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The Solution of Fully Fuzzy Quadratic Equations Based on Restricted Variation

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Abstract

Firstly, in this paper, we apply the Fuzzy Restricted Variation Method to achieve an analytical and approximate unsymmetrical fuzzy solution for Fully Fuzzy Quadratic Equation. In this application, after finding the real root of 1-cut of $\tilde{A}\tilde{X}^2 + \tilde{B}\tilde{X} + \tilde{C} = \tilde{D}$, initial guess is always chosen with possible unknown parameters that leads to highly accurate solution. This technique is applying to solve mentioned equation in four cases via the sign of coefficients and variable that there is not zero in support of them and we solve the problems to find positive or negative solution. This method has been shown to solve effectively, easily and accurately a large class of nonlinear quadratic equations with approximations converging rapidly to accurate solution. In this paper we present the solutions in four cases with formulas, that can be used to write the algorithm for this technique. Finally to illustrate easy application and rich behavior of this method, several examples are given.

Keywords : Fuzzy number; Fully fuzzy quadratic equation; Fuzzy parametric form; Restricted variations; Unsymmetrical fuzzy solution.

1 Introduction

The problem of finding the roots of equations like quadratic has many applications in applied sciences like finance [5, 14] , economy [6, 9, 20, 23], mechanics [13] and etc. Sevastjanov et al. [22] proposed a new method for solving interval and fuzzy linear equations. In [10, 11, 12] Buckly discussed on solving fuzzy equations. Abbasbandy and otadi in [2] obtained the real valued roots of fuzzy polynomials using fuzzy neural networks. In [1, 4, 7, 8] the authors have introduced numerical and neural net solutions to solve fuzzy equations. In [3] Allahviranloo and gerami have proposed new method based on optimization theory to solve fully fuzzy quadratic equations. In the current paper we propose a new method to solve $\tilde{A}\tilde{X}^2 + \tilde{B}\tilde{X} + \tilde{C} = \tilde{D}$ with applying Fuzzy Restricted Variation Method (FRVM). This method has been shown to solve effectively, easily and accurately a large class of nonlinear quadratic equations with approximations converging rapidly to accurate solution. We want to use this efficiency method to find an analytical and approximate unsymmetrical fuzzy solution for Fully Fuzzy Quadratic Equation (FFQE). In FRVM, initial guess is always chosen with a possible unknown parameters that one iteration leads to highly accurate solution. we consider FFQE and show how restricted variations work in FRVM .

The rest of the paper is set out as follows: In second section some related basic definitions of fuzzy mathematics for the analysis are recalled.

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In section 3, new method, for solving fully fuzzy quadratic equation is presented. In section 4, this method is used for an analytical-approximate unsymmetrical fuzzy solution of FFQE. In section 5, the conclusions are drawn.

2 Basic concepts

The basic definitions are given as follows:

Definition 2.1 *[16, 17, 25, 26] A fuzzy number is a function $\tilde{u}:\mathbb{R}\to[0,1]$ which satisfies:*

1. *$\tilde{u}$ is an upper semi-continuous on $\mathbb{R}$,*
2. *$\tilde{u}$ is normal, i.e, $\exists x_0\in\mathbb{R}$ with $\tilde{u}(x_0)=1$,*
3. *$\tilde{u}$ is convex fuzzy set,*
4. *$\overline{\{x\in\mathbb{R}\mid\tilde{u}(x)>0\}}$ is compact, where $\bar{A}$ denotes the closure of A.*

Note: [17] In this paper we consider fuzzy numbers which have a unique $x_0\in\mathbb{R}$ with $\tilde{u}(x_0)=1$.
The set of all these fuzzy numbers is denoted by $\mathcal{F}$. Obviously, $\mathbb{R}\subseteq\mathcal{F}$. For $0<r\leq 1$,we define r-cut of fuzzy number $\tilde{u}$ as $[\tilde{u}]_r=\{x\in\mathbb{R}:\tilde{u}(x)\geq r\}$ and $[\tilde{u}]_0=\overline{\{x\in\mathbb{R}:\tilde{u}(x)\geq 0\}}$. [22] from $(1)-(4)$ it follows that $[\tilde{u}]_r$ is a bounded closed interval for each $r\in[0,1]$. We denote the r-cut of fuzzy number $\tilde{u}$ as $[\tilde{u}]_r=[\underline{u}(r),\overline{u}(r)]$.

