*International Journal of*
*Mathematical Modelling & Computations*
Vol. 10, No. 02, Spring 2020, 95- 101

# The Position of Multiobjective Programming Methods in Fuzzy Data Envelopment Analysis

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**Abstract.** Traditional Data Envelopment Analysis (DEA) models evaluate the efficiency of decision making units (DMUs) with common crisp input and output data. However, the data in real applications are often imprecise or ambiguous. This paper transforms fuzzy fractional DEA model constructed using fuzzy arithmetic, into the conventional crisp model. This transformation is performed considering the goal programming that is one of the Multi Objective Programming (MOP) methods. Therefore, in this research the one linear programming model measures the fuzzy efficiencies of DMUs with fuzzy input and fuzzy output data.

Received: 13 February 2020, Revised: 30 April 2020, Accepted: 20 May 2020.

**Keywords**: Data envelopment analysis, Fuzzy input and fuzzy output data, Fuzzy efficiency, Multiobjective programming, Goal programming.

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## 1. Introduction

Data envelopment analysis (DEA) is a methodology for evaluating the relative efficiency of decision making units (DMUs) where input and output data are precisely known. However, in real world, crisp input and output data may not always be available. Thus, fuzzy DEA models are much more realistic than the commonly used crisp models. Several ways have been recognized to investigate fuzzy data in connection with DEA.

Hosseinzadeh et al. [10] by applying a fuzzy metric and a ranking function obtained from it convert the multiplier fuzzy CCR model to its crisp counterpart. Solving this model yields the optimal solution of fuzzy multiplier model. Hatami-Marbini et al. [8] have provided a taxonomy and review of the fuzzy DEA methods during the past twenty years. They have presented a classification scheme with four primary categories, namely the tolerance approach, the α-level based approach, the fuzzy ranking approach and the possibility approach. Some recent researches in this field are as follows: Chang and Lee [2] have integrated data envelopment analysis (DEA), knapsack formulation and fuzzy set theory to deal with a model for the project selection problem when each project possesses vague input and output data in the selection.

Hsiao et al. [14] have proposed a fuzzy-based SBM model and fuzzy-based super-efficiency SBM model to demonstrate their characteristics theoretically and empirically using the case of 24 commercial banks in Taiwan. Wang and Chin [16] have discussed a "fuzzy expected value approach" for data envelopment analysis (DEA) in

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which fuzzy inputs and fuzzy outputs are first weighted respectively; then their expected values are used to measure the optimistic and pessimistic efficiencies of decision making units (DMUs) in a fuzzy environment. The two efficiencies are finally geometrically averaged for the purposes of ranking and identifying the best performing DMU. Azadeh and Alem [1] have presented a decision making flowchart to choose among DEA, fuzzy DEA and Chance Constraint Data Envelopment Analysis (CCDEA) for the selection of the best DMU under certainty, uncertainty and probabilistic conditions. Hatami-Marbini et al. [9] have integrated the concept of the TOPSIS into a four-phase fuzzy DEA framework based on the theory of displaced ideal to measure the efficiencies of a set of DMUs and to rank them with fuzzy input–output levels. Guo, s [7] fuzzy DEA models evaluate the efficiencies of DMUs with symmetrical L–L fuzzy input and output data. Based on the fuzzy DEA model, an aggregation model for integrating fuzzy attribute values is presented in order to rank objects objectively. Hougaard [13] has provided a simple approximation of (crisp) productivity scores for fuzzy production data in which all calculations can be performed in a spreadsheet and, in any case, do not involve fuzzy programming. Moreover, the entire procedure (with its resulting crisp productivity scores) has an economic interpretation parallel to the original interpretation of the DEA scores. Sanei et al. [15] develops a procedure to measure the efficiencies of DMUs with fuzzy observations. The basic idea is to transform a fuzzy DEA model to the family of conventional crisp DEA models by applying optimistic, intermediate and pessimistic concepts.

