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A new evaluation model for selecting a qualified manager by using fuzzy Topsis approach

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Abstract

Considering the contemporary business settings, managers role is more than essential to the viability and further development of an organization. Managers should possess such skills in order to effectively cope with the competition. Multiple attributes decision making (MADM) is an approach employed to solve problems involving selection from among a finite number of alternatives. The aim of this study is to develop a methodology to evaluate managers based on integrating fuzzy AHP and fuzzy TOPSIS approaches. In this paper, I have taken into consideration some important criteria which affect the process of managers selection. I have calculated the weights for each criterion based on Interval-valued fuzzy AHP and then inputted these weights to the fuzzy TOPSIS method to rank managers. The entire methodology is illustrated with the help of a numerical example and finally the rank of each managers is determined according to its results. The proposed method enables decision analysts to better understand the complete evaluation process and provide a more accurate, effective, and systematic decision support tool. Also, the proposed method provides a useful way for handling fuzzy TOPSIS based on fuzzy numbers.

Keywords : Fuzzy number; Fuzzy TOPSIS; Multiple criteria decision-making.

1 Introduction

Fuzzy numbers and fuzzy arithmetic were introduced in [37, 14, 22]. The fuzzy function was introduced in [6]. Then, the authors [13] presented an elementary calculus based on the [13]. Fuzzy numbers are study a variety of problems ranging from various spaces [7], to control chaotic systems [15, 17, 38], fuzzy differential equations [3], fuzzy integral equations [25, 28], fuzzy linear and nonlinear systems [1, 2] and fuzzy neural network [26, 27, 29].

Methods of approximate inference based on fuzzy set theory allow formal representation to be built for verbal decision-making procedures containing vague, fuzzy premises [16]. These procedures, as a rule, are described by sets of verbal, conditional statements (verbal implications). Methods of approximate inference is based on the extension of the concept of a fuzzy set by an interval-valued fuzzy set (or more precisely: a fuzzy set with an interval-valued membership function, see Zadeh [37]).

It is a clear fact that every organizations management system defines and directs its present and future, in particular under the aforementioned present circumstances. Management policies, processes, tools and structures play a critical role on how to exploit the opportunities and avoid the threats. Improved management skills can be achieved through training and development pro-

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grams inside an organization, as well as through-experienceinpractice.Nevertheless,the initial and decisive step is the selective selection of those managers that possess at a minimum extent a number of contemporary management skills. TOPSIS is a popular approach to multiple criteria decision making (MCDM) problems that was proposed by Hwang and Yoon [21, 36]. The basic principle is that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. In the TOPSIS, the performance ratings and the weights of the criteria are given as crisp values. Chen [8] presented a method for fuzzy group decision making situations based on the ranking of generalized trapezoidal fuzzy numbers. The authors [33] develop a fuzzy version of TOPSIS method based on fuzzy arithmetic operations, which leads to a fuzzy relative closeness for each alternative. Recently fuzzy TOPSIS successfully used for MCDM problems [12, 31, 32]. Ashtiani et al. in [4] used fuzzy TOPSIS method based on interval-valued fuzzy sets for solving MCDM problems, but applying it for some fuzzy MCDM problems leads to the incorrect solution and results. Therefore, Mokhtarian [27] tried to eliminate this problem.

Taking into account the above mentioned, the aim of this paper is to propose a new approach towards managers selection problem and fuzzy AHP is used to determine the preference weights of evaluation. Then, this research adopts the fuzzy TOPSIS to improve the gaps of alternatives between real performance values and pursuing aspired levels in each dimension and criterion and find out the best alternatives.

2   The fuzzy AHP

Gorzalczany [16] proposed the concept of interval-valued fuzzy sets. Then, Yao and Lin [35] represented the interval-valued fuzzy set $[\bar{A}^L, \bar{A}^U]$ where $\bar{A}^L$ denotes the lower interval-valued fuzzy set, $\bar{A}^U$ denotes the upper interval-valued fuzzy set, and $\bar{A}^L \subset \bar{A}^U$. Thereby, the minimum and maximum membership value of $\bar{A}^L$ are $\bar{A}^L$ and $\bar{A}^U$, respectively.

Assume that there are two interval-valued triangular fuzzy numbers $\bar{A}$ and $\bar{B}$, where

$$\bar{A} = [(a_1^L, a_2^L, a_3^L; h_{\bar{A}}^L), (a_1^U, a_2^U, a_3^U; h_{\bar{A}}^U)]$$

and

$$\bar{B} = [(b_1^L, b_2^L, b_3^L; h_{\bar{B}}^L), (b_1^U, b_2^U, b_3^U; h_{\bar{B}}^U)],$$

$a_1^L, a_2^L, a_3^L, a_1^U, a_2^U, a_3^U, b_1^L, b_2^L, b_3^L, b_1^U, b_2^U$ and b_3^U are real values, $0 \le h_{\bar{A}}^L \le h_{\bar{A}}^U \le 1$ and $0 \le h_{\bar{B}}^L \le h_{\bar{B}}^U \le 1$. The arithmetic operations between the triangular fuzzy numbers $\bar{A}$ and $\bar{B}$ are reviewed from [10].

