Probability Distribution Fitting to Maternal Mortality in Nigeria

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Abstract. The consequences of Maternal Mortality (MM) cannot be overemphasized. It inhibits population growth resulting into loss of lives among others. This work tends to obtain the maternal mortality rates (MMR) in Nigeria, identify some fitted distributions to MMR and determine which of the distributions best fits the data. A comprehensive Exploratory Data Analysis (EDA) was carried on MM and the MMRs were obtained. Two parameters Gamma distribution, Weibull and Exponential distributions were fitted for MM. Both BIC and AIC selection criteria were adopted in selecting the most fitted distribution. The AIC for Gamma, Weibull and Exponential distributions fitted for MMR were 1339.396, 1340.161 and 370.5244 respectively. Also, the BIC for Gamma, Weibull and Exponential distributions fitted for MM were 1344.971, 1345.736 and 373.3119 respectively. It can be observed that Exponential distribution has the least AIC (370.5244) and least BIC (373.3119), therefore, it is the most fitted distribution for MM. The estimate (standard error) of exponential distribution on MM is 0.5853 indicating the fitness of the distribution being the one with the least error. Conclusively, the model obtained from this study can be used to study and monitor MM in Nigeria to achieve a better economy and thus brings about local and national development.

Index to information contained in this paper

1. Introduction

The joy of every woman is to get pregnant and give birth to a bouncing baby thereby bringing happiness to the family as a whole. This supposed to be a normal hitch-free physiological process from conception to birth in an ideal society. Most often, the converse is the case in some developing countries of the world like Nigeria. The situation has even worsen to cases where woman are often frightened and scared with conceiving and procreation due to the increase in maternal mortality in developing countries. Over five hundred thousand women and girls die yearly due to pregnancy, childbirths and other related complications (United States Central Intelligence Agency [CIA], 2016; World Health Organization [WHO], 2010). Furthermore, Nigeria has a relatively high HIV prevalence (3.1 percent) and a very low contraceptive prevalence rate (14.1 percent) (CIA, 2016). According to WHO, One hundred and forty-five Nigerian women die in childbirth every day; one every 10 minutes. Determining the actual number of maternal mortality

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might be so difficult in Nigeria depending on the method of measurement. Meanwhile, WHO, UNICEF, UNFPA, and World Bank estimates are closer to 630 per 100,000 (WHO, 2017).

Maternal mortality is defined as the loss of life of a woman while pregnant or within 42 days of termination of pregnancy, irrespective of the site and duration of the pregnancy, from any cause traced or related to the pregnancy or its management but not from accidental or incidental causes. Nigeria has one of the highest maternal mortality rates in the world, second only to India whose population is eight times larger than that of Nigeria (WHO, 2008)

In developing countries today, maternal death or simply called maternal mortality has been identified as one of the major causes of death among women of reproductive age and also remains one of the serious public health issues (WHO, 2008). The causes of maternal deaths can be economical, social, medical and at times political in nature. Fawole et al, (2012) and Oye- Adeniran (2014), identified anemia, HIV infection, malaria and age over thirty five at pregnancy as determinant of high-risk pregnancy that can result into high maternal mortality.

Political intervention is of great importance in reducing maternal mortality in a nation. A need for adjustment of government policies towards improving the health sector as been cited by many researchers on maternal mortality in Nigeria (Adegoke, 2013, Fawole et al, 2012, Ijadunola et al, 2010), Onakewhor, 2011). Bazuaye and Okonofua (2013) identified the major strategies to reduce maternal deaths. These include, good governance, better economy, stoppage of reliance on donor funding for maternal initiatives and emulating other nations for evidence based interventions.

Mairiga et al. (2008) expressed the view that the world's maternal mortality ratio (the number of maternal deaths per 100,000 live births) is declining too slowly to meet Millennium Development Goal (MDG) 5 target which aimed to reduce the number of women who die in pregnancy and childbirth by three quarters by the year 2015. In the MDG framework, two indicators have been specified for monitoring progress towards the maternal health goal namely, the maternal mortality ratio and the proportion of deliveries with a skilled health care provider. A lot of work has been done on maternal mortality in tertiary health institutions, which have a high selection of complicated cases. In order to obtain a true picture of the epidemiology of maternal mortality in Nigeria, this study was carried out in a tertiary health facility to which primary and secondary health care facilities refer patients.

