Center of Mass and its Applications in Ranking

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ABSTRACT

In this paper, a novel method for ranking of fuzzy numbers are proposed. In this method, decision maker is defined based on the center of mass at some $\alpha-$cuts of a pair of fuzzy numbers in discreet version and center of mass on all of $\alpha-$cuts of a pair of fuzzy numbers in continuous version. Our method can rank more than two fuzzy numbers simultaneously. Also, some properties of methods are described. At last, we present some numerical examples to illustrate the proposed method, and compare it with other ranking methods.

Keywords: Fuzzy numbers, Ranking of fuzzy numbers; Center of mass.

1. Introduction

In all area of science, when we want to make a decision, comparison between two or more than two numbers has important roll. On the other hand, since fuzzy logic is more suitable than Euclidean logic for modeling of many of physical applications, we need to find a way for ranking of fuzzy numbers. Since uncertainty and ambiguity are main features of fuzzy logic, different methods for ranking of fuzzy numbers were proposed[1, 2, 3, 6, 8, 9, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 29]. For ranking of fuzzy numbers, a fuzzy number needs to be evaluated and compared with the others, but this may not be easy. Since fuzzy numbers are represented by possibility distributions, they can overlap with each other and it is difficult to determine clearly whether one fuzzy number is larger or smaller than another.

Fuzzy set ranking has been studied by many researchers. Some of these ranking methods have been compared and reviewed by Bortolan and Degain [4]. More recently by Chen and Hwang [5], and it still receives much attention in recent years [7, 13, 22, 23]. Many methods for ranking fuzzy numbers have been proposed, such as representing them with real numbers or using fuzzy relations. Wang and Kerre [23, 15] proposed some axioms as reasonable properties to determine the rationality of a fuzzy ranking method and systematically compared a wide array of existing fuzzy ranking methods. Almost each method, however, has pitfalls in some aspect, such as inconsistency with human intuition, indiscrimination, and difficulty of interpretation. What seems to be clear is that there exists no uniquely best method for comparing fuzzy numbers, and different methods may satisfy different desirable criteria.

In the existing fuzzy number ranking methods, many of them are based on the area measurement with the integral value about the membership function of fuzzy numbers [7, 25, 22, 29, 9, 14, 16, 12, 17, 23, 21, 19]. Yager [10] proposed centroid index ranking method with weighting function. Cheng [6] proposed a centroid index ranking method that calculates the distance of the centroid point of each fuzzy number and original point to improve the ranking method [28]. They also proposed a coefficient of variation (CV index) to improve Lee and Lis method [21]. Recently, Tsu and Tsao [8] pointed out the inconsistent and counter intuition of these two indices and proposed ranking fuzzy numbers with the area between the centroid point and original point. In this paper, a novel method for ranking of fuzzy numbers based on the center of mass at some $\alpha-$cuts of a pair of fuzzy numbers in discreet version and center of mass on all of $\alpha-$cuts of a pair of fuzzy numbers in continuous version is proposed.

The rest of our work is organized as follows. Section 2, contains the basic definitions and notations used in the remaining parts of the paper. In Section 3, introduces the ranking methods and describes some useful properties. In Section 4, solving some examples and compare novel method with other methods. Concluding remarks are finally made in Section 5.

2. Basic definitions and notation

A real fuzzy number can be defined as a fuzzy subset of the real line R, which is convex and normal. That is, for a fuzzy number A of R defined by the membership function $\mu_A(x)$, $x \in R$, The following relations exist:

$$\max_x \mu_A(x) = 1$$

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_A(x_1), \mu_A(x_2)\},$$

where x_1, $x_2 \in R$, $\forall \lambda \in [0,1]$. A fuzzy number A with the membership function $\mu_A(x)$, $x \in R$ the fuzzy number $A = [a,b,c,d;1]$ can be defined as