We will briefly introduce that how to carry out the AHP in this section.

In first time, construct pairwise comparison matrices among all the elements/criteria in the dimensions of the hierarchy system. Assign linguistic terms to the pairwise comparisons by asking which is the more important of each two dimensions, as following matrix $\bar{C}$.

$$\bar{C} = \begin{bmatrix} 1 & \bar{c}_{12} & \dots & \bar{c}_{1n} \\ \bar{c}_{21} & 1 & \dots & \bar{c}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{c}_{n1} & \bar{c}_{n2} & \dots & 1 \end{bmatrix} = \begin{bmatrix} 1 & \bar{c}_{12} & \dots & \bar{c}_{1n} \\ \frac{1}{\bar{c}_{12}} & 1 & \dots & \bar{c}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{\bar{c}_{1n}} & \frac{1}{\bar{c}_{2n}} & \dots & 1 \end{bmatrix} \tag{2.1}$$

where

$$\bar{c}_{ij} = \begin{cases} \frac{1}{7}, \dots, \frac{1}{2}, \frac{1}{1}, \bar{1}, \bar{2}, \dots, \bar{7}, & i \ne j, \\ \\ 1, & i = j. \end{cases}$$

Then to use geometric mean technique to define the fuzzy geometric mean and fuzzy weights of each criterion by Hsieh et al. [18]

$$\bar{r}_i = (\bar{c}_{i1} \otimes \dots \otimes \bar{c}_{ij} \otimes \dots \otimes \bar{c}_{in})^{1/n},$$
$$\bar{w}_i = \bar{r}_i \oslash (\bar{r}_1 \oplus \dots \oplus \bar{r}_i \oplus \dots \oplus \bar{r}_n),$$

where c_{ij} is a IVF comparison value of dimension i to criterion j, thus, $\bar{r}_i$ is a geometric mean of interval-valued fuzzy comparison value of criterion i to each criterion, $\bar{w}_i$ is the IVF weight of the ith criterion, can be indicated by a IVF, $\bar{w}_i = [(w_1^L, w_2^L, w_3^L; h_{\bar{w}}^L), (w_1^U, w_2^U, w_3^U; h_{\bar{w}}^U)]$. The w_1^L, w_2^L and w_3^L stand for the lower, middle, and upper values of the $\bar{w}_i^L$ and w_1^U, w_2^U and w_3^U stand for the lower, middle, and upper values of the $\bar{w}_i^U$ weight of the ith dimension.

There are numerous studies that apply fuzzy AHP method to solve different managerial problems [19, 20, 30].

3 The fuzzy TOPSIS

TOPSIS method is easy to understand and to implement. These issues are of fundamental importance for a direct field implementation of the method by practitioners. Moreover, it allows the straight linguistic definition of importance and ratings under each criterion, without the need of cumbersome pairwise comparisons and the risk of inconsistencies.

Suppose a crisp MCDM problem has n alternatives $(A_1, \ldots, A_m)$ and n decision criteria/attributes $(C_1, \ldots, C_n)$. Each alternative is evaluated with respect to the n criteria/attributes. Due to the fact that, in some cases, determining the exact values for the elements of decision matrix is difficult, so, their values are considered as fuzzy numbers [24]. In other words, in fuzzy MCDM problems, the values of alternatives with respect to each criterion/attribute and the values of relative weights with respect to each criterion/attribute are usually characterized by fuzzy numbers. By considering the fact that, the TOPSIS method can also be used to deal with MCDM problems as a popular, accurate, and easy to use method. The steps of the proposed method can be described as follows:

Step 1: Determine the weighting of evaluation criteria. This research employs fuzzy AHP to find the fuzzy preference weights.

