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Evaluating MBTs Using Fuzzy Measure and Fuzzy Integral

S. Sarabadan,[a,*]

Department of Mathematics, Imam Hossein University, Tehran, Iran.

Abstract. This study introduces a novel evaluation framework utilizing fuzzy analytic hierarchy process and fuzzy integrals to address the inherent uncertainty and subjective nature of decision-making. By employing linguistic values represented by trapezoidal fuzzy numbers, our model effectively manages vagueness. We leverage fuzzy measures and fuzzy integrals key components of multi-criteria decision-making (MCDM) methodologies to systematically rank the subjects under evaluation. This research specifically investigates the performance of 29 model-based techniques (MBTs), providing a detailed case study that demonstrates the practicality and efficacy of the proposed evaluation method.

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1. Introduction

A straightforward and effective method to characterize the relationships among performance attributes of various entities is by substituting traditional additive weights with a non-additive function that operates on the power set of all available information sources. Instead of relying on the conventional weighted average approach, Sugeno [23] introduced the principles of fuzzy measures and fuzzy integrals. Fuzzy measures depart from the additivity constraint found in classical measures [1,29]. Due to the complexity involved in specifying general fuzzy measures, Sugeno developed the λ-fuzzy measure, which adheres to the λ-additive axiom, simplifying the process of identifying fuzzy measures. This λ-fuzzy measure is governed by a parameter, λ, representing the extent of additivity among the elements. In comparison to other fuzzy measurement frameworks, the λ-fuzzy measure is more accessible and has gained popularity for determining measure values. Aggregation is a process of combining several numerical values into a single one which exists in many disciplines, such as image processing [22], pattern recognition [21] and decision making [11, 12]. To achieve a collective agreement based on measurable evaluations, some synthesizing functions have been proposed. For example, arithmetic mean, geometric mean, median can be regarded as a basic class, because they are often used and very classic. However, these operators are not able to model an interaction between criteria. For having a presentation of interaction phenomena between criteria, fuzzy measures have been proposed [23].

[*]Corresponding author. Email: s.sarabadan@yahoo.com

The properties and applications of the fuzzy measures and fuzzy integrals have been studied by many authors. Ralescu and Adams [13] studied the several equivalent definitions of fuzzy integrals. Román-Flores et al. [5], [14, 15, 16, 17] studied the level-continuity of fuzzy integrals, H-continuity of fuzzy measures and geometric inequalities for fuzzy measures and integral, respectively. Wang and Klir [25] had a general overview on fuzzy measurement and fuzzy integration theory. Two main classes of the fuzzy integrals are Choquet and Sugeno integrals. Choquet and Sugeno integrals are idempotent, continuous and monotone operators.Liu et al. [7] proposed a color image encryption scheme based on Choquet fuzzy integral and hyper chaotic system. Chen et al. [1] proposed a fusion recognition scheme based on nonlinear decision fusion, using fuzzy integral to fuse the objective evidence supplied by each modality. Seyedzadeh et al. [20] presented a new RGB color image encryption using keystream generator based on fuzzy integral. Recently, fuzzy measuring and fuzzy integral is newly used to evaluate the interaction of scale factors and to compare the energy performance of buildings in different scale factors [8]. Considering the interactions between the weights of attributes of building energy performance, a multiple attribute decision-making approach, fuzzy measure and fuzzy integral, is adopted to rank the evaluated buildings.

Assessing weapon systems is crucial for developing an efficient defense strategy. This challenge can be addressed through a Multi-Criteria Decision Making (MCDM) framework [24]. In 1994, Cheng and Mon [4] assessed weapon systems with AHP based on fuzzy scales. Cheng and Lin [3] applied fuzzy decision theory to evaluate main battle tanks (MBTs). Yong and Cheng [27] introduced a method for determining the optimal main battle tank (MBT) that enhances efficiency by utilizing the canonical representation of arithmetic operations on fuzzy numbers. This approach allows for straightforward arithmetic calculations with crisp numbers instead of complex operations involving fuzzy numbers. The evaluation of weapon systems presents a Multi-Criteria Decision Making (MCDM) challenge, requiring the consideration of multiple criteria during the decision-making process, while also incorporating elements of subjectivity, uncertainty, and ambiguity in the assessment methodology.

