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Research Article

Ranking Network-Structured Decision-Making Units and Its Application in Bank Branches

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Abstract

Data envelopment analysis (DEA) is a method used for measuring the efficiency of decision-making units. Unlike the standard models, which assume decision-making units to be a black box, network data envelopment analysis focuses on the internal structure of these units. Some researchers have developed a two-stage method where all the inputs are entirely used in the first stage, producing outputs which are subsequently fed as inputs to the second stage. These indices are introduced as intermediate indices. Here, it is assumed that congruent decision-making units have a two-stage serial structure. In this structure, the first stage and second stages act as the supplier and the consumer of resources respectively. Two ranking models based on the efficiency cloud and the common set of weights concepts were developed for ranking network-structured decision-making units. In the practical example presented in this study, 25 bank branches were ranked using the two-stage method.

Keywords : Data Envelopment Analysis (DEA); Two-stage; Ranking.

1 Introduction

The science of management has experienced numerous developments in recent years. For this reason, every organization is in dire need of an assessment system since, in the absence of such a system, the organization can neither grow nor survive. Achieving this goal calls for identification and management of internal and external factors of organizations. In most organizations, the following factors are considered to affect this strategy: customer satisfaction, technology, human force, long- and short-term policies, costs, benefits, etc. As a result of these factors, organizations aim for long-term profitability. Customer satisfaction is the managements most important concern in any organization. Every organization needs performance assessment to become aware of the utility of its activities in todays vague and ambiguous environments. Performance assessment is always one of the important duties of managers. Hence, managers should acquire the knowledge of measurement. If we cannot measure something, we cannot control and manage it. Similar to any other organization, banks need to improve their efficiency to survive. They try to provide desirable services through optimal use of their available facilities so as to satisfy the customers consistently and ensure adequate profitability. As

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providers of public services, banks must be assessed properly, or they will pose serious risks to society. Today, although banks endeavor to improve their performance, they do not have access to an inclusive assessment system for assessing the performance of their branches [2]. Hence, we have to use mathematical methods to assess the performance of banks.

The science of Data Envelopment Analysis [1] is one of the sub-branches of linear programming, focusing on the performance assessment of units. The objective of this subsidiary branch is to assess the performance of decision-making units with congener inputs and outputs. DEA is a powerful instrument with significantly increasing applications, developed for performance assessment of multi-input and multi-output systems. Unit performance can be used as a criterion for ranking units. In other words, a decision-making unit has a higher rank if it has a higher relative efficiency. However, a problem arises when there are decision-making units with equal relative efficiencies. This problem relates to the ranking of the equally efficient units. For this reason, to rank such units, a new technique in addition to the relative efficiency calculation models is required. Ranking efficient units, i.e. units with equal efficiency scores of one, is one of the important issues in DEA. Numerous articles on ranking of decision-making units have been published. One of the most important concepts proposed in these articles was the efficiency cloud [3, 4]. The model proposed by Anderson and Petersen (1993) for ranking efficient units is known as the AP model. In this method, the decision-making unit (DMU_p) in question is deleted from the observations set and a new production possibility set is created. Although the AP model is among the most well-known efficiency cloud models for ranking units, it has many defects. Hence, many other models have been proposed to resolve the defects of this model.

Another way of solving the above problem is using models based on a common set of weights. In this method, all the decision-making units are assessed using a set of optimum weights. Since most of the DEA models use a multi-objective structure to calculate the common set of weights, they are theoretically strong. The model proposed by Charnes et al. (1989) and Korenbluth (1991) is among the very first common set of weights models proposed in DEA. [6, 10, 11]

Multi-stage DEA, also known as the network DEA, measures efficiency and evaluates different phases of a multi-stage production process [8, 9]. In this research, 25 bank branches were analyzed with a two-stage structure. In general, it could be stated that, at the outset, all the inputs are consumed in stage 1, and the outputs of this phase, namely, the intermediate values, are produced. Only these intermediate values are used as the input to the second stage. Various models with constant return to scale have been developed for the network structure. In the majority of these models, efficiency is calculated based on the relevant units and components. The link between efficiency of components and efficiency of all units is consistently considered in such models. Moreover, establishing such a link has been one of the reasons for development of such models.[12]

This paper is presented in the following sections: Section 2 presents a brief review of network DEA. Next, DEA ranking models with a two-stage structure are presented in a multiple form. A practical two-stage example aimed at ranking 25 commercial bank branches with 5 indices is introduced in Section 3. The fourth section analyzes the results of the practical example, and the paper closes with the conclusion.