Step 2: Construct the fuzzy performance/decision matrix and choose the appropriate linguistic variables for the alternatives with respect to criteria

$$\bar{D} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \\ \left[\begin{array}{cccc} \bar{x}_{11} & \bar{x}_{12} & \cdots & \bar{x}_{1n} \\ \bar{x}_{21} & \bar{x}_{22} & \cdots & \bar{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{x}_{m1} & \bar{x}_{m2} & \cdots & \bar{x}_{mn} \end{array} \right] \end{array}.$$
$$i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$$
$$\bar{x}_{ij} = \frac{1}{K}(\bar{x}_{ij}^1 \oplus \cdots \oplus \bar{x}_{ij}^k \oplus \cdots \oplus \bar{x}_{ij}^K) \tag{3.2}$$

where $\bar{x}_{ij}^k$ is the performance rating of alternative A_i with respect to criterion C_j evaluated by kth expert, and

$$\bar{x}_{ij}^k = [((x_{ij}^k)_1^L, (x_{ij}^k)_2^L, (x_{ij}^k)_3^L; h_{\bar{x}_{ij}^k}^L),$$
$$((x_{ij}^k)_1^U, (x_{ij}^k)_2^U, (x_{ij}^k)_3^U; h_{\bar{x}_{ij}^k}^U)].$$

Step 3: Normalize the fuzzy-decision matrix. The normalized fuzzy-decision matrix denoted by $\bar{R}$ is shown as following formula:

$$\bar{R} = [\bar{r}_{ij}]_{m \times n}, \ i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n. \tag{3.3}$$

The normalization process can be performed by following formula:

$$\bar{r}_{ij} = [(\frac{(x_{ij})_1^L}{(x_j^+)_3^U}, \frac{(x_{ij})_2^L}{(x_j^+)_3^U}, \frac{(x_{ij})_3^L}{(x_j^+)_3^U}; h_{\bar{x}_{ij}}^L),$$

$$(\frac{(x_{ij})_1^U}{(x_j^+)_3^U}, \frac{(x_{ij})_2^U}{(x_j^+)_3^U}, \frac{(x_{ij})_3^U}{(x_j^+)_3^U}; h_{\bar{x}_{ij}}^U)], \tag{3.4}$$

$$(x_j^+)_3^U = max_i\{(x_{ij})_3^U | j = 1, 2, \ldots, n\},$$
$$i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n.$$

Also we can set the best aspired level $(x_j^+)_3^U$ and $j = 1, 2, \ldots, n$ is equal one; otherwise, the worst is zero. The weighted fuzzy normalized decision matrix is shown as following matrix $\bar{U}$:

$$\bar{U} = [\bar{u}_{ij}]_{m \times n}, \ i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n, \tag{3.5}$$

where $\bar{u}_{ij} = \bar{r}_{ij} \otimes \bar{w}_j$.

Step 4: Determine the IVF positive-ideal solution (FPIS) and IVF negative-ideal solution (FNIS). According to the weighted normalized fuzzy-decision matrix, we know that the elements $\bar{u}_{ij}$ are normalized positive IVF and

$$0 \leq (u_{ij})_1^L \leq (u_{ij})_2^L \leq (u_{ij})_3^L \leq 1,$$

$$0 \leq (u_{ij})_1^U \leq (u_{ij})_2^U \leq (u_{ij})_3^U \leq 1,$$

$$0 \leq h_{\bar{x}_{ij}}^L \leq h_{\bar{x}_{ij}}^U \leq 1.$$

Then, we can define the FPIS $\bar{A}^+$ (aspiration levels) and FNIS $\bar{A}^-$ (the worst levels) as following formula:

$$\bar{A}^+ = (\bar{u}_1^+, \ldots, \bar{u}_j^+, \ldots, \bar{u}_n^+) \tag{3.6}$$

$$\bar{A}^- = (\bar{u}_1^-, \ldots, \bar{u}_j^-, \ldots, \bar{u}_n^-) \tag{3.7}$$

where $\bar{u}_j^+ = [(1, 1, 1; 1), (1, 1, 1; 1)] \otimes \bar{w}_j$ and $\bar{u}_j^- = [(0, 0, 0; 1), (0, 0, 0; 1)] \otimes \bar{w}_j, j = 1, 2, \ldots, n.$

Step 5: Calculate the degree of similarity based on [11] between the IVF of each alternative from

FPIS and FNIS is now presented as follows:

$$S_i^+ = \sum_{j=1}^n S(\bar{u}_{ij}, \bar{u}_j^+) = \sum_{j=1}^n$$

$$(1 - \frac{\sum_{k=1}^3 |((u_{ij})_k^U - (u_{ij})_k^L) - ((u_j^+)_k^U - (u_j^+)_k^L)|}{3});$$

$$S_i^- = \sum_{j=1}^n S(\bar{u}_{ij}, \bar{u}_j^-) = \sum_{j=1}^n \qquad (3.8)$$

$$(1 - \frac{\sum_{k=1}^3 |((u_{ij})_k^U - (u_{ij})_k^L) - ((u_j^-)_k^U - (u_j^-)_k^L)|}{3});$$
$$i = 1, 2, \ldots, m.$$

Step 6: The relative closeness can be calculated as follows:

$$R_i = \frac{S_i^-}{S_i^+ + S_i^-}, \quad i = 1, 2, \ldots, m. \qquad (3.9)$$

Then, rank alternatives in terms of their relative closeness's.