Evaluating MBTs (Model-Based Testing) refers to the process of assessing the effectiveness, quality, and performance of a model-based testing approach or technique. MBTs involve creating models to represent the behavior and structure of the software system under test, and then using these models to derive test cases.

The primary objective of this research is to apply the Fuzzy Analytic Hierarchy Process (FAHP) and fuzzy integrals to assess and rank 29 different main battle tanks (MBTs). The selection of a Multi-Criteria Decision Making (MCDM) method should be deferred until both analysts and decision-makers thoroughly comprehend the problem, identify the viable MBTs, evaluate various outcomes, resolve conflicts among criteria, and assess the level of uncertainty in the data [10]. A schematic representation of the suggested model for weapon selection is illustrated in Fig. 1.

The structure of this paper is organized as follows: Section 2 presents a brief overview of the proposed methodologies, while Section 3 delves into the steps involved in the proposed approach in greater detail. In Section 4, the proposed model for evaluating the MBTs is presented and how the proposed model is used on a real-world example is explained. Section 5 presents a discussion of the study's conclusions.

2. Methodology

2.1. *Fuzzy sets and fuzzy numbers*

Fuzzy set theory, introduced by Zadeh [28], is a mathematical framework developed to represent the ambiguity and imprecision characteristic of human cognitive processes.

Table 1. Membership function of fuzzy numbers for relative importance.

Linguistic variable	Positive trapezoidal fuzzy numbers	Positive reciprocal trapezoidal fuzzy numbers
Extremely high important/improved (EH)	(7.5, 8.5, 9, 9)	(9^{-1}, 9^{-1}, 8.5^{-1}. 7.5^{-1})
Very high important/improved (VH)	(6.5, 7.5, 7.5, 8.5)	($8{\cdot}5^{-1}$, 7.5^{-1}, 7.5^{-1}, 6.5^{-1})
High important/improved (H)	(5, 5.75, 6.75, 7.5)	(7.5^{-1}, 6.75^{-1}, 5.75^{-1},5^{-1})
Medium high important/improved (M)	(4, 5, 5, 6)	(6^{-1}, 5^{-1}, 5^{-1}, 4^{-1})
Moderately high important/improved (MH)	(2.5, 3.25, 4.25, 5)	(5^{-1}, 4.25^{-1}, 3.25^{-1}, 2.5^{-1})
Little high important/improved (LH)	(1.5, 2.5, 2.5, 3.5)	(3.5^{-1}, 2.5^{-1}, 2.5^{-1}, 1.5^{-1})
Equal important/improved (WMI)	(1, 1, 1.5, 2.5)	(2.5^{-1}, 1.5^{-1},1^{-1},1^{-1})

The fundamental concept of fuzzy set theory is that an element can possess varying degrees of membership within a fuzzy set. A fuzzy set $\tilde{A}$ within a universe of discourse X is defined by a membership function $\mu_{\tilde{A}}(x)$, which maps each element x in X to a real number within the interval [0, 1]. This function value, $\mu_{\tilde{A}}(x)$, indicates the degree of membership of x in the fuzzy set $\tilde{A}$.

In the literature, triangular and trapezoidal fuzzy numbers that are the special forms of fuzzy numbers are usually used to capture the vagueness of the parameters related to the topic. A trapezoidal fuzzy number (TFN) is characterized by four parameters (a_1, a_2, a_3). The corresponding membership function $\mu_{\tilde{A}}(x)$ is defined as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & a_1 \leq x \leq a_2 \\ 1 & a_2 \leq x \leq a_3 \\ \dfrac{a_4 - x}{a_4 - a_3} & a_3 \leq x \leq a_4 \\ 0 & otherwise \end{cases}$$

In order to apply fuzzification techniques to linguistic terms in questionnaires, we use Table 1 to describe the mapping relationship between a nine-linguistic-term scale and its corresponding trapezoidal fuzzy numbers. This study employs trapezoidal fuzzy numbers (TFNs) for conducting pairwise comparisons and determining fuzzy weights. The rationale for selecting TFNs lies in their intuitive accessibility for decision-makers, making calculations straightforward. Furthermore, utilizing TFNs has demonstrated efficacy in addressing decision-making scenarios where the available information is subjective and lacks precision.