2. Methodology

The methodology adopted in this study is the probability distribution fitting approach. Two parameters Gamma distribution, Weibull and Exponential distributions were fitted for MM. Both BIC and AIC selection criteria were adopted in selecting the most fitted distribution.

2.1 *Maternal mortality rate*

Maternal mortality rates can be calculated using

$$MMR = \frac{TMM}{TLB} \; X \; 1000 \; Live \; births$$

where, MMR is maternal mortality rate, TMM is total maternal mortality and TLB is the total live births in a given period of time.

2.2 *Exponential distribution*

The exponential distribution is one of the widely used continuous distributions. It is often used to model the time elapsed between events. A continuous random variable X is said to have an exponential distribution with parameter $\lambda > 0$, shown as $x \sim$ Exponential (λ), if its PDF is given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & otherwise \end{cases}$$

where the variable x and the parameter λ are positive real quantities.

The mean and variance of exponential distributions is $\frac{1}{\lambda}$ and $\frac{1}{\lambda^2}$ respectively. Also, using maximum likelihood, the estimator of the parameter λ is given as $\hat{\lambda} = \frac{n}{\sum_{i=1}^{n} x_i}$.

2.2.1 Parameter estimation for exponential distribution

The probability density function of the exponential distribution is defined as

$$f(x, \lambda) = \begin{cases} \lambda e^{-\lambda x}, & x \leq 0 \\ 0, & otherwise \end{cases}$$

Its likelihood function is

$$L(\lambda, x_1, \ldots, x_n) = \prod_{t=1}^{n} f(x_t, \lambda) = \prod_{t=1}^{n} \lambda e^{-\lambda x} = \lambda^n e^{-\lambda \sum_{t=1}^{n} x_t} \tag{1}$$

To calculate the maximum likelihood estimator, we take the log of equation 1 and differentiate with respect to λ and equate to zero.

$$\frac{d ln\big(L(\lambda, x_1, \ldots, x_n)\big)}{d\lambda} = 0$$

for λ

$$\frac{d ln\big(L(\lambda, x_1, \ldots, x_n)\big)}{d\lambda} = \frac{d ln\big(\lambda^n e^{-\lambda \sum_{t=1}^{n} x_t}\big)}{d\lambda} = \frac{n}{\lambda} - \sum_{t=1}^{n} x_t = 0$$

Finally, we get

$$\lambda = \frac{n}{\sum_{t=1}^{n} x_t} \tag{2}$$

2.3 *Weibull distribution*

The Weibull distribution is named for Waloddi Weibull. Weibull was not the first person to use the distribution, but was the first to study it extensively and recognize its wide use in applications.

The probability density function of a Weibull random variable is given as:

$$f(x) = \frac{\alpha}{\beta} \left(\frac{x}{\beta}\right)^{\alpha-1} \exp\left\{-\left(\frac{x}{\beta}\right)^{\alpha}\right\}$$

Using the method of moment or expectation method, the mean and variance of Weibull distribution is given as $\beta \Gamma\left(1 + \frac{1}{\alpha}\right)$ and $\beta^2 \left(\Gamma\left(1 + \frac{2}{\alpha}\right) - \left[\Gamma\left(1 + \frac{1}{\alpha}\right)\right]^2\right)$ respectively.

2.3.1 Parameter estimation for Weibull distribution

$$f_{\beta,\alpha}(x) = \begin{cases} \dfrac{\alpha}{\beta}\left(\dfrac{x_t}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^{\alpha}}, & x \geq 0 \\[2mm] 0, & x < 0 \end{cases}$$

with $\beta, \alpha > 0$.

Likelihood function becomes:

$$\mathcal{L}_{\bar{x}}(\beta,\alpha) = \prod_{i=1}^{nn} f_{\beta,\alpha}(x_i) = \prod_{i=1}^{n} \frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha-1} e^{\left(\frac{x_i}{\beta}\right)^{\alpha}} = \frac{\alpha^n}{\beta^{n\alpha}} e^{-\Sigma_{i=1}^{N}\left(\frac{x_i}{\beta}\right)^{\alpha}} \prod_{i=1}^{n} x_i^{\alpha-1}$$

Log-likelihood function:

$$\ell_{\hat{x}}(\beta,\alpha) := \ln \mathcal{L}_{\bar{x}}(\beta,\alpha) = n\ln\alpha - n\alpha\ln\beta - \sum_{i=1}^{n}\left(\frac{x_i}{\beta}\right)^{\alpha} + (\alpha-1)\sum_{i=1}^{n}\ln x_i$$

Differentiating with respect to α and β, then equating to zero.