4 Selecting a qualified manager

The purpose of the empirical application is to illustrate the use of the suggested method. Suppose that a company intends to choose a manager from four volunteers named A_1, A_2, A_3 and A_4. The decision making committee assesses the four concerned volunteers based on six criteria which follow:

a) Creative (C_1),
b) proficiency in identifying research areas (C_2),
C) proficiency in administration (C_3),
d) personality (C_4),
e) past experience (C_5),
f) self-confidence (C_6).

The number of the committee members is three, labeled as $D\bar{M}_1, D\bar{M}_2$ and $D\bar{M}_3$ respectively. Each decision maker has presented his assessment based on linguistic variable for rating performance and importance of each criterion by a linguistic variable as depicted in Appendix. The comparison of the importance or preference of one criterion, attribute or alternative over another can be done with the help of the questionnaire. The method of calculating priority weights of the different decision alternatives is discussed below.

Step 1: The weights of evaluation dimensions. We adopt IVF-AHP method to evaluate the weights of different dimensions. Following the construction of IVF-AHP model, it is extremely important that experts fill the judgment matrix. The following section demonstrates the computational procedure of the weights of dimensions.

(1) According to the committee with three representatives about the relative important of dimension, then the pairwise comparison matrices of dimensions will be obtained. We apply the IVFN defined in Table 1 and Table 2. We transfer the linguistic scales to the corresponding IVFN.

(2) Computing the elements of synthetic pairwise comparison matrix by using the geometric mean method suggested by Buckley [5] that is $\bar{a}_{ij} = (\bar{a}_{ij}^1 \otimes \bar{a}_{ij}^2 \otimes \bar{a}_{ij}^3)^{1/3}$, for $\bar{dm}_{12}$ as the example:

$$\bar{c}_{12} = (MH \otimes ML \otimes VH)^{1/3} =$$

$$[((4 \times 2 \times 6)^{1/3}, (5 \times 3 \times 7)^{1/3}, (6 \times 4 \times 8)^{1/3}; 0.5),$$

$$((3 \times 1 \times 5)^{1/3}, (7 \times 5 \times 3)^{1/3}, (9 \times 7 \times 5)^{1/3}; 1)] =$$

$$[(3.63, 4.72, 5.77; 0.5), (2.47, 4.72, 6.80; 1)]$$

It can be obtained the other matrix elements by the same computational procedure, therefore

$$\bar{C} = \begin{array}{c} \\ C_1 \\ C_2 \\ \vdots \\ C_6 \end{array} \begin{array}{c} C_1 \quad C_2 \quad \cdots \quad C_6 \\ \left[\begin{array}{cccc} \bar{c}_{11} & \bar{c}_{12} & \cdots & \bar{c}_{16} \\ \bar{c}_{21} & \bar{c}_{22} & \cdots & \bar{c}_{26} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{c}_{61} & \bar{c}_{62} & \cdots & \bar{c}_{66} \end{array} \right], \end{array}$$

Table 1: Definitions of linguistic variables for the importance of each criterion.

IVF	Linguistic
$\bar{7}$	Very high (VH)
$\bar{6}$	High (H)
$\bar{5}$	Medium high (MH)
$\bar{4}$	Medium (M)
$\bar{3}$	Medium low (ML)
$\bar{2}$	Low (L)
$\bar{1}$	Very low (VL)

Table 2: Definitions of linguistic variables for the importance of each criterion.

IVF	Scale of IVF
$\bar{7}$	$[(6,7,8;0.9),(5,7,9;1)]$
$\bar{6}$	$[(5,6,7;0.9),(4,6,8;1)]$
$\bar{5}$	$[(4,5,6;0.75),(3,5,7;1)]$
$\bar{4}$	$[(3,4,5;0.75),(2,4,6;1)]$
$\bar{3}$	$[(2,3,4;0.5),(1,3,5;1)]$
$\bar{2}$	$[(1,2,3;0.5),(1,2,4;1)]$
$\bar{1}$	$[(1,1,1;1),(1,1,1;1)]$

where

$$\bar{c}_{54} = [(0.2, 0.3, 0.5; 0.5), (0.2, 0.3, 0.7; 1)],$$
$$\bar{c}_{56} = [(0.4, 0.5, 0.6; 0.9), (0.4, 0.5, 0.7; 1)],$$
$$\bar{c}_{61} = [(0.6, 0.7, 1; 0.5), (0.5, 0.7, 1.4; 1)],$$
$$\bar{c}_{62} = [(0.3, 0.4, 0.5; 0.5), (0.3, 0.4, 0.6; 1)],$$
$$\bar{c}_{63} = [(1.6, 2, 2.5; 0.75), (1.3, 2, 3.1; 1)],$$
$$\bar{c}_{64} = [(3.9, 4.9, 5.9; 0.75), (2.8, 4.9, 6.9; 1)],$$
$$\bar{c}_{65} = [(1.6, 1.9, 2.2; 0.9), (1.4, 1.9, 2.5; 1)],$$
$$\bar{c}_{14} = \bar{c}_{15}.$$