2 Ranking Two-Stage Decision-Making Units

Assume n decision-making units (DMU_s), $(X_j, Y_j) j = 1, 2, , n$, with the following input and output vectors:

$$X_j \neq 0, X_j \geq 0, X_j \in R^m \qquad (inputvector)$$

$$Y_j \neq 0, Y_j \geq 0, Y_j \in R^s \qquad (outputvector)$$

Each assumed DMU has a two-stage structure, and thus $Z_j \neq 0, Z_j \geq 0, Z_j \in R^D$ shows the

intermediate products of each DMU. In other words, $Z_j, (j = 1, ..., n)$ refers to the outputs of stage 1 and inputs of stage 2.
The CCR model, which was developed by Charnes, Cooper and Rhodes (1978) for determining the nature of the input for DMU_p assessment, is as follows [5].

To assess n DMUs, Model (2.1) is solved n times. Since in Model (2.1) weights are highly flexible in efficiency calculations, if this model considers a unit inefficient, then that unit is definitely inefficient. The following the serial two-stage structure was assumed for DMU_s. Therefore, efficiency of

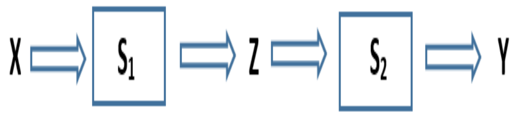

Figure 1

DMU_p is calculated as follows.

$$
\begin{aligned}
\max \quad & z = \sum_{r=1}^{s} u_r y_{rp} \\
s.t. \quad & \sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} \leq 0, \quad j = 1, ..., n, \\
& \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, j = 1, ..., n, \\
& \sum_{i=1}^{m} v_i x_{ip} = 1,
\end{aligned} \tag{2.1}
$$

$u_r, v_i, w_d \geq \varepsilon, r = 1, ..., s, i = 1, ..., m, d = 1, ..., D$

Model (2.2) is the expanded version of Model (2.1). In this model, it is possible to calculate stage 1 and stage 2 efficiencies in addition to the overall efficiency of the system using optimum weights in DMU_p assessment. Since the standard DEA models are highly optimistic (i.e. they incorporate best weights in the assessment of $DMU_p(p = 1, , n)$), in most assessments more than one DMU is introduced as the efficient unit. One of the ways of solving this problem is ranking DMUs. Since the DMUs in this research have a two-stage structure, Model (2.3), which is based on the AP model, was proposed for ranking two-stage units.

$$
\begin{aligned}
\max \quad & z = \sum_{r=1}^{s} u_r y_{rp} \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} \leq 0, \quad j = 1, ..., n, j \neq p \\
& \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, j = 1, ..., n, j \neq p \\
& \sum_{i=1}^{m} v_i x_{ip} = 1,
\end{aligned} \tag{2.2}
$$

$u_r, v_i, w_d \geq \varepsilon, r = 1, ..., s, i = 1, ..., m, d = 1, ..., D$

The ranking scale for stages one and 2 and the overall system is obtained via the following relations.

$$
\begin{aligned}
RankStage1 &= \frac{\sum_{d=1}^{D} w_d z_{dp}}{\sum_{i=1}^{m} v_i x_{ip}}, \\
RankStage2 &= \frac{\sum_{r=1}^{s} u_r y_{rp}}{\sum_{d=1}^{D} w_d z_{dp}}, \\
RankOveral &= \frac{\sum_{r=1}^{s} u_r y_{rp}}{\sum_{i=1}^{m} v_i x_{ip}},
\end{aligned} \tag{2.3}
$$

Where,

$$
\frac{\sum_{r=1}^{s} u_r y_{rp}}{\sum_{i=1}^{m} v_i x_{ip}} = \frac{\sum_{d=1}^{D} w_d z_{dp}}{\sum_{i=1}^{m} v_i x_{ip}} \times \frac{\sum_{r=1}^{s} u_r y_{rp}}{\sum_{d=1}^{D} w_d z_{dp}} \tag{2.4}
$$

Another solution for the optimism of the standard DEA models is to use models based on the common set of weights. The main objective of generating the common set of weights in DEA models is to produce a common criterion for ranking the efficient and inefficient DMUs. In this research, the model proposed for obtaining

the common set of weights was based on multi-objective programming notions.
In data envelopment analysis, the objective of each decision-making unit is to obtain the efficiency of 1. In other words,