$$\mu_A(x) = \begin{cases} \mu_A^L(x) & a \le x \le b \\ 1 & b \le x \le c \\ \mu_A^R(x) & c \le x \le d \\ 0 & otherwise, \end{cases}$$

where $\mu_A^L(x)$ is the left membership function that is an increasing function and $\mu_A^L : [a,b] \to [0,1]$. Meanwhile, $\mu_A^R(x)$ is the right membership function that is an decreasing function and $\mu_A^R : [c,d] \to [0,1]$. A fuzzy number is called trapezoidal fuzzy number if $\mu_A^L(x)$ and $\mu_A^R(x)$ are lines. The trapezoidal fuzzy number is called triangular if $b = c$.

For a fuzzy number A, the $\alpha-$cuts $A_\alpha = \{x \in R \mid \mu_A(x) \ge \alpha\} = [l_\alpha, r_\alpha], \alpha \in [0,1]$, are convex subsets of R. This is obvious that l_α and r_α for trapezoidal fuzzy number are

$l_\alpha = a + (b-a)\alpha$

and

$r_\alpha = d - (d-c)\alpha.$

3. Novel method for Ranking of fuzzy numbers

In this section, novel method for ranking of fuzzy numbers based on the center of mass points of a fuzzy number are proposed. Let A_i, $i = 0,\ldots,l$ be a set of fuzzy numbers. To define the center of mass of any A_i. We calculate $x_{i\alpha}$ for the fuzzy number A_i i.e.

$$x_{i\alpha} = \frac{\overline{m}_{A_{i\alpha}}}{m_{A_{i\alpha}}} = \frac{1}{2}(r_{i\alpha} + l_{i\alpha}).$$

where

$$\overline{m}_{A_{i\alpha}} = \int_{l_{i\alpha}}^{r_{i\alpha}} x\alpha dx = \frac{1}{2}\alpha(r_{i\alpha}^2 - l_{i\alpha}^2),$$

and

$$m_{A_{i\alpha}} = \int_{l_{i\alpha}}^{r_{i\alpha}} \alpha dx = \alpha(r_{i\alpha} - l_{i\alpha}).$$

Proposition 3.1 The center of mass point of a trapezoidal fuzzy number is

$$x_{i\alpha} = \frac{1}{2}[(a+d) + ((b+c)-(a+d))\alpha].$$

Proposition 3.2 *The center of mass point of a triangular fuzzy number* (a,b,c) *is*

$$x_{i\alpha} = \frac{1}{2}[(a+c) + (2b-(a+c))\alpha].$$

A decision maker can rank a pair of fuzzy number A_i and A_j by $D(A_i, A_j)$ as follows:

$$D(A_i, A_j) = \int_0^1 (x_{i\alpha} - x_{j\alpha})dx.$$

Proposition 3.3 *A decision maker for trapezoidal fuzzy numbers* $A_i = (a_i, b_i, c_i, d_i)$ *and* $A_j = (a_j, b_j, c_j, d_j)$ *is*

$$D(A_i, A_j) = \frac{1}{4}[(a_i + b_i + c_i + d_i) - (a_j + b_j + c_j + d_j)].$$

Proposition 3.4 *A decision maker for triangular fuzzy numbers* $A_i = (a_i, b_i, c_i)$ *and* $A_j = (a_j, b_j, c_j)$ *is*

$$D(A_i, A_j) = \frac{1}{4}[(a_i + 2b_i + c_i) - (a_j + 2b_j + c_j)].$$

When l_α or r_α are complex, $D(A_i, A_j)$ is calculated with difficulty and hence we introduce the discreet version of $D(A_i, A_j)$ as follows:

The lower and upper limits of the $k^{th}\alpha-$ cut for the fuzzy number A are defined as

$l_k = \inf\{x \in R \mid \mu_A(x) \ge \alpha_k\}$

$r_k = \sup\{x \in R \mid \mu_A(x) \ge \alpha_k\}$

where l_k and r_k are left and right spreads, respectively where $\alpha_k = \frac{k}{n}, k = 0,\ldots,n$. Now, a discreet version of decision maker for ranking of a pair of fuzzy numbers A_i and A_j by $D(A_i, A_j)$ is:

$$D(A_i, A_j) = \frac{1}{n+1}\sum_{k=0}^{n}(x_{ik} - x_{jk})$$

Definition 3.1 *For* A_i *and* $A_j \in E$*, define the ranking of* A_i *and* A_j *by* $D(.,.)$ *on* E*, i.e.*

$D(A_i, A_j) > 0$ if and only if $A_i > A_j$.