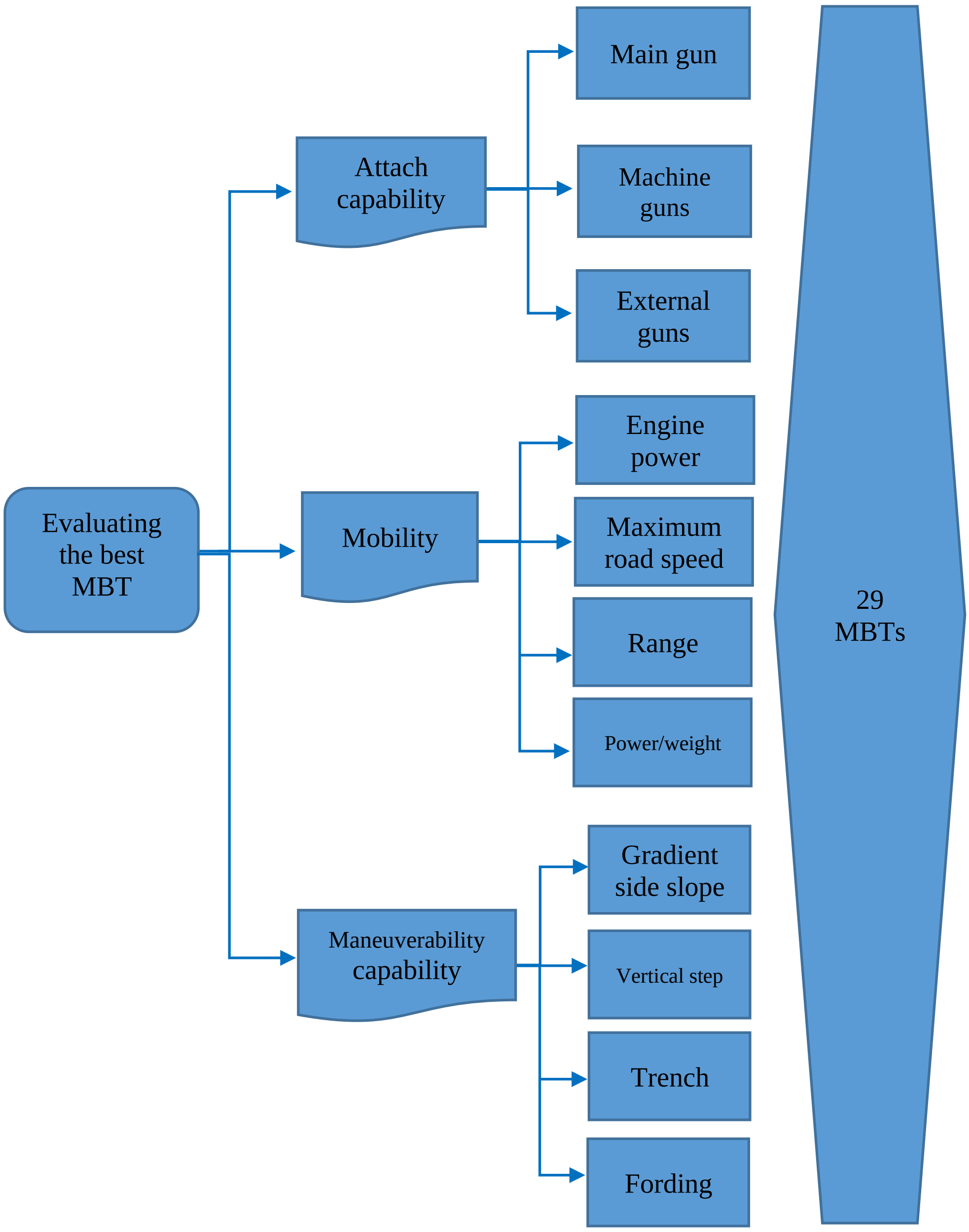

Figure 1: Schematic diagram of the proposed model

2.2. *Fuzzy analytical hierarchy process (FAHP) methodology*

The Analytic Hierarchy Process (AHP) [19] is a quantitative approach that organizes complex problems involving multiple attributes, stakeholders, and time periods into a hierarchical structure to aid in solution development. A significant benefit of this method is its efficiency in managing various criteria. It is capable of effectively addressing both qualitative and quantitative information. Analytic hierarchy process method has the following properties:

- This approach evaluates the relative significance of a set of activities to address complex decision-making issues.

- This methodology is applicable in scenarios where the information is predominantly clear-cut and precise.
- This method generates and addresses a significantly asymmetrical judgment scale.
- This approach does not account for the uncertainties inherent in the involved processes.
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Table 2. Membership function of fuzzy numbers for relative importance.

Linguistic variable	Positive trapezoidal fuzzy numbers
Very good (VG)	(0.75, 0.9, 0.9, 0.9)
Good (G)	(0.65, 0.7, 0.8, 0.85)
Medium good (MG)	(0.45, 0.6, 0.6, 0.75)
Fair (F)	(0.35, 0.4, 0.5, 0.55)
Medium poor (MP)	(0.15, 0.3, 0.3, 0.45)
Poor (P)	(0.05, 0.1, 0.2, 0.25)
Very poor (VP)	(0, 0, 0, 0.15)

Even though the aim of AHP is to capture the expert's knowledge, the conventional AHP still cannot reect the ambiguity in human thinking style. Therefore, fuzzy AHP (FAHP), a fuzzy extension of AHP, was developed to solve the hierarchical fuzzy problems and many fuzzy AHP methods by various authors are proposed. FAHP model is structured such that the objective is in the first level, criteria and sub-criteria are in the second level and objects are on the third level. In the last step of the first stage, the decision hierarchy is approved by decision-making team [1, 19, 18, 9].