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} = 1, \quad j = 1, ..., n, \tag{2.5}$$

Hence, relation (2.7) can be written as:

$$g_j = \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} = 0, \quad j = 1, ..., n, \tag{2.6}$$

That is to say, the aim of each $g_j (j = 1, ..., n)$ is to reach zero [7]. The n-objective problem for assessment of network-structured DMU_s is expressed as:

$$\begin{aligned} \min \quad & W = |(g_1, ..., g_n - (0, ..., 0)|_q \\ s.t. \quad & \sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} \leq 0, \quad j = 1, ..., n, \\ & \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, j = 1, ..., n, \end{aligned} \tag{2.7}$$

$$u_r, v_i, w_d \geq \varepsilon, r = 1, ..., s, i = 1, ..., m, d = 1, ..., D$$

To solve the above problem, we can use different norms $(q = 1, 2, ..., \infty)$ such as the infinity norm.

$$\begin{aligned} \min \quad & W = \theta \\ s.t. \quad & \sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} \leq 0, \quad j = 1, ..., n, \\ & \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, j = 1, ..., n, \\ & \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} \leq \theta, j = 1, ..., n, \end{aligned} \tag{2.8}$$

(2.9)

$$u_r, v_i, w_d \geq \varepsilon, r = 1, ..., s, i = 1, ..., m, d = 1, ..., D,$$

Unlike many of the common set of weights models which are nonlinear, the above model is linear.

3 Practical Example

In this research, 25 branches of an Iranian commercial bank were assessed. Table (2.1) presents the inputs (X), intermediate products (Z), and outputs (Y). The obtained data belong to 2011.

Inputs	Intermediate products	Outputs
Personnel score	Resources	Dividend received
Dividend payout		Commission received

Figure 2

Models (2.3) and (12) were selected for ranking the first stage (resource collection), the second stage (resource allocation and profitability), and the entire unit.

4 Results Analysis

Table (2) shows the results of ranking the banks branches with Model (2.3). These results were obtained in GAMS. According to Table (2), branches 19 and 3 have the top two ranks in stage 1 . The efficiency values of these two branches reveal that these branches are efficient in the first stage. Branch no. 11 has the third place. This

Branch	Stage 1 efficiency	Stage 1 ranking	Stage 2 efficiency	Stage 2 ranking	Overall system efficiency	Overall system rank
DMU 1	0.01576	20	0.35994	4	0.005673	9
DMU 2	0.13318	11	0.04796	13	0.006388	8
DMU 3	1.08705	2	0.03210	16	0.034892	1
DMU 4	0.02912	17	0.12469	7	0.003631	12
DMU 5	0.74016	4	0.00005	24	0.000038	24
DMU 6	0.39012	5	0.00001	25	0.000003	25
DMU 7	0.17845	9	0.00271	23	0.000483	23
DMU 8	0.01935	19	0.54223	3	0.010491	4
DMU 9	0.01052	22	0.29067	5	0.003058	14
DMU 10	0.04119	15	0.04157	14	0.001712	19
DMU 11	0.94722	3	0.01969	19	0.018654	3
DMU 12	0.00187	24	0.89941	2	0.001680	20
DMU 13	0.00181	25	2.49578	1	0.004506	10
DMU 14	0.05952	14	0.03434	15	0.002044	17
DMU 15	0.02808	18	0.08989	9	0.002524	16
DMU 16	0.00633	23	0.17302	6	0.001095	22
DMU 17	0.13518	10	0.02639	17	0.003567	13
DMU 18	0.38258	6	0.01724	20	0.006594	6
DMU 19	1.45835	1	0.01431	21	0.020863	2
DMU 20	0.08058	13	0.07985	10	0.006435	7
DMU 21	0.20134	8	0.04947	12	0.009959	5
DMU 22	0.02996	16	0.06236	11	0.001868	18
DMU 23	0.11692	12	0.02438	18	0.002850	15
DMU 24	0.29704	7	0.01356	22	0.004028	11
DMU 25	0.01401	21	0.10972	8	0.001537	21

