Available online at http://ijim.srbiau.ac.ir/
Int. J. Industrial Mathematics (ISSN 2008-5621)
Vol. 6, No. 4, 2014 Article ID IJIM-00569, 6 pages
Research Article

Science and Research Branch (IAU)

Partial Differential Equations applied to Medical Image Segmentation

B. Bagheri *†, R. Ezzati ‡

Abstract

This paper presents an application of partial differential equations(PDEs) for the segmentation of abdominal and thoracic aortic in CTA datasets. An important challenge in reliably detecting aortic is the need to overcome problems associated with intensity inhomogeneities. Level sets are part of an important class of methods that utilize partial differential equations (PDEs) and have been extensively applied in image segmentation. A kernel function in the level set formulation aids the suppression of noise in the extracted regions of interest and then guides the motion of the evolving contour for the detection of weak boundaries. The speed of curve evolution has been significantly improved with a resulting decrease in segmentation time compared with traditional implementations of level sets, and are shown to be more effective than other approaches in coping with intensity inhomogeneities. We have applied the Courant Friedrichs Levy (CFL) condition as stability criterion for our algorithm.

Keywords : Partial differential equations; Image segmentation; Level-sets; Abdominal; Thoracic aorta.

1 Introduction

PDE are equations that contain some variable functions and their partial derivatives. PDEs are used to formulate problems engaging functions of several variables to explain a large variety of phenomena, like heat, sound, electrostatics, fluid flow , electrodynamics, and elasticity. PDEs are equations that engage rates of change to continuous variables that have many applications but when applied to discrete and noisy data they can be unstable, which may require a numerical remedy. Image analysis provides a rich field for the development of algorithms based on PDE formulation, leading to applications analysis,

with special emphasis virtual reality and robotics to biomedical imaging problems. In addition, image analysis covers the development and implementation of algorithms and strategies based on geometrical, statistical, physical and functional modelling to solve problems such as the representation of pictorial data, visualization, feature extraction, segmentation, texture, shape and motion measurements. This chapter investigates the connection between continuous and discrete variables of PDEs applied to noisy digital images, and in particular the problem of stability.

Image segmentation commonly utilises one of two main approaches to classify pixels belonging to a particular object or region, either edge-based or region-based. Edge-based segmentation looks for discontinuities in image intensity [2]-[11], whilst region-based methods look for uniformity within an image sub-region, based on some consistent property such as intensity, colour or texture [2, 3, 9, 12]. Active contours methods, also referred to as deformable models, evolve an image contour from an initial guess using image

*Corresponding author. bagheri@iaufb.ac.ir
†Department of Mathematics, Firoozkooh Branch, Islamic Azad University, Firoozkooh, Iran.
‡Department of Mathematics, Karaj Branch, Islamic Azad University, Karaj, Iran.

forces derived from region properties to drive the search to locate the boundaries of the desired objects. Level sets provide an implementation of an active contour method based on regions or edges. A local energy functional has been defined in terms of a contour and two fitting functions that locally pproximate the image intensities on either side of the contour. An important characteristic of active contour methods is to identify the appropriate stopping condition for the curve evolution. Mathematically the level set of a real modelled function (f) of n variables is a set where the function takes as a constant value that can describe a boundary or interface. A level set is surrounded implicitly as a constant set in a function defined in a upper dimensional space. This model has significant advantages: first, the topological changes of the contours can be simply handled; second, the numerical implementation and conception can be simply adapted to solve any dimensional problem; third, the areas outside and inside an active contour can be simply determined. In this paper we have used the Courant Friedrichs Levy (CFL) condition to establish the necessary conditions for numerical convergence of the level set PDE's, which also satisfies a criterion for algorithmic stability. The method is applied to the detection of aneurysms in the cardiovascular system imaged by computed tomography angiography (CTA), which uses a contrast dye to enhance detection of the vasculature. The aorta is the major artery which carries blood from the heart and distributes it via many branches to all the organs of the body. The aorta is divided into four sections: the ascending aorta, the aortic arch, the thoracic (descending) aorta and the abdominal aorta. Blockage or weakness in the artery walls can lead to aneurysm, a distension of the vessel wall that is prone to rupture and subsequent haemorrhage in severe cases. Reliable detection of aortic aneurysm must overcome problems of intensity inhomogeneities and image noise. Level sets are part of an important class of methods that utilize partial differential equations (PDEs) and have been extensively applied in image segmentation. The approach uses a kernel function to aid noise suppression and then guides the search motion of the evolving contour, particularly for the detection of weak boundaries. Segmentation time can be significantly reduced by improving the onvergence criteria, for which we have applied the CFL condition. The rest of the paper is organized as follows. Section 2 reviews the level set method, edge based and region-based active contours and introduces the proposed computational methods. Section 3 describes the numerical implementation and experimental results and conclusions are presented in Section 4.