2.3. *ʎ-Fuzzy measure*

This study utilizes the λ-fuzzy measure and fuzzy integral to assess the performance of model-based tests (MBTs) [6]. Key concepts from fuzzy measure theory and fuzzy integrals will be discussed in the subsequent section. Let X = X$(x_1, x_2, \dots, x_{\mathrm{n}})$ be the set of criteria, and let P(X) denote the power set of X or set of all subsets of X. Fuzzy measure g: P(X) $\rightarrow [0; 1]$ is a set function defined on the power set P(X) of satisfying the following properties:

1) $g(\emptyset) = 0 \; and \; g(X) = 1$
2) If $A, B \in P(X) \; and \; A \subseteq B, then \; g(A) \leqq g(B)$
3) If $F_n \in P(X)$ for $n \in \mathbb{N}$ and the sequence $\{F_n\}$ is monotone, then

$$\lim_{n\to\infty} g(F_n) \quad = g(\lim_{n\to\infty} F_n)$$

The ʎ-fuzzy measure g_i (ʎ≩1) is a fuzzy measure with the following property:

$$\forall A, B \in P(X), A \cap B = \emptyset, \; g_\lambda(A \cup B) = g_\lambda(A) + g_\lambda(B) + \lambda g_\lambda(A) g_\lambda(B).$$

It can be noted that g_(xi) for a subset with a single element xi is called a fuzzy density, and can be denoted as $g_i = g_ʎ(x_i)$. ### Revised Text

In other words, the value of g_i corresponds to the weight of x_i. The fuzzy measure can be expressed in the following manner:

$$g(X_\ell) = g(\{x_1, x_2, \dots, x_n\})$$

$$= \sum_{i=1}^{\ell} g_i + \lambda \sum_{i_1=1}^{\ell-1} \sum_{i_2=i_1+1}^{\ell} g_{i_1} g_{i_2} + \cdots + \lambda^{n-1} g_1 g_2 \dots g_\ell$$
$$= \frac{1}{\lambda} [\prod_{i=1}^{\ell} (1 + \lambda g_i) - 1]$$

In the fact $g(X_l) = 1$, it follows that

$$1 + \lambda = \prod_{i=1}^{l} 1 + \lambda g_i \qquad (1)$$

The value of λ can be obtained by the above equation. Using each g_i, the weight of attributes x_i.