Definition 2.2 *[17] A fuzzy number $\tilde{u}$ is positive (negative) if $\tilde{u}(x)=0$ for all $x<0$ $(x>0)$.*

Definition 2.3 *[24, 18] A fuzzy number $\tilde{u}$ in parametric form is a pair $(\underline{u},\overline{u})$ of functions $\underline{u}(r)$, $\overline{u}(r)$, $0\leq r\leq 1$, which satisfy the following requirements:*

1. *$\underline{u}(r)$ is a bounded non-decreasing left continuous function in $[0,1]$,*
2. *$\overline{u}(r)$ is a bounded non-increasing left continuous function in $[0,1]$,*
3. *$\underline{u}(r)\leq\overline{u}(r)$, $0\leq r\leq 1$.*

Definition 2.4 *[24] For arbitrary $\tilde{u}=(\underline{u}(r),\overline{u}(r))$ and $\tilde{v}=(\underline{v}(r),\overline{v}(r))$, $0\leq r\leq 1$, and scalar k, we define addition, subtraction, scalar product by k and multiplication are respectively as following:*
addition: $\underline{u+v}(r)=\underline{u}(r)+\underline{v}(r)$, $\overline{u+v}(r)=\overline{u}(r)+\overline{v}(r)$,
subtraction: $\underline{u-v}(r)=\underline{u}(r)-\overline{v}(r)$, $\overline{u-v}(r)=\overline{u}(r)-\underline{v}(r)$,
scalar product:

$$k\tilde{u}=\begin{cases}(k\underline{u}(r),k\overline{u}(r)), & k\geq 0\\ (k\overline{u}(r),k\underline{u}(r)), & k<0\end{cases}$$

multiplication:
$\underline{uv}(r)=min\{\underline{u}(r)\underline{v}(r),\underline{u}(r)\overline{v}(r),\overline{u}(r)\underline{v}(r),\overline{u}(r)\overline{v}(r)\}$,
$\overline{uv}(r)=max\{\underline{u}(r)\underline{v}(r),\underline{u}(r)\overline{v}(r),\overline{u}(r)\underline{v}(r),\overline{u}(r)\overline{v}(r)\}$.
for two important cases multiplication of two fuzzy number is defined by following terms:
If $\tilde{u}\geq 0$ and $\tilde{v}\geq 0$ then: $\underline{uv}(r)=\underline{u}(r)\underline{v}(r)$ and $\overline{uv}(r)=\overline{u}(r)\overline{v}(r)$,
If $\tilde{u}\leq 0$ and $\tilde{v}\leq 0$ then: $\underline{uv}(r)=\overline{u}(r)\overline{v}(r)$ and $\overline{uv}(r)=\underline{u}(r)\underline{v}(r)$,
If $\tilde{u}\geq 0$ and $\tilde{v}\leq 0$ then: $\underline{uv}(r)=\overline{u}(r)\underline{v}(r)$ and $\overline{uv}(r)=\underline{u}(r)\overline{v}(r)$,
If $\tilde{u}\leq 0$ and $\tilde{v}\geq 0$ then: $\underline{uv}(r)=\underline{u}(r)\overline{v}(r)$ and $\overline{uv}(r)=\overline{u}(r)\underline{v}(r)$.

Arithmetics of r-cuts is similar to arithmetics of parametric form that recalled above [19, 21].

Definition 2.5 *[24] Two fuzzy numbers $\tilde{u}$, $\tilde{v}$ are said to be equal, if and only if $\underline{u}(r)=\underline{v}(r)$ and $\overline{u}(r)=\overline{v}(r)$, for eachr $\in[0,1]$.*

A crisp number α in parametric form is $\underline{u}(r)=\overline{u}(r)=\alpha$, $0\leq r\leq 1$. A triangular fuzzy number is popular and represented by $\tilde{u}=(m,\alpha,\beta)$ where $\alpha>0$, $\beta>0$, which has the parametric form as follows:
$\underline{u}(r)=m-\alpha+r\alpha$, $\overline{u}(r)=m+\beta-r\beta$.