2 Level Set Method and Local Fitting Binary

The level set method developed by Osher and Sethian [2] has been used in the formulation of several region or boundary based approaches for image segmentation and offers highly robust and accurate techniques for tracking interfaces moving under complex motions [4]. Level set segmentation involves solving the energy-based active contour minimization problems by the computation of geodesics or minimal distance curves [2] - [13]. The main idea of the level set method is to represent a closed curve on the plane as a zero level set of a higher dimension function. The motion of the curve is then embedded within the motion of the higher dimension surface. Basically, this means that the closed curves in a two dimensional surface are regarded as a continuous surface of a three- dimensional space [7]. The definition of a smoothing function $\phi(x, y, t)$represents the surface while the set of definitions $\phi(x, y, t) = 0$define curves. Thus the evolution of a curve can be transformed into the evolution of a three- dimensional level set function. Given a level set function whose zero level set corresponds to a curve, with the curve as the boundary; the whole surface can be divided into an internal region and an external region of the curve. The common movement formula of the partial differential equations is:

$$\phi_t(x, y, t) + V\,|\nabla\phi(x, y.t)| = 0,$$
$$\phi(x, y, t = 0) = \phi_0(x, y),$$

where V denotes a constant speed term to move forwards or inwards the contour. A special case is the motion by mean curvature [11] where

$$V = div(\frac{\nabla\phi(x, y, t)}{|\nabla\phi(x, y, t)|})$$

is the curvature of the level-curve ofϕ passing through $(x,\ y)$.

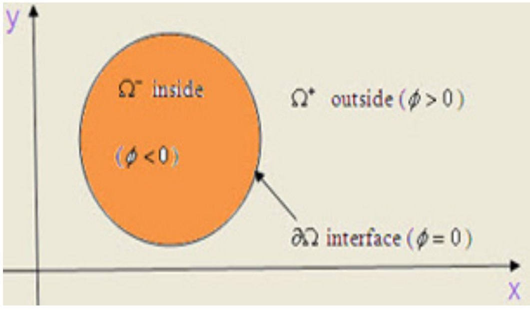

Figure 1: Implicit representation of closed curve.

The purpose of the Heaviside function is to smooth and calculate the length of the interface (initial curve) and the area inside and outside the object. The Heaviside or unit step function, usually denoted by H, is one for positive and zero for negative arguments. The function is used in signal processing to represent a signal that switches on at a specified time and stay switched on indefinitely. The Dirac delta function, or δ function, is zero everywhere except at zero. The Dirac delta function, is the derivative of the Heaviside function, i.e. $\delta(x) = \nabla H(\phi(x)).\vec{N}$ where $\vec{N}$is the normal direction. By defining a domain Ω that is divided into two regions, where Ω^+ refers to the outside portion, $\Omega^-$to the inside portion and $\partial\Omega$ is the interface (boundary) as shown in figure 1.

We consider

$$\partial\Omega = \{(x,y) \in \Omega : \phi(x,y) = 0\}$$

and define the Heaviside function [90] on ϕ , $H(\phi) = \begin{cases} 1 & \phi \geq 0 \\ 0 & \phi < 0 \end{cases}$, the characteristic function χ^- and χ^+ are defined as

$$\chi^+(x) = H(\phi(x))$$

and

$$\chi^-(x) = 1 - H(\phi(x))$$

which shows the exterior and interior regions respectively. The length of interface (initial curve) is calculated by the Heaviside function as follows:

$$Length(\partial\Omega) = |\partial\Omega| = \int_\Omega |\nabla H(\phi)|\, dxdy$$

$$= \int_\Omega \delta(\phi)\, |\nabla\phi|\, dxdy$$

In addition, the area inside the interface is

$$Area(inside(\partial\Omega)) = \int_\Omega H(\phi)dxdy$$

The following regularity of the Heaviside function (H) was introduced by Abramowits and Stegun:

$$H_\varepsilon = \frac{1}{2}[1 + \frac{2}{\pi}\arctan(\frac{x}{\varepsilon})]$$

The derivative of H_ε is

$$\delta_\varepsilon(x) = H'_\varepsilon(x) = \frac{1}{\pi}\frac{\varepsilon}{\varepsilon^2 + x^2}$$

where $\delta_\varepsilon(x)$is the Dirac delta function [6]. As epsilon (ε) tends to zero, both approximations converge to H andδ.