Table 3. Attribute performance for evaluating the MBTs

C	Criteria	SC	Sub-criteria
C_1	Attack capability	SC_{11}	Armament (main gun)
		SC_{12}	Armament (machine guns)
		SC_{13}	External guns
C_2	Mobility	SC_{21}	Engine power
		SC_{22}	Maximum road speed
		SC_{23}	Range
		SC_{24}	Power/Weight
C_3	Maneuverability	SC_{31}	Gradient-side slope
		SC_{32}	Vertical step
		SC_{33}	Trench
		SC_{34}	Fording

2.4. *Choquet fuzzy integral*

The Choquet fuzzy integral is a mathematical concept used in the theory of fuzzy sets to generalize the notion of integration [1]. It was introduced by Gustave Choquet as a means to incorporate non-linear interactions between variables and provide a more flexible aggregation method. In traditional integration, you have a function and you integrate it over a certain range or domain. However, in the fuzzy integral, the focus is on combining multiple fuzzy measures or fuzzy sets. The Choquet fuzzy integral takes into account the importance or weight assigned to each fuzzy set or measure. It captures the interaction between these sets and provides a comprehensive way to aggregate information. The integral is defined as a weighted sum, where the weights represent the importance or relevance of the individual fuzzy sets. These weights are assigned based on the degree to which each set contributes to the overall result. Let g denote a fuzzy measure defined on the set X and its power set P(X), where X is a finite set.

Definition 2.1. Let $f: X \to [0,1]$ and, while maintaining generality, assume that $f(x_i)$ is monotonically non-increasing with respect to i, such that

$$f(x_1) \geq f(x_2) \geq \cdots \geq f(x_n).$$

If needed, reassign numbers to the elements in X. In practical terms, f can be interpreted as the performance measure for a specific attribute associated with the model-based test (MBT), whereas g signifies the level of subjective importance assigned to each attribute. The fuzzy integral of f with respect to g yields a comprehensive assessment of the MBT. We can apply the fuzzy Choquet integral as follows:

$$\int f dg = f(x_n)\, g(H_n) + [f(x_{n-1}) - f(x_n)]g(H_{n-1}) + \cdots + [f(x_1) - f(x_2)]g(H_1), \quad (2)$$

where

$H_1 = \{x_1\}.\ H_2 = \{x_1.x_2\}.\ \ldots\ .H_n = \{x_1.x_2.\ldots.x_n\}$

3. The steps of research

The following outlines the research methodology:

Step 1: Develop an evaluation hierarchy system to identify the optimal model-based test (MBT) among the options, considering various criteria. The best MBT will be determined through this framework.

Step 2: Assign weights to evaluation dimensions using trapezoidal fuzzy numbers. Following the hierarchy's construction, the prioritization process assesses the relative importance of criteria and sub-criteria, starting from the second level and proceeding to the lowest level (model-based tests). In the Fuzzy Analytic Hierarchy Process (FAHP), multiple pairwise comparisons utilize a standardized seven-level scale (refer to Table 4).

Table 4. Weights of criteria and sub-criteria.

