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Design of Accelerated Life Testing Using Geometric Processfor Type-II Censored Pareto Failure Data

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Abstract. Many studies concerning accelerated life testing, the log linear link function between life and stress which is just a simple re-parameterization of the original parameter of the life distribution is used to obtain the estimates of original parameters but from the statistical point of view, it is preferable to obtain the estimates of the original parameters directly instead of developing inferences for the parameters of the log-linear link function. By using geometric process one can deal with original parameters directly in accelerated life testing. In this paper the geometric process is used to estimate the parameters of Pareto distribution with type-II censored data in constant stress accelerated life testing. Assuming that the lifetimes under increasing stress levels form a geometric process, estimates of the parameters are obtained by using the maximum likelihood method. In addition, asymptotic confidence intervals of the parameters using Fisher information matrix are also obtained. Lastly the statistical properties of estimates of the parameters considered in the present are illustrated by a Simulation study.

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1. Introduction

Due to the global competition for the development of new products in a short time and to achieve customer's satisfaction manufacturing industries continuously improving their manufacturing design which makes today's products and materials highly reliable. Since, in life testing experiments, time-to-failure data is used to quantify the product's failure-time distribution and its associated parameters under normal operating conditions, Therefore, Testing under normal operating conditions require a very long period of time and need an extensive number of units under test. So it is usually costly and impractical to perform reliability testing under normal conditions. Under these circumstances accelerated life testing (ALT) may be the best choice to test the products. ALT is a quick way to obtain information about the life distribution of a material, component or product in which products are tested at higher than usual level of stress to yield shorter life but, hopefully, do not change the failure mechanisms. Three types of stress loadings are usually applied in ALTs: constant stress, step stress and linearly increasing stress. The constant stress loading, which is a time-independent test setting, has several advantages over the time-dependent stress loadings. For example, most products are assumed to operate at a constant stress under normal use. Therefore, a constant stress test mimics actual use conditions. Failure data obtained from ALT can be divided into two categories: complete (all failure data are available) or censored (some of failure data are missing). Complete data consist of the exact failure time of test units, which means that the failure time of each sample unit is observed or known. In many cases when life data are analyzed, all units in the sample may not fail. This type of data is called censored or incomplete data.

Constant stress ALT with different type of data and planning has been studied by many authors. For example, Yang [1] proposed an optimal design of 4-level constant-stress ALT plans considering different censoring times. Pan et al. [2] proposed a bivariate constant stress accelerated degradation test model by assuming that the copula parameter is a function of the stress level that can be described by a logistic function. Chen et al. [3] discuss the optimal design of multiple stresses constant ALT plan on non-rectangle test region. Watkins and John [4] considers constant stress ALTs based on Weibull distributions with constant shape and a log-linear link between scale and the stress factor which is terminated by a type-II censoring regime at one of the stress levels. Fan and Yu [5] discuss the reliability analysis of the constant stress ALTs when a parameter in the generalized gamma lifetime distribution is linear in the stress level. Ding et al. [6] dealt with Weibull distribution to obtain ALT sampling plans under type-I progressive interval censoring with random removals. Ahmad et al. [7], Islam and Ahmad [8], Ahmad and Islam [9], Ahmad, et al. [10] and Ahmad [11] discuss the optimal constant stress ALT designs under periodic inspection and type-I censoring.

The concept of geometric process (GP) is introduced by Lam [12], when he studied the problem of repair replacement. Large amount of studies in maintenance problems and system reliability have been shown that a GP model is a good and simple model for analysis of data with a single trend or multiple trends. For example Lam and Zhang [13] apply the GP model in the analysis of a two-component series system with one repairman. Lam [14] studied the GP model for a multistate system and determined an optimal replacement policy to minimize the long run average cost per unit time. Zhang [15] used the GP to model a simple repairable system with delayed repair. So far, there are only three studies that utilize the GP in the analysis of ALT. Huang [16] introduced the GP model for the analysis of ALT with complete and censored exponential samples under the constant stress. Kamal et al. [17] extended the GP model for the analysis of ALT with complete Weibull failure data under constant stress. Zhou et al. [18] considers the GP implementation of the constant stress ALTmodel based on the progressive type-I hybrid censored Rayleigh failure data.

