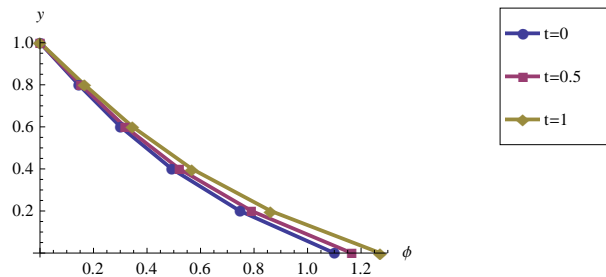


Figure 14. Time evolution of concentration distribution

### 5. Conclusion

Oil that enters the marine environment has distinct effects, according to their composition, concentration and the elements in the environment that are taken into consideration. The analysis of the proposed model presented here is the spread of oil slick resulting from an accidental oil spillage on the sea. It is developed to predict the movement of a surface oil slick with chemical reaction.

The dimensionless governing equations for these investigations are solved using perturbation technique. Graphical results obtained for velocity, temperature and concentration profiles are presented and discussed. From the point of view of ap-

plications in technology, coefficient of skin friction, heat and mass transfer at both the surfaces are obtained numerically and illustrated graphically. Also the study of the chemically reactive pollutant has considerable attention because of its harmful effects.

Having a detailed knowledge of oil slick behavior on water can be important in making operational decision and taking appropriate action against pollution. Better maintenance and emergency response plans has to be developed to protect our sea, ocean waters and air from contamination, since the recovery costs are expensive and complete removal is extremely time consuming.

### Acknowledgment

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