C	C1	C2	C3	SC_{11}	SC_{12}	SC_{13}	SC_{21}	Sc_{22}	SC_{23}	SC_{24}	SC_{31}	SC_{32}	SC_{33}	SC_{34}
C1	1	1.737	1.862											
C2	0.576	1	1.762											
C3	0.537	0.568	1											
SC_{11}	0.470			1	1.880	1.937								
SC_{12}	0.329			0.532	1	1.837								
SC_{13}	0.201			0.516	0.544	1								
SC_{21}		0.306					1	1.427	1.622	1.067				
SC_{22}		0.280					0.701	1	1.662	1.320				
SC_{23}		0.200					0.617	0.602	1	1.130				
SC_{24}		0.214					0.937	0.758	0.885	1				
SC_{31}			0.347								1	1.710	1.660	1.520
SC_{32}			0.251								0.585	1	1.557	1.110
SC_{33}			0.197								0.602	0.642	1	1.100
SC_{34}			0.205								0.658	0.901	0.909	1
Goal	0.458	0.332	0.210	0.215	0.151	0.092	0.102	0.093	0.066	0.071	0.073	0.053	0.041	0.043

Step 3: Construct a sub-network for each criterion based on the standardized comparison scale of seven levels (Table 1). The experts are asked to pairwise compare the importance of the of the sub-criteria with respect to the same upper-level criterion. The linguistic variables of pairwise comparison of each part of the questionnaire from each expert are transformed into trapezoid fuzzy numbers.
Experts are also asked to determine the performance of each MBT with respect to each sub-criterion by a seven-step scale, as shown in Table 2.
Step 4: geometric average approach to aggregate expert's responses and calculate synthetic trapezoid fuzzy numbers. For instance, the synthetic trapezoid fuzzy number for the relative importance between criterion i and criterion j is calculated as follows:

$$\tilde{r}_{ij} = (\tilde{a}_{ij1}, \tilde{a}_{ij2}, \dots, \tilde{a}_{ijk})^{1/k}$$

where $\tilde{a}_{ijk}$ is the pairwise comparison value between criterion i and j determined by expert k.
Step 5: Calculate aggregated crisp pairwise comparison matrices. Defuzzify each fuzzy number into a crisp number using Yager [26] ranking method. For example, fuzzy number $\tilde{r}_{ij}$ is defuzified into a crisp number r_{ij} as follows:

$$r_{ij} = \int_0^1 \frac{1}{2}\left(\left(\tilde{r}_{ij}\right)_\alpha^L + \left(\tilde{r}_{ij}\right)_\alpha^U\right) d\alpha.$$

The α-cuts ($\alpha \in [0.1]$) of the fuzzy numbers $\tilde{a} = (a_1.a_2.a_3.a_4)$ is expressed as:

$$\tilde{a}_\alpha^L = a_1 + (a_2 - a_1)\alpha \quad \tilde{a}_\alpha^U = a_4 - (a_4 - a_3)\alpha$$

The aggregated pairwise comparison matrices for the sub-criteria with respect to the same upper-level criterion are obtained in the same way.
Step 6: The weight for each criterion is determined. This is done by normalizing the comparison matrix A. The relative weights are given by the right eigenvector (w) corresponding to the largest eigenvalue λ_{max}, as $A.w = \lambda_{max}.w$ .If the pairwise comparisons are completely consistent, the matrix A has rank 1 and $\lambda_{max} = n$. In this case, weights can be obtained by normalizing any of the rows or columns of A.
Step 7: Examine the consistency property of the aggregated comparison matrices. The consistency index (CI) and consistency ratio (CR) are defined as

$$CI = (\lambda_{max} - n)/(n-1)$$
$$CR = CI/RI$$

If the value of Consistency Ratio is smaller than or equal to 10%, the inconsistency is acceptable. If the Consistency Ratio is greater than 10%, we need to revise the subjective judgment.
Step 8: Determine the priorities of the MBTs with respect to each sub-criterion. Based on the collected expert's opinions, the membership function of fuzzy numbers for ranking on Table 2, the synthetic trapezoid fuzzy number for the expected performance of a MBT is calculated as follows:

$$\bar{g}_{i_p v} = (\bar{f}_{i_p v1}.\bar{f}_{i_p v2}.\dots.\bar{f}_{i_p vk})^{1/k}$$

where $\bar{f}_{i_p vk}$ is the expected performance of the MBT v under sub-criterion p of criterion i determined by expert k. Defuzzify each fuzzy number into a crisp number using Yager ranking method, and normalize the priorities of the MBTs with respect to each sub-criterion.
Step 9: Find the MBTs performance using fuzzy integrals. This is the main step of work. In this step, we first calculate λ using Equation (1). The weights of attribute performance

calculated in the previous steps are utilized and _finally, using Equation (2.3), the fuzzy indicator of each MBT is computed.