The level set method based on the local fitting binary (LBF) is given by the equation [2]:

$$E(\phi, f_1, f_2) = \sum_{i=1}^{2}$$

$$\lambda_i \int (\int K_\sigma\,(x-y)\,|I(y) - f_i(x)|^2\, M_i(\phi(y))dy)dx$$
$$+$$
$$\nu \int |\nabla H(\phi(x))|\, dx + \mu \int \tfrac{1}{2}(|\nabla\phi(x)| - 1)^2 dx$$

where $k_\sigma$is a kernel function (Gaussian kernel) that decreases and approaches zero as |x-y| increases. Also $f_1(x), f_2(x)$ approximate the image intensities inside and outside the contour. $M_1 = H(\phi)$ and $M_2 = 1 - H(\phi)$. For minimizing the LBF model, first the functional form of model is conformed with level set method, next in order to solve the level set equation, the implicit finite difference scheme is applied and gradient descent will employed to minimize the energy functional with respect to the level set function $\phi(x,y,t)$ which shown as follows:

$$\frac{\partial\phi}{\partial t} = -\delta_\varepsilon(\phi)(\lambda_1 e_1 - \lambda_2 e_2) + \nu\delta_\varepsilon(\phi) div(\tfrac{\nabla\phi}{|\nabla\phi|})$$
$$+ \mu(\nabla^2\phi - div(\tfrac{\nabla\phi}{|\nabla\phi|})) \qquad (1)$$

Where, δ_ε is the smooth Dirac deltas function and $e_1, e_2$are the functions as follows:

$$e_i(x) = \int K_\sigma\,(y-x)\,|I(x) - f_i(y)|^2\, dy$$

$i = 1, 2.$

The term $-\delta_\varepsilon(\phi)(\lambda_1 e_1 - \lambda_2 e_2)$ drives the active contour toward the object's boundary and the second term has a length shortening (arc length) term [5]-[13]. The third term is called a level set regularization term [3], which maintains the regularity of the level set function.

3 Implement and Experimental Results

We have developed a code based on the LBF model to segment the object boundary in medical images. The active contour works well to detect the boundary images inhomogeneity intensity. And also we have applied CFL condition as stability criterion on our algorithm. CFL number is:

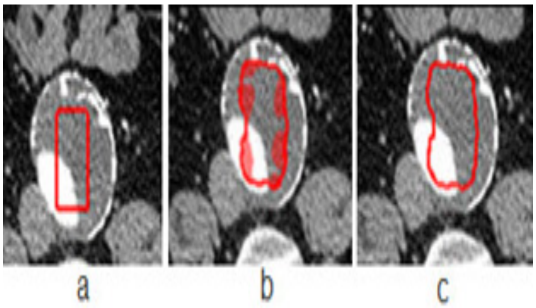

Figure 2: Segmentation result for a CT slice of AAA, (a) initial curve (red rectangle) and the original image, (b) segmentation result after 40 iterations with $\Delta t = 4.0$ and $\mu = 0.259/\Delta t$ that it is unstable,(c) segmentation result after 40 iteration with $\Delta t = 4.0$ and $\mu = 0.249/\Delta t$ where the evolving curve is stable.

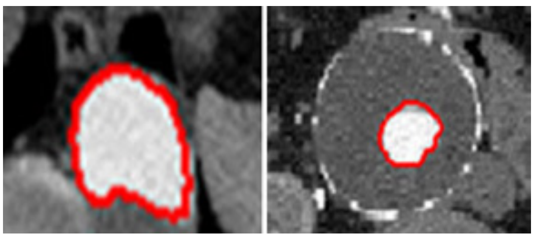

Figure 3: Result of segmentation for the descending thoracic aorta with the final contour that are shown.

$$c = (\frac{v\Delta t}{\Delta x})$$

Where v is the velocity, Δt is the time step, Δx is the length interval. CFL condition is a necessary condition for convergence while solving certain partial differential equations numerically. Consequently, we have segmented the ascending aorta and the descending thoracic aorta in CTA data. A time step (Δt) can be selected that is larger than the time step used in the region-based techniques of Chan - Vese and Li et.al [3]-[12] as using a larger time step can speed up the evolution