In this paper, the GP model is implemented in the analysis of ALT for the Pareto life distribution under constant stress with type-II censored data. Maximum likelihood (ML) estimates of parameters and their asymptotic confidence intervals (CIs) are obtained. The performance of the estimates is evaluated by a simulation study.

2. The Model and Test Procedure

2.1 The Geometric Process

A GP describes a stochastic process $\{X_n, n = 1,2,...\}$, where there exists a real valued $\lambda > 0$ such that $\{\lambda^{n-1} X_n, n = 1,2,...\}$ forms a renewal process.The positive number $\lambda > 0$ is called the ratio of the GP. It is clear to see that a GP is stochastically increasing if $0 < \lambda < 1$ and stochastically decreasing if $\lambda > 1$. Therefore, the GP is a natural approach to analyze data from a series of events with trend.

It can be shown that if $\{X_n, n = 1,2,...\}$ is a GP and the probability density function (pdf) of X_1 is $f(x)$ with mean μ and variance σ^2 then the pdf of X_n will be given be $\lambda^{n-1} f(\lambda^{n-1} x)$ with $E(X_n) = \mu / \lambda^{n-1}$ and $Var(X_n) = \sigma^2 / \lambda^{2(n-1)}$. Thus λ, μ and σ^2 are three important parameters of a GP.

2.2 The Pareto Distribution

The pdf, cumulative distribution function and survival function of the Pareto distribution with scale parameter θ and shape parameter α are given respectively by

$$f(x) = \frac{\alpha\theta^{\alpha}}{(\theta + x)^{\alpha+1}}; \quad x > 0, \theta > 0, \alpha > 0 \tag{1}$$

$$F(x) = 1 - \frac{\theta^{\alpha}}{(\theta + x)^{\alpha+1}}; \quad x > 0, \theta > 0, \alpha > 0$$

$$S(x) = \frac{\theta^{\alpha}}{(\theta + x)^{\alpha+1}}$$

2.3 Assumptions and Test Procedure

I. Suppose that we are given an ALT with s increasing stress levels. A random sample of n identical items is placed under each stress level and start to operate at the same time. Let $x_{ki}, i = 1,2,...,n,\ k = 1,2,...,s$ denote observed failure time of i^{th} test item under k^{th} stress level. Whenever an item fails, it will be removed from the test and the test is continue until a prespecified number of failures r at each stress level (type-II censoring). Here total numbers of observed failure are r and can be written as $x_{k(1)} \le x_{k(2)} \le \dots \le x_{k(r)}$.

II. The product life follows a Pareto distribution given by (1) at any stress.

III. The shape parameter α is constant, i.e. independent of stress.

IV. The scale parameter is a log-linear function of stress thatis $\log\theta_i = a + bS_i$, where a and b are unknown parameters depending on the nature of the product and the test method.

V. Let random variables $X_0, X_1, X_2,..., X_s$, denote the lifetimes under each stress level, where X_0 denotes item's lifetime under the design stress at which items will operate ordinarily and sequence $\{X_k, k=1,2,...,s\}$ forms a GP with ratio $\lambda > 0$.

The assumption (V) which will be used in this study may be stronger than the commonly used Assumptions (I-IV) in usual discussion of ALTs in literature without increasing the complexity of calculations. The next theorem discusses how the assumption of GP (assumption V) is satisfied when there is a log linear relationship between a life characteristic and the stress level (assumption IV).

Theorem 2.1: *If the stress level in an ALT is increasing with a constant difference then the lifetimesunder each stress level forms a GP. That is, If $S_{k+1} - S_k$ is constant for $k=1,2,...,s-1$, then $\{X_k, k=0,1,2,...,s\}$ forms a GP.*

Or Log Linear and GP model are equivalent when the stress increases arithmetically.