Wang et al. [17] have tackled fuzziness in input and output data in DEA with two fuzzy DEA models that are formulated as linear programming (LP) models from the fuzzy arithmetic perspective and can be solved to determine fuzzy efficiencies of a group of DMUs. In their approach, three LP models must be solved for determining the fuzzy efficiency score of each DMU. Therefore, three separate sets of weight corresponding to inputs and outputs are provided for computing the fuzzy efficiency of each DMU. To reduce the computational effort and provide one weight set corresponding to inputs and outputs for measuring the fuzzy efficiency score, in this paper, multiobjective programming (MOP) is used to deal with fuzziness in input and output data in DEA. This new method can evaluate fuzzy efficiencies of DMUs with one linear crisp model. Accordingly, the obtained fuzzy efficiency score from one weight vector is more acceptable and has a better interpretation.

This paper is organized in the following form: In section 2, the fuzzy fractional DEA model for evaluating the efficiency of each DMU is introduced, and some present deficiencies in Wang et al.'s method [17] are discussed. In Section 3, a new method based upon MOP is provided for transforming the fuzzy fractional DEA model into the linear crisp one. Finally, a numerical example and concluding remarks are illustrated in sections 4 and 5 respectively.

## 2. The fractional DEA model for measuring the efficiencies of DMUs with fuzzy numbers

Consider $n$ DMUs to be evaluated, each with $m$ inputs and $s$ outputs. Suppose $\tilde{x}_{ij}$ ($i = 1,\dots,m$) and $\tilde{y}_{rj}$ ($r = 1,\dots,s$) are the input and output data of DMU$_j$ ($j = 1,\dots,n$), where $\tilde{x}_{ij} = (x_{ij}^L, x_{ij}^M, x_{ij}^U)$ and $\tilde{y}_{rj} = (y_{rj}^L, y_{rj}^M, y_{rj}^U)$ are positive triangular fuzzy numbers. Therefore, $x_{ij}^L > 0$ and $y_{rj}^L > 0$ for $i = 1,\dots,m$ , $r = 1,\dots,s$ and $j = 1,\dots,n$ and the membership functions of $\tilde{x}_{ij}$ and $\tilde{y}_{rj}$ are respectively defined as:

$$\mu_{\tilde{x}_{ij}}(x)=\begin{cases}\dfrac{x-x_{ij}^L}{x_{ij}^M-x_{ij}^L}, x_{ij}^L\le x\le x_{ij}^M,\\ \\ \dfrac{x_{ij}^U-x}{x_{ij}^U-x_{ij}^M}, x_{ij}^M\le x\le x_{ij}^U,\\ \\ 0, Otherwise.\end{cases}\qquad \mu_{\tilde{y}_{rj}}(y)=\begin{cases}\dfrac{y-y_{rj}^L}{y_{rj}^M-y_{rj}^L}, y_{rj}^L\le y\le y_{rj}^M,\\ \\ \dfrac{y_{rj}^U-y}{y_{rj}^U-y_{rj}^M}, y_{rj}^M\le y\le y_{rj}^U,\\ \\ 0, Otherwise.\end{cases}$$

Assume that $\tilde{x}_{ij}=(x_{ij}^L,x_{ij}^M,x_{ij}^U)$ and $\tilde{x}_{kj}=(x_{kj}^L,x_{kj}^M,x_{kj}^U)$, the fuzzy arithmetic operations on $\tilde{x}_{ij}$ and $\tilde{x}_{kj}$ are defined as follows:

$$\tilde{x}_{ij}+\tilde{x}_{kj}=\left(x_{ij}^L+x_{kj}^L, x_{ij}^M+x_{kj}^M, x_{ij}^U+x_{kj}^U\right),$$
$$\tilde{x}_{ij}-\tilde{x}_{kj}=\left(x_{ij}^L-x_{kj}^U, x_{ij}^M-x_{kj}^M, x_{ij}^U-x_{kj}^L\right),$$
$$\tilde{x}_{ij}\times\tilde{x}_{kj}=\left(x_{ij}^Lx_{kj}^L, x_{ij}^Mx_{kj}^M, x_{ij}^Ux_{kj}^U\right),$$
$$\frac{\tilde{x}_{ij}}{\tilde{x}_{kj}}=\left(\frac{x_{ij}^L}{x_{kj}^U},\frac{x_{ij}^M}{x_{kj}^M},\frac{x_{ij}^U}{x_{kj}^L}\right)$$