$D(A_i, A_j) < 0$ if and only if $A_i < A_j$.

$D(A_i, A_j) = 0$ if and only if $A_i = A_j$.

Proposition 3.5 *For two arbitrary fuzzy numbers* A_i *and* A_j*, it is obvious that:*

$D(A_i, A_i) = 0$.

$D(A_i, A_j) = -D(A_j, A_i)$.

If $D(A_i, A_j) > 0$ and $D(A_j, A_k) > 0$ then $D(A_i, A_k) > 0$
$D(A_i, A_k) = D(A_i, A_j) + D(A_j, A_k)$.
If $D(A_i, A_j) < \varepsilon$ then $|D(A_i, A_k) - D(A_j, A_k)| < \varepsilon$.

We consider the following reasonable properties for the ordering approaches, see [24].

A_1: For an arbitrary finite subset Γ of E and $A \in \Gamma, A \pm A$.

A_2: For an arbitrary finite subset Γ of E and $(A, B) \in \Gamma^2, A \pm B$ and $B \pm A$, we should have $A : B$.

A_3: For an arbitrary finite subset Γ of E and $(A, B, C) \in \Gamma^3, A \pm B$ and $B \pm C$, we should have $A \pm C$.

A_4: For an arbitrary finite subset Γ of E and $(A, B) \in \Gamma^2$, $\inf\{supp(A)\} > \sup\{supp(B)\}$, we should have $A \succ B$.

A_4': For an arbitrary finite subset Γ of E and $(A, B) \in \Gamma^2$, $\inf\{supp(A)\} > \sup\{supp(B)\}$, we should have $A \pm B$.

A_5: Let Γ and Γ' be two arbitrary finite subsets of E also A and B are in $\Gamma \cap \Gamma'$. We obtain the ranking order $A \succ B$ by novel method on Γ' if and only if $A \succ B$ by novel method on Γ.

A_6: Let A, B, $A+C$ and $B+C$ be elements of E. If $A \succ B$, then $A+C \succ B+C$, when $C \neq 0$.

A_6': Let A, B, $A+C$ and $B+C$ be elements of E. If $A \pm B$, then $A+C \pm B+C$. A_7: Let A, B, AC and BC be elements of E and $C \pm 0$. If $A \pm B$, then $AC \pm BC$.

Theorem 3.1 *The novel ranking method* $D(A, B)$ *has the properties* $A_1, A_2, A_3, A_4, A_4', A_5, A_6, A_6'$.

Proof: It is easy to verify that the properties $A_1, A_2, A_3, A_4, A_4', A_5, A_6, A_6'$ are hold.

4. Numerical Examples

Example 4.1 *Consider the following sets.*

Set 1: $A_1 = (0.3, 0.5, 0.7)$, $A_2 = (0.1, 0.2, 0.3)$ (See Fig. 1).
Set 2: $B_1 = (0.7, 0.8, 0.9)$, $B_2 = (0, 0.4, 1)$ (See Fig. 2).
Set 3: $C_1 = (0.4, 0.6, 0.8)$, $C_2 = (0.3, 0.6, 0.8)$ (See Fig. 3).
Set 4: $G_1 = (0.3, 0.5, 0.7)$, $G_2 = (0.1, 0.5, 0.9)$ (See Fig. 4).