Proof: From assumption (IV), it can easily be shown that

$$\log\left(\frac{\theta_{k+1}}{\theta_k}\right) = b(S_{k+1} - S_k) = b\Delta S \tag{2}$$

This shows that the increased stress levels form an arithmetic sequence with a constant difference ΔS.

Now (2) can be rewritten as

$$\frac{\theta_{k+1}}{\theta_k} = e^{b\Delta S} = \frac{1}{\lambda} \text{ (Assumed)} \tag{3}$$

It is clear from (3) that

$$\theta_k = \frac{1}{\lambda}\theta_{k-1} = \frac{1}{\lambda^2}\theta_{k-2} = ... = \frac{1}{\lambda^k}\theta$$

The PDF of the product lifetime under the k^{th} stress level is

$$f_{X_k}(x) = \frac{\alpha\theta_k{}^{\alpha}}{(\theta_k + x)^{\alpha+1}}$$

$$= \frac{\alpha\left\{\frac{1}{\lambda^k}\theta\right\}^{\alpha}}{\left(\frac{1}{\lambda^k}\theta + x\right)^{\alpha+1}} = \lambda^k \frac{\alpha\theta^{\alpha}}{(\theta + \lambda^k x)^{\alpha+1}}$$

This implies that

$$f_{X_k}(x) = \lambda^k f_{X_0}(\lambda^k x) \tag{4}$$

Now, the definition of GP and (4) have the evidence that, if density function of X_0 is $f_{X_0}(x)$, then the pdf of X_k will be given by $\lambda^k f(\lambda^k x)$, $k=0,1,2,...,s$. Therefore, it is clear that lifetimes under a sequence of arithmetically increasing stress levels form a GP with ratio λ. Now the pdf of a test item at k^{th} stress level by using theorem 2.1 can be written as

$$f_{X_k}(x \mid \alpha, \theta, \lambda) = \frac{\lambda^k \alpha\theta^{\alpha}}{(\theta + \lambda^k x)^{\alpha+1}} \tag{5}$$

3. The Maximum Likelihood Method of Estimation

Here the ML method of estimation is used because ML method is very robust and gives the estimates of parameter with good statistical properties. In this method, the estimates of parameters are those values which maximize the sampling distribution of data. However, ML estimation method is very simple for one parameter distributions but its implementation in ALT is mathematically more intense and, generally, estimates of parameters do not exist in closed form, therefore, numerical techniques such as Newton Method, Some computer programs are used to compute them.

Let $r_k (\leq n)$ failures times $x_{k(1)} \leq x_{k(2)} \leq \ldots \leq x_{k(r)}$ are observed before test termination at the r^{th} failure and $(n-r)$ units are still survived at each stress level. Here, r is fixed in advance and t is random. Therefore the likelihood function in constant stress ALT at one of the stress level using GP for the Pareto distribution with type-II censored data is given by

$$L_k = \frac{n!}{(n-r)!} (\lambda^k \alpha \theta^\alpha)^r \left[\prod_{i=1}^{r} \frac{1}{\left(\theta + \lambda^k x_{k(i)}\right)^{\alpha+1}} \right] \left[\frac{\theta^\alpha}{\left(\theta + \lambda^k x_{k(r)}\right)^{\alpha+1}} \right]^{n-r}$$

Now the likelihood function of observed data in a total s stress levels is:

$L = L_1 \times L_2 \times \ldots \times L_s$

$$= \prod_{k=1}^{s} \left\{ \frac{n!}{(n-r)!} (\lambda^k \alpha \theta^\alpha)^r \left[\prod_{i=1}^{r} \frac{1}{\left(\theta + \lambda^k x_{k(i)}\right)^{\alpha+1}} \right] \left[\frac{\theta^\alpha}{\left(\theta + \lambda^k x_{k(r)}\right)^{\alpha+1}} \right]^{n-r} \right\} \tag{6}$$