Considering the above relations, the following fuzzy DEA model that has been proposed by Wang et al. [17], measures the fuzzy efficiency of a DMU (denoted by DMU$_o$):

$$\begin{aligned}&\text{Max}\quad \tilde{\theta}_o\approx[\theta_o^L,\theta_o^M,\theta_o^U]=\left[\frac{\sum_{r=1}^s u_r y_{ro}^L}{\sum_{i=1}^m v_i x_{io}^U},\frac{\sum_{r=1}^s u_r y_{ro}^M}{\sum_{i=1}^m v_i x_{io}^M},\frac{\sum_{r=1}^s u_r y_{ro}^U}{\sum_{i=1}^m v_i x_{io}^L}\right]\\ &\text{s.t.}\quad \tilde{\theta}_j\approx[\theta_j^L,\theta_j^M,\theta_j^U]=[\frac{\sum_{r=1}^s u_r y_{rj}^L}{\sum_{i=1}^m v_i x_{ij}^U},\frac{\sum_{r=1}^s u_r y_{rj}^M}{\sum_{i=1}^m v_i x_{ij}^M},\frac{\sum_{r=1}^s u_r y_{rj}^U}{\sum_{i=1}^m v_i x_{ij}^L}]\le 1,\ j=1,\ldots,n,\\ &\qquad u_r,v_i\ge 0,\ r=1,\ldots,s,\ \ i=1,\ldots,m.\end{aligned}\tag{1}$$

They simplify the former model as:

$$\begin{aligned}&\text{Max}\quad \tilde{\theta}_o\approx[\theta_o^L,\theta_o^M,\theta_o^U]=[\frac{\sum_{r=1}^s u_r y_{ro}^L}{\sum_{i=1}^m v_i x_{io}^U},\frac{\sum_{r=1}^s u_r y_{ro}^M}{\sum_{i=1}^m v_i x_{io}^M},\frac{\sum_{r=1}^s u_r y_{ro}^U}{\sum_{i=1}^m v_i x_{io}^L}]\\ &\text{s.t.}\quad \theta_j^U=\frac{\sum_{r=1}^s u_r y_{rj}^U}{\sum_{i=1}^m v_i x_{ij}^L}\le 1,\ j=1,\ldots,n,\\ &\qquad u_r,v_i\ge 0,\ r=1,\ldots,s,\ i=1,\ldots,m.\end{aligned}\tag{2}$$

Wang et al. [17] have calculated the best possible values of $\theta_o^L$, $\theta_o^M$ and $\theta_o^U$ by the following three fractional programming models:

$$\begin{aligned}&\text{Max}\quad \theta_o^L=\frac{\sum_{r=1}^s u_r y_{ro}^L}{\sum_{i=1}^m v_i x_{io}^U}\\ &\text{s.t.}\quad \theta_j^U=\frac{\sum_{r=1}^s u_r y_{rj}^U}{\sum_{i=1}^m v_i x_{ij}^L}\le 1,\ j=1,\ldots,n,\\ &\qquad u_r,v_i\ge 0,\ \ r=1,\ldots,s,\ \ i=1,\ldots,m.\end{aligned}\tag{3}$$

$$\begin{aligned}&\text{Max}\quad \theta_o^M=\frac{\sum_{r=1}^s u_r y_{ro}^M}{\sum_{i=1}^m v_i x_{io}^M}\\ &\text{s.t.}\quad \theta_j^U=\frac{\sum_{r=1}^s u_r y_{rj}^U}{\sum_{i=1}^m v_i x_{ij}^L}\le 1,\ j=1,\ldots,n,\end{aligned}\tag{4}$$

$$u_r, v_i \geq 0, \quad r = 1,\dots,s\,, \quad i = 1,\dots,m.$$

$$\begin{aligned} \text{Max} \quad & \theta_o^U = \frac{\sum_{r=1}^{s} u_r y_{ro}^U}{\sum_{i=1}^{m} v_i x_{io}^L} \\ \text{s.t.} \quad & \theta_j^U = \frac{\sum_{r=1}^{s} u_r y_{rj}^U}{\sum_{i=1}^{m} v_i x_{ij}^L} \leq 1,\ j = 1,\dots,n, \\ & u_r, v_i \geq 0, \quad r = 1,\dots,s, \quad i = 1,\dots,m. \end{aligned} \tag{5}$$

By transforming the above models into three linear programming (LP) models, they have obtained the best fuzzy efficiency of $DMU_o$.