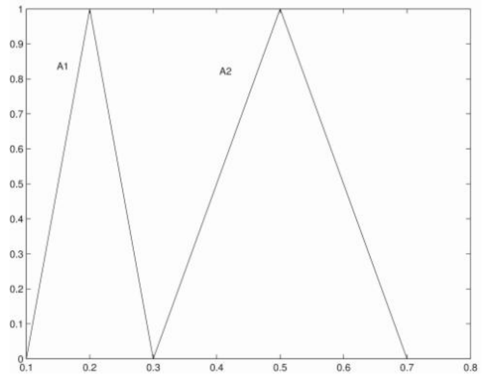

Figure 1. Set 1.

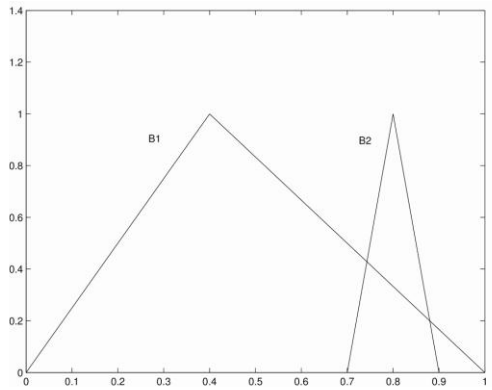

Figure 2. Set 2.

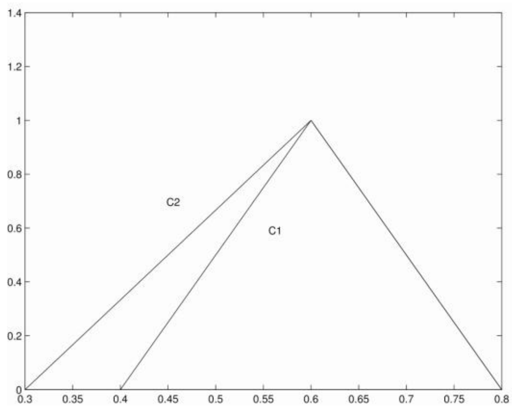

Figure 3. Set 3.

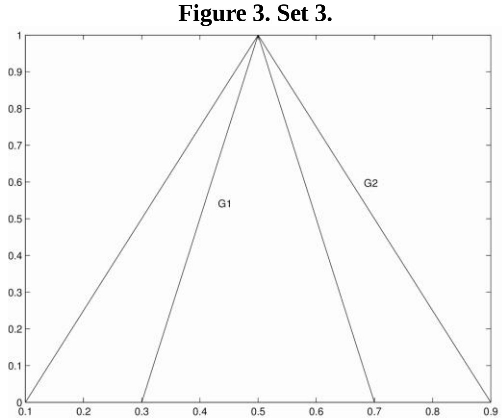

Figure 4. Set 4.

By using novel method we obtain the following results:

Table 1. results of eample 4.1

set 1	set 2	set 3	set 4
$D(A_1, A_2)$	$D(B_1, B_2)$	$D(C_1, C_2)$	$D(G_1, G_2)$
.0727	0.0591	0.000	-0.0091

Hence the ranking order for our method are as following:
$A_1 > A_2$, $B_1 > B_2$, $C_1 = C_2$, $G_1 < G_2$.

Example 4.2 *Consider the three triangular fuzzy numbers,* $A = (5,6,7)$, $B = (5.9,6,7)$ *and* $C = (6,6,7)$. *(See Fig. 5).*

By using our method:
$D(A, B) = -0.0409$, $D(B, C) = -0.0046$ and
$D(A, C) = -0.0455$.