Definition 2.6 *[15] Let $D:\mathcal{F}\times\mathcal{F}\to\mathbb{R}\cup\{0\}$, $D(\tilde{u},\tilde{v})=sup_{r\in[0,1]}max\{|\ \underline{u}(r)-\underline{v}(r)\ |,|\ \overline{u}(r)-\overline{v}(r)\ |\}$ be the Hausdorff distance between fuzzy numbers, where $[\tilde{u}]_r=[\underline{u}(r),\overline{u}(r)]$, $[\tilde{v}]_r=[\underline{v}(r),\overline{v}(r)]$. The following properties are well known:*

1. *$D(\tilde{u}\bigoplus\tilde{w},\tilde{v}\bigoplus\tilde{w})=D(\tilde{u},\tilde{v}),\forall\tilde{u},\tilde{v},\tilde{w}\in\mathcal{F}$,*
2. *$D(k\bigodot\tilde{u},k\bigodot\tilde{v})=|\ k\ |\ D(\tilde{u},\tilde{v}),\forall k\in\mathbb{R}$ and $\tilde{u},\tilde{v}\in\mathcal{F}$,*
3. *$D(\tilde{u}\bigoplus\tilde{v},\tilde{w}\bigoplus\tilde{e})\leq D(\tilde{u},\tilde{w})+D(\tilde{v},\tilde{e}),\forall\tilde{u},\tilde{v},\tilde{w},\tilde{e}\in\mathcal{F}$.*

Therefore $(\mathcal{F},D)$ is a complete metric space.

3 Fuzzy Restricted Variation Method (FRVM)

Let $\tilde{A} = (\underline{a}(r), \overline{a}(r))$, $\tilde{B} = (\underline{b}(r), \overline{b}(r))$, $\tilde{C} = (\underline{c}(r), \overline{c}(r))$, $\tilde{D} = (\underline{d}(r), \overline{d}(r))$, $\tilde{X} = (\underline{x}(r), \overline{x}(r))$ and

$$F(\tilde{X}) = \tilde{D}, \tag{3.1}$$

$$F(\tilde{X}) = (\underline{F}(\underline{x}, \overline{x}, r), \overline{F}(\underline{x}, \overline{x}, r)). \tag{3.2}$$

Where $F(\tilde{X}) = \tilde{A}\tilde{X}^2 + \tilde{B}\tilde{X} + \tilde{C}$. In this section, we solve $F(\tilde{X}) = \tilde{D}$ and show how restricted variables work in FRVM. This method is applying to solve FFQE in four cases that there is not zero in support of coefficients and variable. These four cases are as follows:

1. $\tilde{A} > 0, \tilde{B} > 0, \tilde{X} > 0,$
2. $\tilde{A} < 0, \tilde{B} < 0, \tilde{X} > 0,$
3. $\tilde{A} > 0, \tilde{B} < 0, \tilde{X} < 0,$
4. $\tilde{A} < 0, \tilde{B} > 0, \tilde{X} < 0.$

Without loss of generality, we use this method to solve FFQE considering case 1. Another cases are the same.

3.1 Case (1):

At first we write FFQE to parametric form:
$(\underline{a}(r), \overline{a}(r))(\underline{x}^2(r), \overline{x}^2(r)) + (\underline{b}(r), \overline{b}(r))(\underline{x}(r), \overline{x}(r))$
$+(\underline{c}(r), \overline{c}(r))$
$=(\underline{d}(r), \overline{d}(r))$
Then we set

$$\begin{cases} \underline{a}(r)\underline{x}^2(r) + \underline{b}(r)\underline{x}(r) + \underline{c}(r) = \underline{d}(r) \\ \overline{a}(r)\overline{x}^2(r) + \overline{b}(r)\overline{x}(r) + \overline{c}(r) = \overline{d}(r) \end{cases} \tag{3.3}$$