To describe the problem in their approach, one more time, we consider Wang, Luo and Liang's simple numerical illustration where eight manufacturing enterprises (DMUs) with two inputs and two outputs are assumed. The Corresponding data set is given in Table 1.

Note that model (2) is a multiobjective problem, but by converting it to separate fractional problems and solving linear models for $DMU_A$, the fuzzy efficiency is obtained as $(\theta_A^L, \theta_A^M, \theta_A^U) =$ (0.8124, 0.9033, 1.0000) with three weight sets as listed in Table 2.

Table 1. DMUs' data (extracted from [17]).

| DMUs | Input1 | Input2 | Output1 | Output2 |
|---|---|---|---|---|
| A | (2120,2170,2210) | 1870 | (14500, 14790, 14860) | (3.1, 4.1, 4.9) |
| B | (1420,1460,1500) | 1340 | (12470,12720,12790) | (1.2, 2.1, 3.0) |
| C | (2510,2570,2610) | 2360 | (17900,18260,18400) | (3.3, 4.3, 5.0) |
| D | (2300,2350,2400) | 2020 | (14970,15270,15400) | (2.7, 3.7, 4.6) |
| E | (1480,1520,1560) | 1550 | (13980,14260,14330) | (1.0,1.8,2.7) |
| F | (1990,2030,2100) | 1760 | (14030,14310,14400) | (1.6,2.6,3.6) |
| G | (2200,2260,2300) | 1980 | (16540,16870,17000) | (2.4,3.4,4.4) |
| H | (2400,2460,2520) | 2250 | (17600,17960,18100) | (2.6,3.6,4.6) |

Table 2. Fuzzy efficiency and optimal weights

| $\tilde{\theta}$ | $(u_1^*, u_2^*, v_1^*, v_2^*)$ |
|---|---|
| $\theta_A^L$=0.812383 | (0.000056,0,0,0.000535) |
| $\theta_A^M$=0.903316 | (0.000028,0.118382,0,0.000535) |
| $\theta_A^U$=1.000000 | (0,0.204082,0.000472,0) |

In multiobjective programming, the optimal solution of each model is called the ideal solution for its corresponding objective function. Therefore, the ideal points for models (3), (4) and (5) define the maximum feasible value of $\theta_A^L, \theta_A^M$ and $\theta_A^U$ , respectively. However, each ideal point does not optimize all objective functions simultaneously.

In the next section, for removing this ambiguity, a new method for measuring the fuzzy efficiency of each DMU, considering Goal Programming (GP), is proposed.

## 3. Application of interactive goal programming (IGP) for investigating the fuzzy DEA

Since formulation (1) is equivalent to a multiobjective problem, an interactive method can be used to solve it. Interactive multiobjective programming methods constitute techniques that allow the Decision Maker (DM) to search for different efficient solutions, so that the DM can reach the most preferred solution or at least to a good solution, in the sense that it is acceptable by the DM. Therefore, interactive methods are powerful tools for solving multiobjective programming problems (see [5,6,11,12,18,19]). There are several various interactive methods. It is evident that the selection of the interactive method would indeed influence the final solution.

For determining the efficient solutions in model (1), a new method is proposed. This method is based upon the goal programming (GP). By means of minimizing deviations from the set goals provided by the DM, a preferred solution is obtained. GP method has been widely used in many multiobjective programming problems (see [3]).

First, by using the Charnes-Cooper transformation for the third objective function (see Charnes and Cooper [4]), the model (2) can be transformed to the following program:

$$\begin{aligned} \text{Max} \quad & \left[\frac{\sum_{r=1}^{s} u_r y_{ro}^{L}}{\sum_{i=1}^{m} v_i x_{io}^{U}}, \frac{\sum_{r=1}^{s} u_r y_{ro}^{M}}{\sum_{i=1}^{m} v_i x_{io}^{M}}, \sum_{r=1}^{s} u_r y_{ro}^{U}\right] \\ \text{s.t.} \quad & \sum_{r=1}^{s} u_r y_{rj}^{U} - \sum_{i=1}^{m} v_i x_{ij}^{L} \le 0, \quad j = 1,\dots,n, \\ & \sum_{i=1}^{m} v_i x_{io}^{L} = 1 \\ & u_r, v_i \ge 0, \quad r = 1,\dots,s, \ i = 1,\dots,m. \end{aligned} \tag{6}$$