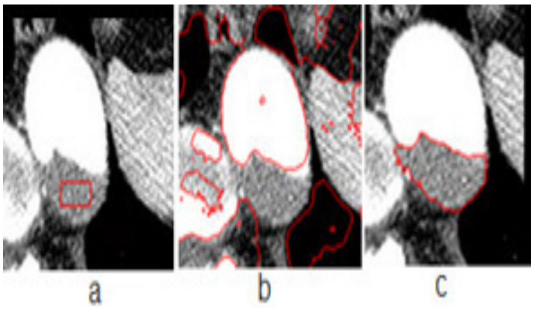

Figure 4: Segmentation result for a CT slice of AAA, (a) Initial contour (red rectangle) and the original image,(b)segmentation result with $\lambda_1 = 3.2$ and $\lambda_2 = 1.2$,(c) segmentation result with $\lambda_1 = 1.2$ and $\lambda_2 = 3.2$.

but may cause errors in the border place. The time step Δt and coefficient m must satisfy the condition ($\Delta t.\mu < 0.25$) in order to maintain the stability of the level set evolution [3]. A range of time steps usually less than ten has been used for most images.

The coefficient of length term (ν) from equation (1) is a constant that controls the curvature term in the evolving function. Chan - Vese investigated a range of values for the coefficient (ν) as it depends on image size and object size, so the length term functions like a scale parameter: if ν is large, then larger objects(for example grouping objects); if $\nu$is small, then smaller objects (such as points due to noise) will be detected [12]. The results used values between 3 and 7 for aorta thrombus segmentation.

To validate and assess the robustness of the level set segmentation algorithm, we have applied the algorithm to 2D image slices of a dataset of twelve CTA scans supplied by Prof S. Qanaldi (Lausanne University). The scan data have dimensions of between 512*512*201 and 512*512*897 voxels and the voxel size ranged from 0.53*0.53*0.63 to 0.94*0.94*2.50 mm. The acquisition protocol used an x- ray tube voltage that ranged from 120 - 140 KV and a mean tube current from 241 - 350ma. Eight scans contain thrombus, with an average of 79 slices (including an aortal thrombus) and four scans without thrombus.
The main steps of the algorithm can be expressed as follow: Initialize the level set function to be binary function as follows:

$$\phi(x,y,t=0)=\begin{cases} -c & x\in\Omega_0-\partial\Omega_0 \\ 0 & x\in\partial\Omega_0 \\ c & x\in\Omega-\Omega_0 \end{cases}$$

Where c>0 is a constant, $\Omega_0$is a subset of the image domain $\Omega$and $\partial\Omega_0$ is the boundary ofΩ_0. Evolve the level set function according to (1).

3. Check with iteration number whether the evolution is stationary or no.

All partial derivatives can be discretized as central finite differences and also the temporal derivative is discretized as a forward difference. Therefore there are in total six convolutions, two convolutions $k_\sigma * I$ and $k_\sigma * 1$ can be computed only once before the iterations and four convolutions must be computed in each iteration. To validate and assess the robustness of the proposed method, we used computed tomography angiography (CTA) images to detect aorta and thoracic abdominal (AA and TA). The CTA images were collected at Lausanne University. This model is very encouraged. So, for the next work, we would like to validate this model in 3-D. The methodology has been tested with several data with good result for images by intensity inhomogeneity, rather noisy and part of boundary is weak that are shown in figures 3.

The PC model [1]-[12] generally fails to segment images with intensity inhomogeneity. Therefore some part of the background/foreground is incorrectly identified as the foreground/ background, can be seen the difficulties in segmenting images with intensity inhomogeneity.

The benefits of using a binary function over a SDF as the initial level set are with the use of positive values outside and negative values inside the boundary. To compute the convolutions in $f_1(x)$ and $f_2(x)$ more efficiently, the smoothing kernel is a w * w mask, where w is the smallest odd number not less than 4 σ [9]. In this study, the result of segmentation is gained for the smallest scale parameter i.e. $\sigma = 4$ as a larger scale is more strong to the initial curve location and can be as insensitive to the initialization. Setting $\sigma = 12$ would make the process further robust to initialization, nevertheless the result of segmentation may not be as correct as using a small scale when there are intensity inhomogeneities in the image [9]. The coefficients λ_1 and λ_2 are the weights of the two integrals in (1) over the regions outside and inside the initial curve, respectively. In most cases λ_1=λ_2, which leads to a fair opposition between the areas outside and inside the zero level curve throughout the evolution. But in thrombus segmentation, when λ_2 is larger than λ_1 the evolving curve can be stable since the outside of the initial curve has different tissue with the same intensity, as shown in figure 2. The final range of values used were 0.8 <λ_1< 1.8 and 2.2 <λ_2< 4.2 to prevent the emergence of new curves far away from the initial curve. Figure 4 shows the segmentation result using the same initial contour and different values of $\lambda_1$and λ_2; 2b shows that curve evolution is unstable when λ_1 is larger than λ_2 and 2c shows curve evolution is stable when λ_2 is larger than λ_1.