The 0-gradient is obtained as:

$$\frac{\partial l}{\partial \beta} = -n\alpha\frac{1}{\beta} + \alpha\sum_{i=1}^{N} x_i^{\alpha}\frac{1}{\beta^{\alpha+1}}$$

$$\frac{\partial l}{\partial \alpha} = \frac{n}{\alpha} - n\ln\beta - \sum_{i=1}^{n}\ln\left(\frac{x_i}{\beta}\right)e^{\alpha\ln\left(\frac{x_i}{\beta}\right)} + \sum_{i=1}^{n}\ln x_i$$

It follows

$$-n\alpha\frac{1}{\beta} + \alpha\sum_{i=1}^{n} x_i^{\alpha}\frac{1}{\beta^{\alpha+1}} = 0$$

$$-\alpha\frac{1}{\beta}n + \alpha\frac{1}{\beta}\sum_{i=1}^{n} x_i^{\alpha}\frac{1}{\beta^{\alpha}} = 0$$

$$-1 + \frac{1}{n}\sum_{i=1}^{N} x_i^{\alpha}\frac{1}{\beta^{\alpha}} = 0$$

$$\frac{1}{n}\sum_{i=1}^{N} x_i^{\alpha} = \beta^{\alpha}$$

Plugging β^* into the second 0-gradient condition, we have

$$\Rightarrow \alpha^* = \left[\frac{\sum_{i=1}^{N} x_i^{\alpha^*}\ln x_i}{\sum_{i=1}^{N} x_i^{\alpha^*}} - \ln x\right]^{-1}$$

This equation is only numerically solvable, e.g. Newton-Raphson algorithm $\hat{\alpha}^*$ can be placed into $\hat{\beta}^*$ to complete the ML estimator for the Weibull distribution.

2.4 *Gamma Distribution*

This is generally known as a distribution frequently used in waiting time modeling. Its Pdf is given as:

$$f(x) = \frac{x^{\alpha-1}e^{-\frac{x}{\beta}}}{\Gamma\alpha\beta^{\alpha}}, \qquad x > 0, \alpha > 0, \beta > 0$$

where the parameters α and β are positive real quantities as is the variable x. Note the parameter α is simply a scale factor.

The mean and variance of gamma distribution is $\alpha\beta$ and $\alpha\beta^2$.

3. Model selection criterion

The selection criterion used in this research is AIC (Alkaike Information Criterion) and BIC (Bayesian Information Criterion).

The correct formula for the AIC for a model with parameters $\beta 0, \beta 1, \cdots, \beta p - 1$ and σ^2 is

$$AIC = -2 \; loglikelihood + 2p$$

and the correct formula for BIC is

$$BIC = n + n log 2\pi + n log(RSS/n) + (logn)(p + 1)$$

When comparing AIC or BIC between two models, however, it makes no difference which formula you use because the differences will be the same regard less which choice you make.

4. Results and discussion

Here we present and discuss the analysis and results obtained. This is broadly divided into two, the descriptive section and the probability modeling section.

Descriptive statistics summarizes the data, Histogram help us to know the pattern of the data and the Box-plot assist in checking for outliers. Lastly, Exponential, Gamma and Weibull distributions were also fitted to Maternal mortality and the best model was selected.

4.1 *Summary of Maternal Mortality Data*

Table 1. Summary of the data.

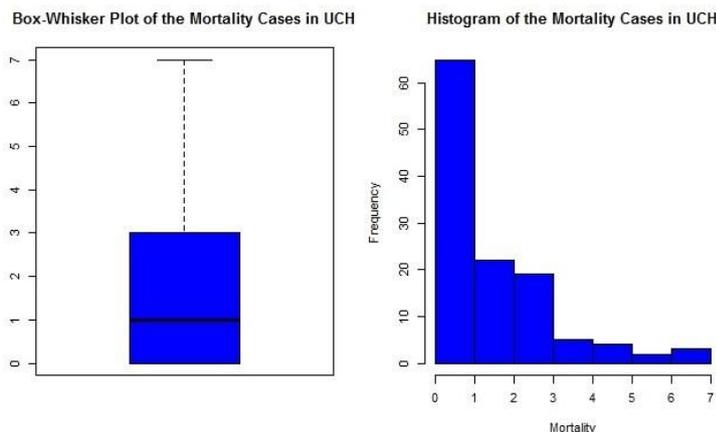| Min.	1st Qu.	Median	3rd Qu	Max
0	0	1	3	7

Figure 1. Box-plot and histogram of the live birth data.