The log-likelihood function corresponding (6) takes the form

$$l = \sum_{k=1}^{s} \left\{ \begin{array}{l} \ln\left(\dfrac{n!}{(n-r)!}\right) + kr\ln\lambda + r\ln\alpha + \alpha r \ln\theta - (\alpha+1)\displaystyle\sum_{i=1}^{r} \ln(\theta + \lambda^k x_{k(i)}) \\ + \alpha(n-r)\ln\theta - (\alpha+1)(n-r)\ln(\theta + \lambda^k x_{k(r)}) \end{array} \right\}$$

ML estimates of α, θ and λ are obtained by solving the equations $\frac{\partial l}{\partial \alpha} = 0$, $\frac{\partial l}{\partial \theta} = 0$ and $\frac{\partial l}{\partial \lambda} = 0$, where

$$\frac{\partial l}{\partial \alpha} = \sum_{k=1}^{s} \frac{r}{\alpha} + \sum_{k=1}^{s} \left\{ r\ln\theta - \sum_{i=1}^{r} \ln(\theta + \lambda^k x_{k(i)}) + (n-r)\ln\left(\frac{\theta}{(\theta + \lambda^k x_{k(r)})}\right) \right\} = 0 \tag{7}$$

$$\frac{\partial l}{\partial \theta} = \sum_{k=1}^{s} \left\{ \frac{\alpha r}{\theta} - (\alpha+1)\sum_{i=1}^{r} \frac{1}{(\theta + \lambda^k x_{k(i)})} + \frac{\alpha(n-r)}{\theta} - \frac{(\alpha+1)(n-r)}{(\theta + \lambda^k x_{k(r)})} \right\} = 0 \tag{8}$$

$$\frac{\partial l}{\partial \lambda} = \sum_{k=1}^{s} \left\{ \frac{kr}{\lambda} - k(\alpha+1)\sum_{i=1}^{r} \frac{\lambda^{k-1} x_{k(i)}}{(\theta + \lambda^k x_{k(i)})} - \frac{k(n-r)(\alpha+1)\lambda^{k-1} x_{k(r)}}{(\theta + \lambda^k x_{k(r)})} \right\} = 0 \tag{9}$$

Equations (7), (8) and (9) are nonlinear; therefore, it is very difficult to obtain a closed form solution. So, Newton-Raphson method is used to solve these equations simultaneously to obtain $\hat{\alpha}$, $\hat{\theta}$ and $\hat{\lambda}$.

4. Asymptotic Confidence Interval Estimates

According to large sample theory, the ML estimators, under some appropriate regularity conditions, are consistent and normally distributed. Since ML estimates of parameters are not in closed form, therefore, it is impossible to obtain the exact CIs, so asymptotic CIs based on the asymptotic normal distribution of ML estimators instead of exact CIs are obtained here.

The Fisher-information matrix composed of the negative second partial derivatives of log likelihood function can be written as

$$F=\begin{bmatrix} -\frac{\partial^2 l}{\partial\alpha^2} & -\frac{\partial^2 l}{\partial\alpha\partial\theta} & -\frac{\partial^2 l}{\partial\alpha\partial\lambda} \\ -\frac{\partial^2 l}{\partial\theta\partial\alpha} & -\frac{\partial^2 l}{\partial\theta^2} & -\frac{\partial^2 l}{\partial\theta\partial\lambda} \\ -\frac{\partial^2 l}{\partial\lambda\partial\alpha} & -\frac{\partial^2 l}{\partial\lambda\partial\theta} & -\frac{\partial^2 l}{\partial\lambda^2} \end{bmatrix}$$