$$\bar{c}_{11} = \bar{c}_{22} = \bar{c}_{33} = \bar{c}_{44} = \bar{c}_{55} = \bar{c}_{66} = 1,$$
$$\bar{c}_{13} = [(1.4, 1.8, 2.2; 0.75), (1.1, 1.8, 2.8; 1)],$$
$$\bar{c}_{14} = [(2.8, 3.9, 4.9; 0.5), (1.8, 3.9, 5.9; 1)],$$
$$\bar{c}_{16} = [(1, 1.3, 1.6; 0.5), (0.7, 1.3, 1.9; 1)],$$
$$\bar{c}_{21} = [(0.2, 0.2, 0.3; 0.5), (0.2, 0.2, 0.4; 1)],$$
$$\bar{c}_{23} = [(0.7, 0.8, 1; 0.5), (0.5, 0.8, 1.2; 1)],$$
$$\bar{c}_{24} = [(1, 1.6, 2.1; 0.5), (1, 1.6, 2.5; 1)],$$
$$\bar{c}_{25} = [(0.2, 0.2, 0.3; 0.5), (0.2, 0.2, 0.4; 1)],$$
$$\bar{c}_{26} = [(1.8, 2.2, 2.6; 0.75), (1.5, 2.2, 3.3; 1)],$$
$$\bar{c}_{31} = [(0.4, 0.5, 0.7; 0.75), (0.3, 0.5, 0.9; 1)],$$
$$\bar{c}_{32} = [(1, 1.2, 1.4; 0.5), (0.8, 1.2, 1.9; 1)],$$
$$\bar{c}_{34} = [(4.9, 5.9, 6.9; 0.75), (3.9, 5.9, 7.9; 1)],$$
$$\bar{c}_{35} = [(0.9, 1.1, 1.2; 0.75), (0.8, 1.1, 1.4; 1)],$$
$$\bar{c}_{36} = [(0.4, 0.5, 0.6; 0.75), (0, 0.5, 0.7; 1)],$$
$$\bar{c}_{41} = [(0.2, 0.2, 0.3; 0.5), (0.1, 0.2, 0.5; 1)],$$
$$\bar{c}_{42} = [(0.5, 0.6, 1; 0.5), (0.4, 0.6, 1; 1)],$$
$$\bar{c}_{43} = [(0.1, 0.2, 0.2; 0.75), (0.1, 0.2, 0.3; 1)],$$
$$\bar{c}_{45} = [(1.8, 2.9, 3.9; 0.5), (1.2, 2.8, 4.9; 1)],$$
$$\bar{c}_{46} = [(0.1, 0.2, 0.2; 0.75), (0.1, 0.2, 0.3; 1)],$$
$$\bar{c}_{51} = [(0.2, 0.2, 0.3; 0.5), (0.1, 0.2, 0.5; 1)],$$
$$\bar{c}_{52} = [(3.6, 4.7, 5.8; 0.5), (2.5, 4.7, 6.8; 1)],$$
$$\bar{c}_{53} = [(0.8, 0.9, 1; 0.75), (0.7, 0.9, 1.2; 1)],$$

(3) To calculate the fuzzy weights of dimensions, the computational procedures are displayed as following parts

$$\bar{r}_1 = [(1.8, 2.3, 2.8; 0.5), (1.3, 2.3, 2.8; 1)],$$
$$\bar{r}_2 = [(0.58, 0.71, 0.86; 0.5), (0.51, 0.71, 1.08; 1)],$$
$$\bar{r}_3 = [(0.96, 1.12, 1.3; 0.5), (0.56, 1.12, 1.56; 1)],$$
$$\bar{r}_4 = [(0.4, 0.5, 0.64; 0.5), (0.34, 0.5, 0.79; 1)],$$
$$\bar{r}_5 = [(0.65, 0.77, 0.95; 0.5), (0.54, 0.77, 1.17; 1)],$$
$$\bar{r}_6 = [(1.17, 1.38, 1.62; 0.5), (0.98, 1.38, 1.94; 1)].$$