4 Conclusion

We have presented an active contour model based on local binary fitting and which is better adapted to the problem of intensity inhomogeneities in the image. The method was demonstrated to segment the ascending and descending thoracic aorta and the abdominal aorta with desirable performance in the presence of intensity inhomogeneties and weak object boundaries. The time required for segmentation was significantly decreased through more effective convergence criteria. These methodologies are the key elements of solutions to more systems-oriented problems, which include disease diagnosis, image guided intervention/surgery, atlas-based description of anatomical regions, deformation analysis, and visualization of anatomical and physiological processes Finally, the effectiveness of the algorithm has been validated on a CTA dataset to assess its performance in terms of efficiency and accuracy. Further work will be to extend the level set algorithm to 3D which can then be applied to CTA voxel data.

References

[1] A. Tsai, A Yezzi, A. S. Willsky, *Curve evolution implementation of Mumford-Shah functional for image segmentation, denoising, interpolation, and magnification.* IEEE Trans. Image. Proc. 10:1169- 1186, 2001.

[2] C. Li, C. Kao, J. Gore, Z. Ding, *Minimization of region-scalable fitting energy for im-*

age segmentation, IEEE Transactions on Image Processing 17 (2008) 1940-1949.

[3] C. M. Li, C.Y. Xu, C. F. Gui, M. D. Fox, *Level set evolution without reinitialization: a new variational formulation*, in: IEEE Conference on Computer Vision and Pattern Recognition, San Diego (2005) 430- 436.

[4] Feng Zhao, Xianghua Xie., *Interactive Segmentation of Medical Images: A Survey*, Computer Science Swansea University, Singleton Park Swansea SA2 8PP, UK (2012) 263-270.

[5] G. P. Zhu, Sh.Q. Zhang, Q. SH. Zeng, H. Ch, *Wang, Boundary-based image segmentation using binary level set method*, SPIE OE Letters 46 (2007).

[6] Hartman, W.M., Signals, sound, and sensation. Springer, 1997.

[7] I. Macia, M. A. Jon Haitz Legarreta, I. Robin, *Fast AAA Thrombus Segmentation from CTA Images for Endovascular Repair Follow-up*, MICCAI-STENT (2012) 72-79.

[8] J. A. Sethian, *Level Set Methods and Fast Marching methods*, Cambridge University Press, 1999.

[9] J. Lie, M. Lysaker, X. C. Tai, *A binary level set model and some application to Mumford-Shah image segmentation*, IEEE Transaction on Image Processing 15 (2006) 1171-1181.

[10] M. Kass, A. Witkin, D. Terzopoulos, *Snakes: active contour models*, International Journal of Computer Vision 1 (1988) 321-331.

[11] S. J. Osher and J. A. Sethian, *Fronts propagating with curvature dependent speed*, J. Comput. Pysc. 79 (1988) 12-49.

[12] T. F. Chan, L. A. Vese, *Active Contours without Edges*, IEEE Trans Image Proc. 10 (2001) 266-277.

[13] V. Caselles, R. Kimmel, G. Sapiro, *Geodesic active contours*, International Journal of Computer Vision 22 (1997) 61-79.

Bashir Bagheri Nakhjavanlo is working as a member of academic board at Islamic Azad University, Firoozkooh Branch, Iran. His current area of research is medical image analysis , partial differential equation and numerical analysis.

Reza Ezzati was born in Tabriz, Iran in 1974. He received his B.S. degree in pure mathematics in 1997 and M.S. degree in applied mathematics from Azerbaijan Teacher Education University and Tarbyat Modarres University of Tehran, respectively, and his Ph.D degree in applied mathematics from IAU- Science and Research Branch, Tehran, Iran in 2006. He is an Associate Professor in the Department of Mathematics at Islamic Azad University, Karaj Branch, (Iran). He has published about 80 papers in international journals, and he also is the Associate Editor of Mathematical Sciences (a Springer Open Journal). He is the Member of Editorial Board of The Scientific Journal, Journal of Applied Mathematics. He is a Reviewer in the Journals of Iranian Journal of Fuzzy systems, Information Science (Elsevier), Applied Mathematics Letters (Elsevier), Intelligent and Fuzzy Systems, Applied Mathematical Modelling, Applied Mathematics and Computations, and others. His current interests include fuzzy mathematics, especially, on numerical solution of fuzzy systems, fuzzy integral equations and fuzzy interpolation.