Elements of the Fisher Information matrix are

$$\frac{\partial^2 l}{\partial\alpha^2}=-\frac{rs}{\alpha^2}$$

$$\frac{\partial^2 l}{\partial\theta^2}=\sum_{k=1}^{s}\left\{-\frac{\alpha r}{\theta^2}+(\alpha+1)\sum_{i=1}^{r}\frac{1}{(\theta+\lambda^k x_{k(i)})^2}-\frac{\alpha(n-r)}{\theta^2}+\frac{(\alpha+1)(n-r)}{(\theta+\lambda^k x_{k(r)})^2}\right\}$$

$$\frac{\partial^2 l}{\partial\lambda^2}=\sum_{k=1}^{s}\left\{-\frac{kr}{\lambda^2}-k(\alpha+1)\sum_{i=1}^{r}x_{k(i)}\left[\frac{(k-1)(\theta+\lambda^k x_{k(i)})\lambda^{k-2}-k\lambda^{2k-2}x_{k(i)}}{(\theta+\lambda^k x_{k(i)})^2}\right]\right.$$
$$\left.-k(n-r)(\alpha+1)\lambda^{k-1}x_{k(r)}\left[\frac{(k-1)(\theta+\lambda^k x_{k(r)})\lambda^{k-2}-k\lambda^{2k-2}x_{k(r)}}{(\theta+\lambda^k x_{k(r)})^2}\right]\right\}$$

$$\frac{\partial^2 l}{\partial\alpha\partial\theta}=\sum_{k=1}^{s}\left\{\frac{r}{\theta}-\sum_{i=1}^{r}\frac{1}{(\theta+\lambda^k x_{k(i)})}+\frac{(n-r)}{\theta}-\frac{(n-r)}{(\theta+\lambda^k x_{k(r)})}\right\}$$

$$\frac{\partial^2 l}{\partial\theta\partial\alpha}=\frac{\partial^2 l}{\partial\alpha\partial\theta}=\sum_{k=1}^{s}\left\{\frac{r}{\theta}-\sum_{i=1}^{r}\frac{1}{(\theta+\lambda^k x_{k(i)})}+\frac{(n-r)}{\theta}-\frac{(n-r)}{(\theta+\lambda^k x_{k(r)})}\right\}$$

$$\frac{\partial^2 l}{\partial\alpha\partial\lambda}=\frac{\partial^2 l}{\partial\lambda\partial\alpha}=\sum_{k=1}^{s}\left\{-k\sum_{i=1}^{r}\frac{\lambda^{k-1}x_{k(i)}}{(\theta+\lambda^k x_{k(i)})}-\frac{k(n-r)\lambda^{k-1}x_{k(r)}}{(\theta+\lambda^k x_{k(r)})}\right\}$$

$$\frac{\partial l}{\partial\theta\partial\lambda}=\frac{\partial l}{\partial\lambda\partial\theta}=\sum_{k=1}^{s}\left\{k(\alpha+1)\sum_{i=1}^{r}\frac{\lambda^{k-1}x_{k(i)}}{(\theta+\lambda^k x_{k(i)})^2}+\frac{k(n-r)(\alpha+1)\lambda^{k-1}x_{k(r)}}{(\theta+\lambda^k x_{k(r)})^2}\right\}$$

Now, the variance covariance matrix can be written as

$$\Sigma = \begin{bmatrix} -\frac{\partial^2 l}{\partial \alpha^2} & -\frac{\partial^2 l}{\partial \alpha \partial \theta} & -\frac{\partial^2 l}{\partial \alpha \partial \lambda} \\ -\frac{\partial^2 l}{\partial \theta \partial \alpha} & -\frac{\partial^2 l}{\partial \theta^2} & -\frac{\partial^2 l}{\partial \theta \partial \lambda} \\ -\frac{\partial^2 l}{\partial \lambda \partial \alpha} & -\frac{\partial^2 l}{\partial \lambda \partial \theta} & -\frac{\partial^2 l}{\partial \lambda^2} \end{bmatrix}^{-1} = \begin{bmatrix} AVar(\hat{\alpha}) & ACov(\hat{\alpha}\hat{\theta}) & ACov(\hat{\alpha}\hat{\lambda}) \\ ACov(\hat{\theta}\hat{\alpha}) & AVar(\hat{\theta}) & ACov(\hat{\theta}\hat{\lambda}) \\ ACov(\hat{\lambda}\hat{\alpha}) & ACov(\hat{\lambda}\hat{\theta}) & AVar(\hat{\lambda}) \end{bmatrix}$$

where $AVar$ and $ACov$ are stand for asymptotic variance and covariance respectively.