The weights of each dimension are:

$$\bar{w}_1 = [(0.23, 0.35, 0.5; 0.5),$$
$$(0.14, 0.35, 0.67; 1)],$$
$$\bar{w}_2 = [(0.07, 0.1, 0.15; 0.5),$$
$$(0.05, 0.1, 0.25; 1)],$$
$$\bar{w}_3 = [(0.12, 0.16, 0.23; 0.5),$$
$$(0.06, 0.16, 0.36; 1)],$$
$$\bar{w}_4 = [(0.05, 0.07, 0.11; 0.5),$$
$$(0.04, 0.07, 0.18; 1)],$$
$$\bar{w}_5 = [(0.08, 0.11, 0.17; 0.5),$$
$$(0.06, 0.11, 0.27; 1)],$$
$$\bar{w}_6 = [(0.14, 0.2, 0.29; 0.5),$$
$$(0.1, 0.2, 0.45; 1)].$$

Each decision maker has presented his assessment based on linguistic variable for A_1, A_2, A_3 and A_4 as depicted in Table 3.

Now the proposed approach to develop the TOPSIS for fuzzy TOPSIS can be defined as follows:

1) According to the committee with three representatives, the importance of the criteria and the rating of alternatives with respect to each criterion can be calculated as:

$$\bar{x}_{ij} = \tfrac{1}{3}[\bar{x}_{ij}^1 \oplus \bar{x}_{ij}^2 \oplus \bar{x}_{ij}^3],$$

$$i = 1, \dots, 4; \ j = 1, \dots 6.$$

Therefore, we have:

$$\bar{D} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{cccc} C_1 & C_2 & \cdots & C_6 \\ \left[\begin{array}{cccc} \bar{x}_{11} & \bar{x}_{12} & \cdots & \bar{x}_{16} \\ \bar{x}_{21} & \bar{x}_{22} & \cdots & \bar{x}_{26} \\ \bar{x}_{31} & \bar{x}_{32} & \cdots & \bar{x}_{36} \\ \bar{x}_{41} & \bar{x}_{42} & \cdots & \bar{x}_{46} \end{array} \right], \end{array} \quad (4.10)$$

where

$$\bar{x}_{11} = [(5.83, 6.33, 7.33; 0.9),$$
$$(5.33, 6.33, 8.33; 1)],$$
$$\bar{x}_{12} = [(3.67, 4.33, 5.33; 0.75),$$
$$(3, 4.33, 6.33; 1)],$$
$$\bar{x}_{13} = [(6.5, 7, 8; 0.9),$$
$$(6, 7, 9; 1)],$$
$$\bar{x}_{14} = [(1, 1.33, 1.5; 0.5),$$
$$(1, 1.33, 1.67; 1)],$$
$$\bar{x}_{15} = [(1.67, 2.67, 3.5; 0.5),$$
$$(1, 2.67, 4.33; 1)],$$

$$\bar{x}_{16} = [(2.67, 3.33, 4.17; 0.5),$$
$$(2.33, 3.33, 5; 1)],$$
$$\bar{x}_{21} = [(6.176.67, 7.67; 0.9),$$
$$(5.67, 6.67, 8.67; 1)],$$
$$\bar{x}_{22} = [(1.67, 2.67, 3.33; 0.5),$$
$$(1, 2.67, 4.33; 1)],$$
$$\bar{x}_{23} = [(4.83, 5.33, 6.33; 0.75),$$
$$(4.33, 5.33, 7.33; 1)],$$
$$\bar{x}_{24} = [(1.33, 2.33, 3; 0.5),$$
$$(1, 2.33, 3.67; 1)],$$
$$\bar{x}_{25} = [(2.5, 3.33, 4.33; 0.5),$$
$$(1.67, 3.33, 5.33; 1)],$$
$$\bar{x}_{26} = [(5.33, 6, 7; 0.75),$$
$$(4.67, 6, 8; 1)],$$
$$\bar{x}_{31} = [(6.17, 6.67, 7.67; 0.9),$$
$$(5.67, 6.67, 8.67; 1)],$$
$$\bar{x}_{32} = [(2.17, 3, 3.83; 0.5),$$
$$(1.67, 3, 4.67; 1)],$$
$$\bar{x}_{33} = [(4.33, 5, 6; 0.75),$$
$$(3.67, 5, 7; 1)],$$
$$\bar{x}_{34} = [(3.67, 4.33, 5.33; 0.5),$$
$$(3, 4.33, 6.33; 1)],$$
$$\bar{x}_{35} = [(3.67, 4.33, 5.33; 0.75),$$
$$(3, 4.33, 6.33; 1)],$$
$$\bar{x}_{36} = [(5.83, 6.33, 7.33; 0.9),$$
$$(5.33, 6.33, 8.33; 1)],$$
$$\bar{x}_{41} = [(5.83, 6.33, 7.33; 0.9),$$
$$(5.33, 6.33, 8.33; 1)],$$
$$\bar{x}_{42} = [(3.83, 4.67, 5.67; 0.75),$$
$$(3, 4.67, 6.67; 1)],$$
$$\bar{x}_{43} = [(5, 5.67, 6.67; 0.75),$$
$$(4.33, 5.67, 7.67; 1)],$$
$$\bar{x}_{44} = [(5, 5.67, 6.67; 0.75),$$
$$(4.33, 5.67, 7.67; 1)],$$
$$\bar{x}_{45} = [(6.5, 7, 8; 0.9),$$
$$(6, 7, 9; 1)],$$
$$\bar{x}_{46} = [(5.33, 6, 7; 0.75),$$
$$(4.67, 6, 8; 1)].$$