4. Evaluating the MBTs

The case study involves the selection of best MBT using the FAHP and fuzzy Choquet integral and consists of three basic stages: (1) identify the criteria to be used in the model, (2) FAHP computations, (3) evaluation of MBTs with fuzzy integrals and determination of the final rank. In the first stage, MBTs and the criteria which will be used in their evaluation are determined and the decision hierarchy is formed.

Upon confirmation of the decision hierarchy, the next step involves assigning weights to the criteria and sub-criteria utilized for system evaluation through the Fuzzy Analytic Hierarchy Process (FAHP). During this phase, pairwise comparison matrices are developed to establish the weights of the criteria. Five experts from the decision-making team carry out individual assessments using the scale outlined in Table 1 to derive values for the elements within the pairwise comparison matrices. The geometric mean of these values is calculated to create a consensus-based pairwise comparison matrix (see Table 4). For instance,

$$\tilde{a}_{12} = ((1.5\times2.5\times1.5\times2.5\times2.5)^{(1/5)}, (2.5\times3.25\times2.5\times3.25\times3.25)^{(1/5)},$$
$$(2.5\times4.25\times2.5\times4.25\times4.25)^{(1/5)}, (3.5\times5\times3.5\times5\times5)^{(1/5)})$$
$$=(1.60, 1.71, 1.78, 1.86).$$

The weights of the criteria are calculated based on this final comparison matrix. In the last step of this phase, calculated weights of the criteria are approved by decision making team. The weights were determined utilizing the FAHP (Fuzzy Analytic Hierarchy Process) methodology. As illustrated in Table 4, the weight values indicate that the most significant performance sub-criteria for assessing the Main Battle Tanks (MBTs) are armament (main gun) with a weight of 0.215, followed by armament (machine guns) at 0.151. Furthermore, the maximum eigenvalue (λ_max) was computed to be 3.0377, resulting in a consistency index (CI) of 0.0188. The consistency ratio derived from the pairwise comparison matrix was calculated as 0.0325, which is less than the threshold of 0.1. Therefore, it can be concluded that the weights exhibit a satisfactory level of consistency and are applicable in the evaluation process. The value of λ by Equation (2:2) is equal to 0:02305. Based on Choquet fuzzy integral, the best MBT among 29 MBTs is M3 with the fuzzy indicator 0:5321 (Table 5).

5. Conclusion

This research introduces a scientific framework for evaluating Model-Based Testing (MBT) methodologies by employing trapezoidal fuzzy numbers to depict linguistic values, which are reflective of the subjective assessments made by evaluators. A fuzzy multiple criteria decision-making technique is utilized to consolidate the collective decision-making process. Specifically, the Fuzzy Analytic Hierarchy Process (FAHP) identifies the preference weights associated with the evaluation metrics. Subsequently, these weights are integrated into Choquet fuzzy integrals to address the discrepancies between actual performance metrics and the desired levels across various criteria. This approach ultimately identifies the optimal MBT that aligns with the desired standards among a set of 29 different MBTs.

It is important to emphasize that we do not utilize direct scores provided by experts; instead, we construct membership functions based on data reflecting the performance of Model-Based Testing (MBT) methodologies to derive grade values. These grade values serve as representations of performance scores. The proposed model aims to enhance rationality and transparency in defense expenditure by reinforcing the justification for military procurement decisions. Furthermore, with minor adaptations, this model can be applied to various other decision-making scenarios. Additionally, the integration of mathematical models with this framework is planned, which has the potential to enhance the methodologies and represents a key avenue for our future research endeavors.

Table 5. Weighted evaluation for the MBTs based on Choquet fuzzy integrals.