and

$$\underline{F}(\underline{x}, \overline{x}, r) = \underline{a}(r)\underline{x}^2(r) + \underline{b}(r)\underline{x}(r) + \underline{c}(r), \tag{3.4}$$

$$\overline{F}(\underline{x}, \overline{x}, r) = \overline{a}(r)\overline{x}^2(r) + \overline{b}(r)\overline{x}(r) + \overline{c}(r), \tag{3.5}$$

$$\underline{G}(\underline{x}, \overline{x}, r) = \underline{F}(\underline{x}, \overline{x}, r) - \underline{d}(r), \tag{3.6}$$

$$\overline{G}(\underline{x}, \overline{x}, r) = \overline{F}(\underline{x}, \overline{x}, r) - \overline{d}(r). \tag{3.7}$$

In this technique, rewrite Eqs (3.3) in the next equalities:

$$\begin{cases} \underline{a}(r)\underline{x}(r)\underline{\tilde{x}}(r) + \underline{b}(r)\underline{x}(r) + \underline{c}(r) = \underline{d}(r) \\ \overline{a}(r)\overline{x}(r)\overline{\tilde{x}}(r) + \overline{b}(r)\overline{x}(r) + \overline{c}(r) = \overline{d}(r) \end{cases} \tag{3.8}$$

Where $\underline{\tilde{x}}(r)$ and $\overline{\tilde{x}}(r)$ are called restricted variables. The value of $\underline{\tilde{x}}(r)$ and $\overline{\tilde{x}}(r)$ is assumed to be known (initial guess) with $\underline{\tilde{x}}(r) = \alpha - \alpha_1(r)$ and $\overline{\tilde{x}}(r) = \alpha - \alpha_2(r)$ that α is the positive real root of 1-cut equation of FFQE and $\alpha_1(r)$ and $\alpha_2(r)$ are free parameters to be calculated from $\underline{x}(r) = \underline{\tilde{x}}(r)$ and $\overline{x}(r) = \overline{\tilde{x}}(r)$ where $\alpha_1(r), \alpha_2(r) \geq 0$ and $\alpha_1'(r) \leq 0$, $\alpha_2'(r) \geq 0$.

Lemma 3.1 *Suppose that $ll_+(r) = \alpha\, \underline{a}(r) + \underline{b}(r)$ and $uu_+(r) = \alpha\, \overline{a}(r) + \overline{b}(r)$ and $K(r) = \frac{\underline{a}(r)\alpha_1(r)}{ll_+(r)}, L(r) = \frac{\overline{a}(r)\alpha_2(r)}{uu_+(r)}$. If $|K(r)| < 1$, $|L(r)| < 1$ then with FRVM we obtain:*

$$\alpha_1(r) = \frac{ll_+(r)\underline{G}(\alpha, \alpha, r)}{ll_+^2(r) + (\underline{d}(r) - \underline{c}(r))\underline{a}(r)},$$

$$\alpha_2(r) = \frac{uu_+(r)\overline{G}(\alpha, \alpha, r)}{uu_+^2(r) + (\overline{d}(r) - \overline{c}(r))\overline{a}(r)}$$

Proof. Solving (3.8) via $\underline{x}(r)$, $\overline{x}(r)$ we get

$$\underline{x}(r) = \frac{\underline{d}(r) - \underline{c}(r)}{ll_+(r)(1 - K(r))},$$

$$\overline{x}(r) = \frac{\overline{d}(r) - \overline{c}(r)}{uu_+(r)(1 - L(r))}.$$

with the assumption $|K(r)| < 1$, $|L(r)| < 1$ we have

$$\underline{x}(r) = \frac{\underline{d}(r) - \underline{c}(r)}{ll_+(r)}(1 + K(r) + O(K^2(r))),$$

$$\overline{x}(r) = \frac{\overline{d}(r) - \overline{c}(r)}{uu_+(r)}(1 + L(r) + O(L^2(r))).$$

then

$$\underline{x}(r) \simeq \frac{\underline{d}(r) - \underline{c}(r)}{ll_+(r)}(1 + K(r)),$$

$$\overline{x}(r) \simeq \frac{\overline{d}(r) - \overline{c}(r)}{uu_+(r)}(1 + L(r)).$$