2) The normalization process can be performed by Eq. (3.4):

$$\bar{r}_{11} = [(0.67, 0.73, 0.85; 0.9),$$
$$(0.61, 0.73, 0.96; 1)],$$
$$\bar{r}_{12} = [(0.55, 0.65, 0.8; 0.75),$$
$$(0.45, 0.65, 0.95; 1)],$$
$$\bar{r}_{13} = [(0.72, 0.78, 0.89; 0.9),$$
$$(0.67, 0.78, 1; 1)],$$

$\bar{r}_{14} = [(0.13, 0.17, 0.2; 0.5),$
$(0.13, 0.17, 0.22; 1)],$
$\bar{r}_{15} = [(0.19, 0.3, 0.39; 0.5),$
$(0.11, 0.3, 0.48; 1)],$
$\bar{r}_{16} = [(0.32, 0.4, 0.5; 0.5),$
$(0.28, 0.4, 0.6; 1)],$
$\bar{r}_{21} = [(0.71, 0.77, 0.88; 0.9),$
$(0.65, 0.77, 1; 1)],$
$\bar{r}_{22} = [(0.25, 0.4, 0.5; 0.5),$
$(0.15, 0.4, 0.65; 1)],$
$\bar{r}_{23} = [(0.54, 0.6, 0.7; 0.75),$
$(0.48, 0.6, 0.81; 1)],$
$\bar{r}_{24} = [(0.17, 0.3, 0.39; 0.5),$
$(0.13, 0.3, 0.48; 1)],$
$\bar{r}_{25} = [(0.28, 0.37, 0.48; 0.5),$
$(0.19, 0.37, 0.59; 1)],$
$\bar{r}_{26} = [(0.64, 0.72, 0.84; 0.75),$
$(0.56, 0.72, 0.96; 1)],$
$\bar{r}_{31} = [(0.71, 0.77, 0.88; 0.9),$
$(0.66, 0.77, 1; 1)],$
$\bar{r}_{32} = [(0.33, 0.45, 0.57; 0.5),$
$(0.25, 0.45, 0.7; 1)],$
$\bar{r}_{33} = [(0.48, 0.56, 0.67; 0.75),$
$(0.41, 0.56, 0.78; 1)],$
$\bar{r}_{34} = [(0.48, 0.56, 0.69; 0.5),$
$(0.45, 0.56, 0.83; 1)],$
$\bar{r}_{35} = [(0.4, 0.48, 0.59; 0.75),$
$(0.33, 0.48, 0.7; 1)],$
$\bar{r}_{36} = [(0.7, 0.76, 0.88; 0.9),$
$(0.64, 0.76, 1; 1)],$
$\bar{r}_{41} = [(0.67, 0.73, 0.85; 0.9),$
$(0.61, 0.73, 0.96; 1)],$
$\bar{r}_{42} = [(0.57, 0.7, 0.85; 0.75),$
$(0.45, 0.7, 1; 1)],$
$\bar{r}_{43} = [(0.56, 0.63, 0.74; 0.75),$
$(0.48, 0.63, 0.85; 1)],$
$\bar{r}_{44} = [(0.65, 0.74, 0.87; 0.75),$
$(0.56, 0.74, 1; 1)],$
$\bar{r}_{45} = [(0.72, 0.78, 0.89; 0.9),$
$(0.67, 0.78, 1; 1)],$
$\bar{r}_{46} = [(0.64, 0.72, 0.84; 0.75),$
$(0.56, 0.72, 0.96; 1)].$

3) The weighted fuzzy normalized decision matrix can be performed by Eq. (3.5):

$\bar{u}_{11} = [(0.16, 0.26, 0.42; 0.5),$
$(0.09, 0.26, 0.64; 1)],$

$\bar{u}_{12} = [(0.04, 0.06, 0.12; 0.5),$
$(0.02, 0.06, 0.24; 1)],$
$\bar{u}_{13} = [(0.09, 0.12, 0.2; 0.5),$
$(0.04, 0.12, 0.36; 1)],$