Now the $100(1-\gamma)\%$ asymptotic CIs for θ, α and λ can be obtained respectively as

$$\left[\hat{\theta} \pm Z_{1-\frac{\gamma}{2}} \sqrt{AVar(\hat{\theta})}\right], \left[\hat{\alpha} \pm Z_{1-\frac{\gamma}{2}} \sqrt{AVar(\hat{\alpha})}\right] \text{ and } \left[\hat{\lambda} \pm Z_{1-\frac{\gamma}{2}} \sqrt{AVar(\hat{\lambda})}\right]$$

5. Simulation Study

The performance of the estimates is evaluated through simulation study in which MLEs, lower and upper CI limits (LCL and UCL) of parameters and the coverage rate of asymptotic CIs for different sample sizes are obtained.

Now to perform the simulation study, first different samples x_{ki}, $k = 1,2,...,s$, $i = 1,2,...,r$ of sizes $n = 20, 50, 100$ are generated from Pareto distribution which is censored at $r = 12, 15$. The combinations $(\lambda, \alpha, \theta)$ of values of the parameters are chosen to be $(1.1, 1.25, 0.25)$, $(1.2, 1.5, 0.5)$ and $(1.3, 1.75, 0.75)$. The number of stress levels s is assumed to be 5 throughout the study. For different sample sizes, stress levels and numbers of censored units, the lower and upper CI limits (LCL and UCL) and the coverage rate of the 95% CI of parameters based on 400 simulations are obtained by our proposed model and summarized in Tables 1, 2, 3, 4, 5 and 6.

Table 1: Results based on constant stress ALT for type-II censored Pareto Data using GP with $\lambda = 1.1, \alpha = 1.25, \theta = 0.25, s = 5$ and $r = 12$

Sample Size n	Parameter	MLE	LCL	UCL	95% Asymptotic CI Coverage
	λ	1.1887	0.4181	1.8054	0.9666
20	α	1.2505	0.9679	1.4111	0.9777
	θ	0.2501	0.1935	0.2822	0.9837
	λ	1.1865	0.1896	0.9948	0.9878
50	α	1.2556	1.1309	1.7210	0.9634
	θ	0.2515	0.2261	0.3442	0.9583
	λ	1.1676	0.2313	1.1838	0.9887
100	α	1.2612	1.1251	1.7034	0.9662
	θ	0.2522	0.2250	0.3406	0.9562

Table 2: Results based on constant stress ALT for type-II censored Pareto Data using GP with $\lambda = 1.1, \alpha = 1.25,\ \theta = 0.25,\ s = 5$ and $r = 15$

Sample Size *n*	Parameter	MLE	LCL	UCL	95% Asymptotic CI Coverage
	λ	1.1814	0.6140	1.9407	0.9782
20	α	1.2470	0.9513	1.3254	0.9347
	θ	0.2489	0.1902	0.2650	0.9274
	λ	1.1574	0.3224	1.2337	0.9574
50	α	1.2533	1.1172	1.5997	0.9680
	θ	0.2506	0.2234	0.3199	0.9590
	λ	1.1456	0.2453	0.9802	0.9891
100	α	1.2622	1.2230	1.7699	0.9782
	θ	0.2518	0.2446	0.3539	0.9780

Table 3: Results based on constant stress ALT for type-II censored Pareto Data using GP with $\lambda = 1.2, \alpha = 1.5,$ $\theta = 0.5\ \ s = 5$ and $r = 12$