F1	MBT	SC_{11}	SC_{12}	SC_{13}	SC_{21}	Sc_{22}	SC_{23}	SC_{24}	SC_{31}	SC_{32}	SC_{33}	SC_{34}
0.4741	M1	0.60	0.43	0.00	0.66	0.56	0.40	0.48	0.41	0.60	0.49	0.44
0.4891	M2	0.57	0.46	0.00	0.66	0.60	0.32	0.73	0.41	0.60	0.49	0.44
0.5321	M3	0.57	0.75	0.19	0.66	0.54	0.31	0.72	0.45	0.51	0.38	0.35
0.4373	M4	0.60	0.31	0.00	0.51	0.52	0.46	0.47	0.41	0.51	0.49	0.47
0.4246	M5	0.47	0.22	0.31	0.42	0.60	0.35	0.49	0.45	0.51	0.44	0.64
0.4728	M6	0.53	0.35	0.31	0.66	0.72	0.28	0.47	0.45	0.32	0.49	0.54
0.5052	M7	0.67	0.56	0.57	0.42	0.45	0.40	0.24	0.45	0.51	0.49	0.36
0.4268	M8	0.49	0.35	0.31	0.42	0.52	0.35	0.47	0.45	0.32	0.49	0.54
0.3959	M9	0.47	0.35	0.00	0.23	0.60	0.61	0.36	0.44	0.38	0.42	0.64
0.3545	M10	0.37	0.39	0.00	0.23	0.47	0.61	0.26	0.45	0.35	0.38	0.54
0.4473	M11	0.53	0.51	0.00	0.27	0.60	0.46	0.45	0.45	0.60	0.49	0.35
0.4533	M12	0.43	0.42	0.31	0.51	0.54	0.40	0.55	0.45	0.62	0.38	0.50
0.4574	M13	0.50	0.48	0.00	0.30	0.60	0.74	0.57	0.41	0.51	0.56	0.49
0.4454	M14	0.51	0.47	0.00	0.42	0.52	0.45	0.55	0.45	0.51	0.44	0.64
0.4933	M15	0.60	0.43	0.00	0.66	0.63	0.40	0.47	0.41	0.60	0.49	0.44
0.3894	M16	0.46	0.45	0.38	0.23	0.45	0.37	0.28	0.41	0.38	0.35	0.31
0.4867	M17	0.56	0.45	0.38	0.58	0.63	0.35	0.46	0.53	0.43	0.29	0.40
0.4611	M18	0.48	0.42	0.31	0.42	0.60	0.32	0.70	0.45	0.38	0.49	0.58
0.4275	M19	0.48	0.34	0.31	0.33	0.63	0.46	0.44	0.45	0.32	0.44	0.54
0.4704	M20	0.54	0.32	0.31	0.47	0.60	0.28	0.73	0.45	0.51	0.44	0.64
0.4375	M21	0.49	0.35	0.33	0.33	0.60	0.40	0.57	0.45	0.38	0.42	0.54
0.4804	M22	0.61	0.45	0.00	0.58	0.60	0.40	0.77	0.41	0.51	0.31	0.36
0.4248	M23	0.34	0.36	0.31	0.42	0.45	0.65	0.47	0.51	0.51	0.42	0.64
0.4466	M24	0.49	0.32	0.31	0.42	0.52	0.45	0.55	0.45	0.51	0.44	0.64
0.3581	M25	0.42	0.31	0.00	0.23	0.52	0.52	0.35	0.45	0.32	0.38	0.58
0.4823	M26	0.61	0.28	0.00	0.66	0.63	0.46	0.74	0.41	0.60	0.49	0.44
0.4403	M27	0.60	0.63	0.19	0.42	0.34	0.35	0.28	0.41	0.43	0.35	0.31
0.4398	M28	0.63	0.53	0.00	0.39	0.65	0.40	0.26	0.41	0.43	0.35	0.31
0.4538	M29	0.48	0.37	0.25	0.42	0.63	0.61	0.74	0.41	0.23	0.27	0.56

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