$\bar{u}_{14} = [(0.01, 0.01, 0.02; 0.5),$
$(0.01, 0.01, 0.04; 1)],$
$\bar{u}_{15} = [(0.02, 0.03, 0.07; 0.5),$
$(0.01, 0.03, 0.13; 1)],$

$\bar{u}_{16} = [(0.04, 0.08, 0.14; 0.5),$
$(0.03, 0.08, 0.27; 1)],$
$\bar{u}_{21} = [(0.16, 0.27, 0.44; 0.5),$
$(0.09, 0.27, 0.67; 1)],$
$\bar{u}_{22} = [(0.02, 0.04, 0.07; 0.5),$
$(0.01, 0.04, 0.16; 1)],$
$\bar{u}_{23} = [(0.06, 0.1, 0.16; 0.5),$
$(0.03, 0.1, 0.29; 1)],$
$\bar{u}_{24} = [(0.01, 0.02, 0.04; 0.5),$
$(0.01, 0.02, 0.09; 1)],$
$\bar{u}_{25} = [(0.02, 0.04, 0.08; 0.5),$
$(0.01, 0.04, 0.16; 1)],$

$\bar{u}_{26} = [(0.09, 0.14, 0.24; 0.5),$
$(0.06, 0.14, 0.43; 1)],$
$\bar{u}_{31} = [(0.16, 0.27, 0.44; 0.5),$
$(0.09, 0.27, 0.67; 1)],$
$\bar{u}_{32} = [(0.02, 0.04, 0.08; 0.5),$
$(0.01, 0.04, 0.17; 1)],$
$\bar{u}_{33} = [(0.06, 0.09, 0.15; 0.5),$
$(0.02, 0.09, 0.28; 1)],$
$\bar{u}_{34} = [(0.02, 0.04, 0.07; 0.5),$
$(0.02, 0.04, 0.15; 1)],$
$\bar{u}_{35} = [(0.03, 0.05, 0.1; 0.5),$
$(0.02, 0.05, 0.19; 1)],$
$\bar{u}_{36} = [(0.1, 0.15, 0.25; 0.5),$
$(0.06, 0.15, 0.45; 1)],$

$\bar{u}_{41} = [(0.15, 0.25, 0.42; 0.5),$
$(0.08, 0.25, 0.64; 1)],$
$\bar{u}_{42} = [(0.04, 0.07, 0.13; 0.5),$
$(0.02, 0.07, 0.25; 1)],$
$\bar{u}_{43} = [(0.07, 0.1, 0.17; 0.5),$
$(0.03, 0.1, 0.31; 1)],$
$\bar{u}_{44} = [(0.03, 0.05, 0.09; 0.5),$
$(0.02, 0.05, 0.18; 1)],$
$\bar{u}_{45} = [(0.06, 0.08, 0.15; 0.5),$
$(0.04, 0.08, 0.27; 1)],$
$\bar{u}_{46} = [(0.09, 0.14, 0.24; 0.5),$
$(0.06, 0.14, 0.43; 1)].$

4) We obtain define the FPIS $\bar{A}^+$ and FNIS $\bar{A}^-$ by means of Eq. (3.6) and Eq. (3.7).

5) Finally, we calculate the relative closeness by Eq. (3.8) and Eq. (3.9):

$$R_1 = 0.491, \quad R_2 = 0.492,$$
$$R_3 = 0.489, \quad R_4 = 0.487.$$

Table 3: Definitions of linguistic variables for the ratings.

IVF	Scale of IVF
$\bar{7}$	[(6.5,7,8;0.9),(6,7,9;1)]
$\bar{6}$	[(5.5,6,7;0.9),(5,6,8;1)]
$\bar{5}$	[(4,5,6;0.75),(3,5,7;1)]
$\bar{4}$	[(3.5,4,5;0.75),(3,4,6;1)]
$\bar{3}$	[(2,3,4;0.5),(1,3,5;1)]
$\bar{2}$	[(1,2,2.5;0.5),(1,2,3;1)]
$\bar{1}$	[(1,1,1;1),(1,1,1;1)]

Therefore, the final ranking is $A_2 \succ A_1 \succ A_3 \succ A_4$.

5   Conclusion

In this paper, we have presented a new method for fuzzy AHP and fuzzy TOPSIS model to evaluate different alternatives based on the proposed similarity measure between interval- valued fuzzy numbers and the proposed interval-valued fuzzy number arithmetic operators. The proposed fuzzy AHP and fuzzy TOPSIS method provides a useful way for handling fuzzy MCDM problems based on fuzzy numbers. Utilizing the proposed fuzzy AHP and fuzzy TOPSIS method, a manager selection problem was examined and the results are demonstrated.

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