4.2 *Maternal mortality rates*

Table 2. Maternal mortality rate.

Year	Time(t)	TMM	TLB	MMR
2007	1	25	3389	7.3768 =7
2008	2	16	3334	4.7990=5
2009	3	14	3071	4.5588=5
2010	4	18	2790	6.4516 =7
2011	5	24	3005	7.9866=8
2012	6	27	3329	8.1105=8
2013	7	20	3339	5.9898=6
2014	8	14	3448	4.0603=4
2015	9	23	3387	6.7906=7
2016	10	24	3325	7.2180 =7

Source: University College Hospital (UCH), Ibadan.

Table 2 shows the maternal mortality rates in Nigeria. It can be deduced that high maternal mortality rates were recorded in the year 2011 and 2012 with 8 per 1000 live births

4.3 *Distribution for maternal mortality cases*

A two parameters Gamma, Weibull, and Exponential distribution is fitted into the total number maternal mortality cases. The estimated parameters and their standard errors are obtained in preceding subsections.

4.3.1 Two Parameters Gamma Distribution

Using Maximum Likelihood Approach, the estimated parameters are derived from fitted distribution density.

Table 3. Estimated Gamma parameter values with standard errors.

	Shape	Rate
Values	1.6538964	0.01613350
Standard Errors	0.9751826	0.03245623

$$f(x_t, \alpha, \beta) = \frac{x_t^{1.6538964-1} e^{-\frac{x_t}{0.01613350}}}{0.01613350^{1.6538964}\Gamma(1.6538964)}, \qquad x_t > 0.$$

The estimated for shape (α) is 1.6538964 with a standard error of 0.0161335 and the estimated value for rate (β) is 0.0161335 with a standard error of 0.194383987. The standard error or β is quite smaller than that of α.

4.3.2 Weibull Distribution

Adopting a Weibull distribution, the estimated shape and scale standard deviation with their standard error are given below:

Table 4. Estimated Weibull parameter values with standard errors.

	Shape	Scale
Values	1.401204	114.010785
Standard Errors	0.1020607	7.8179819

$$f(x_t, \alpha, \beta) = \frac{1.401204}{x_t} \left(\frac{x_t}{114.010785}\right)^{1.401204} e^{-\left(\frac{x_t}{114.010785}\right)}, \qquad x_t > 0.$$

The estimated for shape (α) is 1.401204 with a standard error of 0.1020607 and the estimated value for scale (β) is 114.010785 with a standard error of 7.8179819. The standard error of shape is smaller than that of scale parameter.

4.3.3 Exponential distribution

Fitting an exponential distribution into the data, the estimated rate parameter is given below:

Table 5. Estimated parameter value with standard error.

	Estimate	Standard Error
Rate	0.5853659	0.05343619

The estimated rate (parameter) occurrences via maximum likelihood method is 0.5853659. Thus, the distribution of the number of live births recorded over the study is given below

$$f(x_t, \lambda) = \frac{1}{0.5853659} e^{\frac{x_t}{0.5853659}}, \qquad x_t > 0.$$

where t is period between year understudy.
The standard error of the parameter is 0.0008682863 which is quite small and it implies that the rate estimate is very close to the parameter.

4.4 *Model selection*

In this section, we compute the **AIC** (Alkaike Information Criterion) and the **BIC** (Bayesian Information Criterion) to select the best model that fits the data. This will be done on the basis of the minimum AIC and BIC.

Table 6. AIC and BIC values of the distributions considered for MMR.

	Gamma	Weibull	Exponential
AIC	1339.396	1340.161	370.5244
BIC	1344.971	1345.736	373.3119

The Table 6 above shows the computation of AIC and BIC of the four distribution used to fit the maternal mortality rates. It reviewed that Exponential Distribution is the appropriate model for the data due to the smallest AIC and BIC when compared the AIC and BIC with one another, with the value of 370.5244 and 373.3119 respectively.