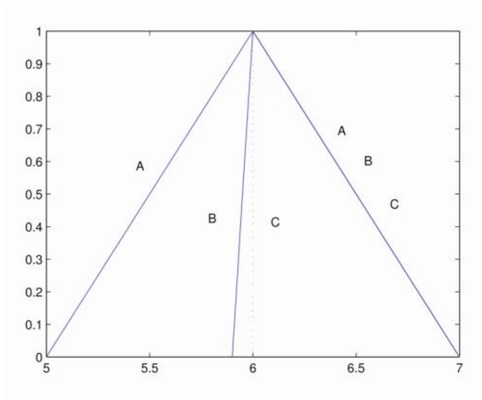

Figure 5. Triangular fuzzy numbers

$A = (5,6,7)$, $B = (5.9,6,7)$ **and** $C = (6,6,7)$.

Thus the ranking order is $A \prec B \prec C$.
As you see in Table 2, the results of Chu-Tsao method and Cheng CV index are unreasonable. The results of sign distance method [1] and Cheng distance method, are the same as our new approach.

Table 2. Comparative results of Example 4.2

Fuzzy numbers	Sign Distance P=1	Sign Distance P=2	Chu and Tsao	Cheng Distance	CV index
A	6.12	8.52	3	6.021	0.028
B	12.45	8.82	3.126	6.349	0.0098
C	12.5	8.85	3.085	6.3519	0.0089
Results	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \prec C$	$C \prec B \prec A$	$A \prec B \prec C$

Example 4.3 *Consider the following sets, see Yao and Wu [8].*

Set 1: A=(0.4,0.5,1), B=(0.4,0.7,1), C=(0.4,0.9,1), (Fig.6).

Set 2: A=(0.3,0.4,0.7,0.9)(trapezoidal fuzzy number), B=(0.3,0.7,0.9), C=(0.5,0.7,0.9), (Fig.7).

Set 3: A=(0.3,0.5,0.7), B=(0.3,0.5,0.8,0.9), C=(0.3,0.5,0.9), (Fig.8).

Set 4: A=(0,0.4,0.7,0.8), B=(0.2,0.5,0.9), C=(0.1,0.6,0.8), (Fig.9).

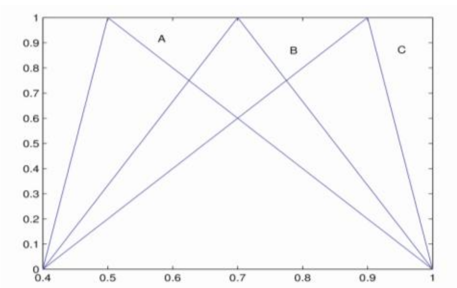

Figure 6. Set 1

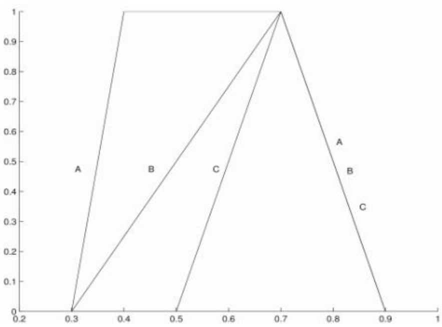

Figure 7. Set2

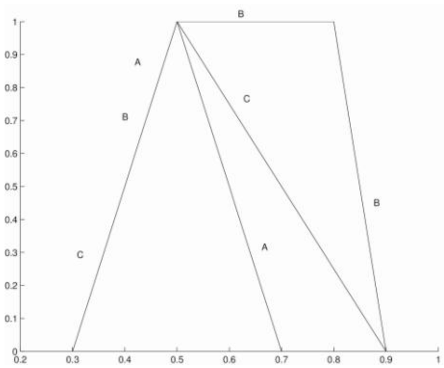

Figure 8. Set 3

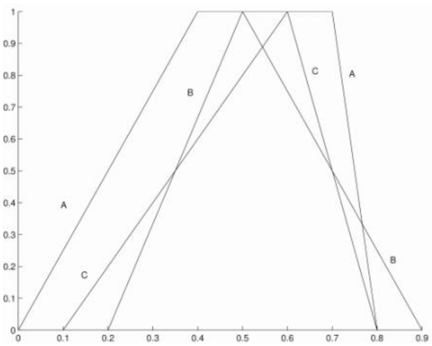

Figure 9. Set 4

From novel method we obtain the following results:

Set 1: $D(A,B) = -0.0182$ $D(B,C) = -0.0182$ $D(A,C) = -0.0364$,

Set 2: $D(A,B) = -0.0136$ $D(B,C) = -0.0091$ $D(A,C) = -0.0227$,

Set 3: $D(A,B) = -0.0318$ $D(B,C) = +0.0136$ $D(A,C) = -0.0182$,

Set 4: $D(A,B) = -0.0136$ $D(B,C) = -0.0045$ $D(A,C) = -0.0091$.

Hence the ranking order for our method are as follows:

Set 1: $A \prec B \prec C$,
Set 2: $A \prec B \prec C$,
Set 3: $A \prec C \prec B$,
Set 4: $A \prec B \prec C$.

To compare with other method we refer the reader to Table 3.(see [1]).

Table 3. Comparative results of Example 4.3

Methods	Fuzzy number	set 1	set 2	set 3	set 4
Sign Distance	A	1.2	1.15	1	0.95
method	B	1.4	1.3	1.25	1.05
p=1	C	1.6	1.4	1.1	1.05
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B : C$
Sign Distance	A	0.8869	0.8756	0.7257	0.7853
method	B	1.0194	0.9522	0.9416	0.7958
p=2	C	1.1605	1.0033	0.8165	0.8386
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \prec C$
Choobineh	A	0.333	0.458	0.333	0.50
and Li	B	0.50	0.583	0.4167	0.5833
	C	0.667	0.667	0.5417	0.6111
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$
	A	0.60	0.575	0.5	0.45
Yager	B	0.70	0.65	0.55	0.525
	C	0.80	0.7	0.625	0.55
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$
	A	0.3375	0.4315	0.375	0.52
Chen	B	0.50	0.5625	0.425	0.57
	C	0.667	0.625	0.55	0.625
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$
Baldwin	A	0.30	0.27	0.27	0.40
and Guild	B	0.33	0.27	0.37	0.42
	C	0.44	0.37	0.45	0.42
Results		$A \prec B \prec C$	$A : B \prec C$	$A \prec B \prec C$	$A \prec B : C$
Chu	A	0.299	0.2847	0.25	0.24402
and Tsao	B	0.350	0.32478	0.31526	0.26243

Methods	Fuzzy number	set 1	set 2	set 3	set 4
	C	0.3993	0.350	0.27475	0.2619
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec C \prec B$
Yao	A	0.6	0.575	0.5	0.475
and Wu	B	0.7	0.65	0.625	0.525
	C	0.8	0.7	0.55	0.525
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B : C$
Cheng	A	0.79	0.7577	0.7071	0.7106
Distance	B	0.8602	0.8149	0.8037	0.7256
	C	0.9268	0.8602	0.7458	0.7241
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec C \prec B$
Cheng CV	A	0.0272	0.0328	0.0133	0.0693
uniform	B	0.0214	0.0246	0.0304	0.0385
distribution	C	0.0225	0.0095	0.0275	0.0433
Results		$B \prec C \prec A$	$C \prec B \prec A$	$A \prec C \prec B$	$B \prec C \prec A$
Cheng CV	A	0.0183	0.026	0.008	0.0471
proportional	B	0.0128	0.0146	0.0234	0.0236
distribution	C	0.0137	0.0057	0.0173	0.0255
Results		$B \prec C \prec A$	$C \prec B \prec A$	$A \prec C \prec B$	$B \prec C \prec A$

5. Conclusions

In spite of many ranking methods, no one can rank fuzzy numbers with human intuition consistently in all cases. In this work, we have presented a simple ranking method for fuzzy numbers. The proposed ranking method has some properties. It does not imply much computational effort and does not require a priori knowledge of the set of all alternatives. Finally, comparative examples were presented to illustrate the advantage of our new approach.

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