To obtain $\alpha_1(r), \alpha_2(r)$, According to FRVM,we set $\underline{x}(r) = \underline{\tilde{x}}(r)$ and $\overline{x}(r) = \overline{\tilde{x}}(r)$, then we find

$$\frac{\underline{d}(r) - \underline{c}(r)}{ll_+(r)}(1 + \frac{\underline{a}(r)\alpha_1(r)}{ll_+(r)}) = \alpha - \alpha_1(r),$$

$$\frac{\overline{d}(r) - \overline{c}(r)}{uu_+(r)}(1 + \frac{\overline{a}(r)\alpha_2(r)}{uu_+(r)}) = \alpha - \alpha_2(r).$$

Now we have linear equations with respect to $\alpha_1(r), \alpha_2(r)$, after solving them:

$$\alpha_1(r) = \frac{ll_+(r)(\alpha^2\underline{a}(r) + \alpha\underline{b}(r) + \underline{c}(r) - \underline{d}(r))}{ll_+^2(r) + (\underline{d}(r) - \underline{c}(r))\underline{a}(r)},$$

$$\alpha_2(r) = \frac{uu_+(r)(\alpha^2\overline{a}(r) + \alpha\overline{b}(r) + \overline{c}(r) - \overline{d}(r))}{uu_+^2(r) + (\overline{d}(r) - \overline{c}(r))\overline{a}(r)}.$$

Using (3.5), (3.6), proof is completed.
In this method the updated approximates are $\underline{x}(r) = \alpha - \alpha_1(r)$ and $\overline{x}(r) = \alpha - \alpha_2(r)$. In another cases similar to case 1 we find:

3.2 Case (2):

With $\tilde{A} < 0, \tilde{B} < 0, \tilde{X} > 0$:

Lemma 3.2 *Suppose that $ll_+(r) = \alpha\ \underline{a}(r) + \underline{b}(r)$ and $uu_+(r) = \alpha\ \overline{a}(r) + \overline{b}(r)$ and $K(r) = \frac{\overline{a}(r)\alpha_1(r)}{uu_+(r)}, L(r) = \frac{\underline{a}(r)\alpha_2(r)}{ll_+(r)}$. If $|K(r)| < 1$, $|L(r)| < 1$ then with FRVM we obtain:*

$$\alpha_1(r) = \frac{uu_+(r)\overline{G}(\alpha, \alpha, r)}{uu_+^2(r) + (\overline{d}(r) - \overline{c}(r))\overline{a}(r)},$$

$$\alpha_2(r) = \frac{ll_+(r)\underline{G}(\alpha, \alpha, r)}{ll_+^2(r) + (\underline{d}(r) - \underline{c}(r))\underline{a}(r)}.$$

3.3 Case (3):

With $\tilde{A} > 0, \tilde{B} < 0, \tilde{X} < 0$:

Lemma 3.3 *Suppose that $ul_+(r) = \alpha\ \overline{a}(r) + \underline{b}(r)$ and $lu_+(r) = \alpha\ \underline{a}(r) + \overline{b}(r)$ and $K(r) = \frac{\overline{a}(r)\alpha_1(r)}{ul_+(r)}, L(r) = \frac{\underline{a}(r)\alpha_2(r)}{lu_+(r)}$. If $|K(r)| < 1$, $|L(r)| < 1$ then with FRVM we obtain:*

$$\alpha_1(r) = \frac{ul_+(r)\overline{G}(\alpha, \alpha, r)}{ul_+^2(r) + (\overline{d}(r) - \overline{c}(r))\overline{a}(r)},$$

$$\alpha_2(r) = \frac{lu_+(r)\underline{G}(\alpha, \alpha, r)}{lu_+^2(r) + (\underline{d}(r) - \underline{c}(r))\underline{a}(r)}.$$

3.4 Case (4):

With $\tilde{A} < 0, \tilde{B} > 0, \tilde{X} < 0$:

Lemma 3.4 *Suppose that $lu_+(r) = \alpha\ \underline{a}(r) + \overline{b}(r)$ and $ul_+(r) = \alpha\ \overline{a}(r) + \underline{b}(r)$ and $K(r) = \frac{\underline{a}(r)\alpha_1(r)}{lu_+(r)}, L(r) = \frac{\overline{a}(r)\alpha_2(r)}{ul_+(r)}$. If $|K(r)| < 1$, $|L(r)| < 1$ then with FRVM we obtain:*

$$\alpha_1(r) = \frac{lu_+(r)\underline{G}(\alpha, \alpha, r)}{lu_+^2(r) + (\underline{d}(r) - \underline{c}(r))\underline{a}(r)},$$

$$\alpha_2(r) = \frac{ul_+(r)\overline{G}(\alpha, \alpha, r)}{ul_+^2(r) + (\overline{d}(r) - \overline{c}(r))\overline{a}(r)}.$$

4 Numerical Examples

In the next examples we round numbers with approximation less than 10^{-4}.

Example 4.1 *Let $\tilde{A} = (4/1/1)$, $\tilde{B} = (2/1/1)$, $\tilde{C} = (1/1/1)$, $\tilde{D} = (3/2/2)$.*
We will look for a solution where $\tilde{X} > 0$.
The equation $F(\tilde{X}) = \tilde{D}$ becomes in parametric form as follows :
$(3 + r, 5 - r)(\underline{x}^2(r), \overline{x}^2(r)) + (1 + r, 3 - r)(\underline{x}(r), \overline{x}(r)) + (r, 2 - r) = (1 + 2r, 5 - 2r)$.
The positive real root of 1-cut is $\alpha = \underline{x}(1) = \overline{x}(1) = 0.5$.
With proposed method and based on lemma 3.1 we obtain:

$$\alpha_1(r) = \frac{-0.75r^2 - 0.5r + 1.25}{6.5r^2 + 23r + 18.5},$$

$$\alpha_2(r) = \frac{-0.75r^2 + 3.5r - 2.75}{6.5r^2 - 49 + 90.5},$$

$$\underline{x}(r) = 0.5 - \frac{-0.75r^2 - 0.5r + 1.25}{6.5r^2 + 23r + 18.5},$$

$$\overline{x}(r) = 0.5 - \frac{-0.75r^2 + 3.5r - 2.75}{6.5r^2 - 49 + 90.5}.$$

Because of $|K(r)| < 1$, $|L(r)| < 1$ we accept these spreads.
In this problem Hausdorff metric is $D(F(\tilde{X}), \tilde{D}) = 0.0018$.

Example 4.2 *Let $\tilde{A} = (-2/1/1)$, $\tilde{B} = (-3/2/1)$, $\tilde{D} = (-5/3/0)$.*
We will look for a solution where $\tilde{X} > 0$. The equation $F(\tilde{X}) = \tilde{D}$ becomes in parametric form as follows :
$(-3 + r, -1 - r)(\underline{x}^2(r), \overline{x}^2(r)) + (2r - 5, -2 - r)(\underline{x}(r), \overline{x}(r)) = (-8 + 3r, -5r)$

The positive real root of 1-cut is $\alpha = \underline{x}(1) = \overline{x}(1) = 1$:
With proposed method and based on lemma 3.2 we obtain:

$$\alpha_1(r) = \frac{-6r^2 - 3r + 9}{9r^2 + 17r + 9},$$

$$\alpha_2(r) = 0,$$

$$\underline{x}(r) = 1 - \frac{-6r^2 - 3r + 9}{9r^2 + 17r + 9},$$
$$\overline{x}(r) = 1.$$

Because of $|K(r)| < 1$, $|L(r)| < 1$ *we accept these spreads. In this problem Hausdorff metric is* $D(F(\tilde{X}), \tilde{D}) = 0.0519$

5 coclusion

In this paper we use Fuzzy Restricted Variation Method (FRVM) to solve a Fully Fuzzy Quadratic Equation (FFQE). To this purpose we use restricted variables that lead to highly accurate solution. we can apply this method to write an algorithm to calculate solutions of a FFQE in four cases based on the sign of coefficients and variable.

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