Table 2: Results of ranking bank branches with Model (2.3)

branch is inefficient and is the best branch among the inefficient branches. In the first stage, branch no. 13 is the worst branch because it has the lowest rank. In view of stage 2 rankings, branch no. 13 has the highest rank and is also efficient. Ranks 2 and 3 belong to branches no. 12 and 8 respectively. Considering the efficiency values obtained in stage 2, branch no. 12 is the best inefficient branch, whereas branch no. 6 is the worst inefficient branch because it has the 25^{th} rank. These results suggest that branches 19 and 13 are the best and worst resource suppliers respectively. In addition, branches no. 13 and 6 are the best and worst resource consumers respectively. As seen, branch no. 13 has the lowest efficiency in stage 1, and shows the highest efficiency in stage 2. The sixth column in Table (2) indicates that relation (2.5) can be applied to each one of the DMUs. In the overall system ranking, branch no. 3 has the highest rank and branch no. 6 has the lowest rank. The average efficiencies of stages one and 2 are 0.256 and 0.222 respectively. These values show that the overall efficiency of these 25 branches is weaker in the second stage as compared to the first stage.

Table (3) presents the ranks of bank branches obtained with the common set of weights model (2.9). These results were obtained using GAMS. According to Table (3), branches no. 3, 11, and

Branch	Stage 1 efficiency	Stage 1 ranking	Stage 2 efficiency	Stage 2 ranking	Overall system efficiency	Overall system ranking
DMU 1	0.01367	21	0.14540	2	0.003115	6
DMU 2	0.13207	10	0.01362	15	0.001893	12
DMU 3	0.99261	1	0.01038	18	0.010384	1
DMU 4	0.02530	17	0.03243	9	0.000996	18
DMU 5	0.49748	4	0.00002	24	0.000009	24
DMU 6	0.37572	6	0.00001	25	0.000002	25
DMU 7	0.02455	18	0.00031	23	0.000016	23
DMU 8	0.01921	19	0.13566	3	0.003563	5
DMU 9	0.01047	22	0.10064	5	0.001486	15
DMU 10	0.04102	14	0.02168	13	0.000945	20
DMU 11	0.93958	2	0.00631	21	0.005976	2
DMU 12	0.00186	24	0.13128	4	0.000959	19
DMU 13	0.00179	25	0.14881	1	0.001785	13
DMU 14	0.05881	13	0.02194	12	0.001531	14
DMU 15	0.02774	16	0.05522	7	0.002122	11
DMU 16	0.00629	23	0.08351	6	0.000934	21
DMU 17	0.13442	9	0.02127	14	0.002969	7
DMU 18	0.38131	5	0.01091	17	0.004188	3
DMU 19	0.84755	3	0.00433	22	0.003698	4
DMU 20	0.07961	12	0.02647	11	0.002407	10
DMU 21	0.19943	8	0.01296	16	0.002689	8
DMU 22	0.02825	15	0.02658	10	0.000830	22
DMU 23	0.11622	11	0.00915	19	0.001110	17
DMU 24	0.29577	7	0.00876	20	0.002627	9
DMU 25	0.01385	20	0.04678	8	0.001177	16

Table 3: Results of ranking bank branches with Model (2.9)

19 have the top three ranks in the first stage. On account of the efficiency values obtained in the second stage, branches no. 13, 1, and 8 have the top three ranks. These results indicate that branches no. 3 and 13 are the best and worst resource suppliers respectively. Moreover, branches no. 13 and 6 are the best and worst resource consumers respectively. As seen, branch no. 13 has the lowest efficiency in the first stage and the highest efficiency in the second stage. In the overall system ranking, branch no. 3 has the highest rank and branch no. 6 has the lowest rank. The average efficiencies of stages one and 2 are 0.210 and 0.043 respectively. Hence, the overall performance of these 25 branches is considerably weaker in the second stage as compared to the first stage. Moreover, the efficiency results obtained from both of these models are almost equal.

5 Conclusion

In this paper, two ranking models with a two-stage structure were developed for bank branches. The first stage involves collection of resources for the investment and business purposes. This stage usually incurs costs, but it is necessary for supplying the resources required for investment and business in the second stage. The rank of each branch in the resource absorption, profitability, and overall performance sections was calculated with two models. In the first proposed model, branches no. 19, 13, and 3 had the highest ranks in the first stage, second stage, and overall system stage respectively. In the second proposed model, branches no. 3, 13, and 3 had the highest ranks in the first stage, second stage, and overall system stage respectively. As seen, the ranking results obtained from the two models are similar. That is to say, the ranks of some branches are completely equal while there is a slight difference between the ranks of other branches. In general, it could be concluded that the supplier section had a better performance than the consumers section.

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