Sample Size *n*	Parameter	MLE	LCL	UCL	95% Asymptotic CI Coverage
	λ	1.2762	0.6406	2.3259	0.9462
20	α	1.4998	1.1393	1.6045	0.9569
	θ	0.5002	0.3871	0.5644	0.9777
	λ	1.2656	0.2888	1.2364	0.9673
50	α	1.5086	1.3683	2.0067	0.9782
	θ	0.5037	0.4523	0.6884	0.9756
	λ	1.2582	0.3675	1.4278	0.9784
100	α	1.5129	1.2630	1.8096	0.9563
	θ	0.5045	0.4500	0.6813	0.9662

Table 4: Results based on constant stress ALT for type-II censored Pareto Data using GP with $\lambda = 1.2, \alpha = 1.5,$ $\theta = 0.5\ \ s = 5$ and $r = 15$

Sample Size *n*	Parameter	MLE	LCL	UCL	95% Asymptotic CI Coverage
	λ	1.2888	0.6699	2.3052	0.9782
20	α	1.4964	1.1415	1.5905	0.9347
	θ	0.4988	0.3805	0.5301	0.9347
	λ	1.2626	0.3517	1.3459	0.9574
50	α	1.5039	1.3406	1.9197	0.9680
	θ	0.5013	0.4468	0.6399	0.9680
	λ	1.2614	0.2676	1.0693	0.9891
100	α	1.5089	1.4677	1.7916	0.9782
	θ	0.5037	0.4892	0.7079	0.9780

Table 5: Results based on constant stress ALT for type-II censored Pareto Data using GP with $\lambda = 1.3, \alpha = 1.75, \ \theta = 0.75 \ \ s = 5$ and $r = 12$

Sample Size *n*	Parameter	MLE	LCL	UCL	95% Asymptotic CI Coverage
	λ	1.4145	0.4941	2.1337	0.9560
20	α	1.7475	1.3550	1.9756	0.9670
	θ	0.7503	0.5807	0.8467	0.9777
	λ	1.3862	0.2241	1.1756	0.9753
50	α	1.7746	1.5833	2.4094	0.9629
	θ	0.7544	0.6785	1.0326	0.9753
	λ	1.3798	0.2734	1.3990	0.9887
100	α	1.7657	1.5752	2.3847	0.9662
	θ	0.7567	0.6750	1.0220	0.9662

Table 6: Results based on constant stress ALT for type-II censored Pareto Data using GP with $\lambda = 1.3, \alpha = 1.75, \ \theta = 0.75 \ \ s = 5$ and $r = 15$

Sample Size *n*	**Parameter**	**MLE**	**LCL**	**UCL**	**95% Asymptotic CI Coverage**
	λ	1.3904	0.7257	1.9356	0.9784
20	α	1.7473	1.3318	1.8556	0.9354
	θ	0.7482	0.5707	0.5707	0.9347
	λ	1.3616	0.3810	1.4580	0.9473
50	α	1.7572	1.5641	2.2396	0.9684
	θ	0.7531	0.6703	0.9598	0.9684
	λ	1.3555	0.2899	1.1584	0.9891
100	α	1.7668	1.7123	2.4779	0.9782
	θ	0.7556	0.7338	1.0619	0.9780

6. Discussion and Conclusions

This paper deals with use of GP model in the analysis of constant stress ALT plan for Pareto distribution with type-II censored data. The ML estimates of the model parameters were obtained. Based on the asymptotic normality, the lower and upper CI limits (LCL and UCL) and the coverage rate of 95% CI of the model parameters were also obtained.

From the results in Table 1, 2, 3, 4, 5 and 6, it is easy to find that estimates of θ, α and λ perform well. For fixed θ, α and λ we find that as sample size n increases, the CIs get narrower. For the fixed sample sizes n, as the number of failures r gets larger the CIs get narrower also. This is very usual because more failures increase the efficiency of the estimators. It is also notice that the coverage probabilities of the asymptotic CI are close to the nominal level and do not change much as sample size and the number of failures increases. From these results, it may be concluded that the present model work well under type-II censored data.

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