5. Conclusion

The data used in this research contains no outlier. The histogram plot showed that the distribution is also not normal (skewed). The maternal mortality rates were so high in 2011 and 2012 but a decline was noticed in the succeeding years indicating the implementation of some policy to reduce maternal death rates. However, a higher rate was also noticed in 2016 suggesting the failure in some of the policies introduced in the preceding months. Of all the fitted distribution to MMR, exponential distribution has shown to be the best due to its least AIC, BIC and low standard error.

A numerous factor and causes to Maternal death have also been identified in this research work. Adequate care must be given to women during pregnancy and delivery. Implementation of good government policy towards health sector, priority of women's health, administration of quality obstetric care to women and so on are hereby recommended in reducing maternal mortality rates in Nigeria.

Maternal mortality rate is an important factor that affects the national economy, so its control must be put into consideration. Hence, the model obtained from this study can be used to monitor and study Maternal mortality in Nigeria to achieve a better economy and thus brings about local and national development at large.

Acknowledgement

References

[1]   C. Abouzahr and T. Wardlaw, Maternal Mortality in 2000: Estimates Developed by WHO, UNICEF and UNFPA, Geneva, (2003).
[2]   A. A. Adegoke et al., Community study of maternal mortality in south west Nigeria: How applicable is the sisterhood method, Maternal and Child Health Journal, **17 (2)** (2013) 319-329.
[3]   A. Bazuaye and O. Friday, Tackling maternal mortality in Africa after 2015: What should the priorities be?, African Journal of Reproductive Health, **17 (2)** (2013) 9-17.
[4]   A. Fawole, A. Shah and A. Fabanwo et al., Predictors of maternal mortality in institutional deliveries in Nigeria, African Health Sciences, **12 (1)** (2012) 32-40.
[5]   https://www.who.int/news-room/fact-sheets/detail/maternal-mortality (2019).
[6]   K. T. Ijadunola, M. Y. Ijadunola, O. A. Esimai, and T. C. Abiona, New paradigm old thinking: the case for emergency obstetric care in the prevention of maternal mortality in Nigeria, BMC Women's Health, **6 (10)** (2010), doi: 10.1186/1472-6874-10-6.
[7]   A. M. Mairiga, M. B. Kawuwa and A. Kulima, Community perception of maternal mortality in northeastern Nigeria, African Journal of Reproductive Health, **12 (3)** (2008) 27-34.
[8]   J. U. Onakewhor, B. N. Olagbuji, A. B. Ande, M. C. Ezeanochie, O. E. Olokor and F. E. Okonofua, HIV-AIDS related maternal mortality in Benin city, Nigeria, Ghana Medical Journal, **46 (2)** (2011) 54-59.
[9]   B. Oye-Adeniran, O. Kofoworola, A. Gbadegesin, O. Akin-Adenekan, O. Akinsola, E. Ekanem and O. Osilaja, Causes of maternal mortality in Lagos state, Nigeria, Annals of Tropical Medicine and Public Health, **7 (3)** (2014) 177-181.
[10] C. Ronsmans and W. Graham, Maternal mortality: Who, when, where, and why, The Lancet, **368** (2006) 1189-1200.
[11] I. A. O. Ujah, O. A. Aisien, J. T. Mutihir, D. J. Vanderagt, R. H. Glew and V. E. Uguru, Factors contributing to maternal mortality in north-central Nigeria: A seventeen-year review, African Journal Reproduction Health, **9 (3)** (2005) 27-40.
[12] United States Central Intelligence Agency (CIA), World Fact Book [Online]. https://www.cia.gov/library/publications/the-world-factbook/, (Accessed July 29, 2016).
[13] WHO Available at http://www.who.int/reproductivehealth/publications/maternalmortality2000/mme.pdf, (Accessed April 30, 2017).
[14] WHO; UNICEF; UNFPA; The World Bank, Trends in maternal mortality 1990-2008: estimates developed by WHO, UNICEF, UNFPA and The World Bank, Geneva: World Health Organization, (2010).
[15] World Health Organization, Factsheet, Maternal Mortality, Department of Making Pregnancy Safer, (2008).