Then a simple GP formation for the three objective functions is applied. Therefore, there are:

$$\begin{aligned} \text{Min} \quad & \Delta_1 + \Delta_2 + \Delta_3 \\ \text{s.t.} \quad & \sum_{r=1}^{s} u_r y_{ro}^{L} - \sum_{i=1}^{m} v_i x_{io}^{U} + \Delta_1 = 0 \\ & \sum_{r=1}^{s} u_r y_{ro}^{M} - \sum_{i=1}^{m} v_i x_{io}^{M} + \Delta_2 = 0 \\ & \sum_{r=1}^{s} u_r y_{ro}^{U} + \Delta_3 = 1 \\ & \sum_{r=1}^{s} u_r y_{rj}^{U} - \sum_{i=1}^{m} v_i x_{ij}^{L} \le 0, \ j = 1,\dots,n, \\ & \sum_{i=1}^{m} v_i x_{io}^{L} = 1, \\ & \Delta_1 - \Delta_2 \ge 0, \\ & \Delta_2 - \Delta_3 \ge 0, \\ & u_r, v_i \ge 0, \quad r = 1,\dots,s, \ i = 1,\dots,m, \\ & \Delta_1, \Delta_2, \Delta_3 \ge 0. \end{aligned} \tag{7}$$

Where the goal of each objective that the DM wishes to attain is one, and $\Delta_j$, $j$=1,2,3 are under-achievement of the $j$th goal. The last two constraints reflect this fact: $\theta_o^L \le \theta_o^M \le \theta_o^U$.

To sum up, a new method in fuzzy efficiency evaluation from the perspective of MOLP is employed. For each $DMU_o$, a linear model is solved to obtain weights that optimize all objective functions simultaneously; therefore, the fuzzy efficiency of $DMU_o$ is achieved.

## 4. Numerical example

Model (7) is applied to the data set used in Wang et al. [17]. Table 1 presents the input and output data for the eight manufacturing enterprises. Table 3 reports the results from model (7), i.e., the fuzzy efficiency and optimal weights of each DMU.

As it can be seen, the efficiency of each DMU is positive triangular fuzzy number.

Furthermore, this fuzzy efficiency has been obtained from one optimal set of weight.

Table 3. The obtained results from solving model (7).

| DMUs | $(\theta^L, \theta^M, \theta^U)$ | $(u_1^*, u_2^*, v_1^*, v_2^*)$ |
|---|---|---|
| A | (0.772637,0.899082,0.995704) | (0.000028,0.118382,0,0.000535) |
| B | (0.973010,0.992517,0.997979) | (0.000078,0,0,0.000746) |
| C | (0.787096,0.802926,0.809082) | (0.000044,0,0,0.000424) |
| D | (0.685187,0.802591,0.904614) | (0.000026,0.109592,0,0.000495) |
| E | (0.960202,0.987183,0.999941) | (0.000070,0,0.000199,0.000455) |
| F | (0.842070,0.858875,0.864277) | (0.000060,0,0,0.000568) |
| G | (0.876708,0.894199,0.901090) | (0.000053,0,0,0.000505) |
| H | (0.828028,0.844965,0.851552) | (0.000047,0,0,0.000444) |

## 5. Conclusion

The main purpose of this paper is investigating the DEA models in fuzzy environment to provide their extended applications. To deal with this problem, the goal programming method which is the method in multiobjective linear programming (MOLP) has been applied. Accordingly, the fuzzy efficiency assessment can be performed by a linear model. The new method is illustrated with a numerical example which has been used in Wang, Luo and Liang's article. Wang et al. [17] have made a significant contribution by using a fuzzy ranking approach to compare and rank the fuzzy efficiencies of DMUs. But their approach has some deficiencies such as solving three LP models and providing three sets of weights corresponding inputs and outputs for evaluating DMUs. Therefore, the proposed approach in this study which is based on GP method eliminates these deficiencies.

Investigating the other fuzzy DEA models from the view point of MOLP can be the base for the future research issues. It is possible that other researches to be found comparing the different fuzzy DEA models for evaluating DMUs. Finally, it is hoped that this study makes a small contribution to fuzzy DEA.

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