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Research Article

Environmental Assessment and Congestion for DMUs With Undesirable Outputs

M. Khodadadi *†, H. Zare Haghighi ‡

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Abstract

In recent years, the assessment of environmental performance has been received considerable attention by environmental policy indicators and decision makers. In this regard, this paper applies data envelopment analysis (DEA) as a management tool for evaluating the environmental performance of the firms and uses the extended bounded adjusted measure (extended BAM) for including undesirable outputs in the environmental assessment. On the other hand, congestion points to a situation where inputs are excessively used. The previous studies of congestion mainly considered the framework of desirable outputs, and undesirable outputs have been disregarded in the evaluation. To overcome these shortcomings, this paper introduces an alternative definition and approach to treat congestion in the simultaneous presence of desirable and undesirable outputs. Then, the presented method is applied on 31 administrative regions of China. As a result of this application, it is observed that seven industries are environmentally efficient and also do not show congestion. Four industries evidence congestion in one or both of their inputs. By reducing the amount of congestion in each input, these industries can increase their desirable output and decrease their undesirable outputs.

Keywords : Data Envelopment Analysis; Environmental performance; Congestion; Undesirable output.

1 Introduction

In recent years, evaluation of environmental performance has been received a great deal of attention by environmental policy analysts and decision makers. Air and water pollution and climate change are now universal policy issues all over the world. As a result, development of organizations with less CO_2 emission and various types of pollutants is a major topic of concern. The important problem is that in the real world, pollutants or wastes are unavoidably produced along with desirable outputs. Therefore, there should exist approaches for studying the environmental performance of organizations leading to the reduction of pollutants and environmental protection issues.

Data envelopment analysis (DEA) is a popular management tool concerned with assessing the performance of comparable decision making units (DMUs) with multiple inputs and outputs. Recently, DEA has been widely employed as an evaluation method to study the environmental performance of organizations. DEA usually supposes that generating more desirable outputs and consuming less input resources is a gauge of efficiency. In DEA-based environmental assessment, however, the efficiency of organizations is influ-

*Corresponding author. m.khodadadi@iaurmia.ac.ir, Tel: +989141464712

†Department of Mathematics, Urmia Branch, Islamic Azad University, Urmia, Iran.

‡Department of Mathematics, Urmia Branch, Islamic Azad University, Urmia, Iran.

enced by the amounts of pollutants and wastes (called undesirable outputs or environmental outputs) that they produced.

In the past few decades, many scholars have tried to model undesirable outputs within the DEA framework. A common treatment of undesirable outputs is to think of them as inputs and apply the traditional DEA models. Some of the literature in this regard include Dyson et al. [26], Hailu and Veeman [1], Dyckhoff and Allen [4], Sueyoshi and Goto [32], Rashidi et al. [10], and Chen et al. [12]. Nevertheless, these procedures prompt concern about two issues. First, the free disposability postulate between inputs and bad (undesirable) outputs implies that a finite amount of inputs can produce an infinite amount of undesirable outputs, while this is physically impossible [20]. Second, the free disposability principle does not determine the link between desirable and undesirable outputs [30].

Seiford and Zhu [14] employed another approach which first multiplies each undesirable output by -1 and then adds a big enough positive number to them to let all negative undesirable outputs be positive. Nevertheless, this approach is only valid under the variable returns to scale condition and furthermore does not reflect a rational production possibility set.

Scheel [5] inverted the undesirable output amounts, and dealt with them as desirable output. However, this nonlinear transformation may change the efficiency frontiers, and hence result in wrong efficiency scores.

In fact, this subject was first methodically treated by Fare et al. [20, 22] publications. They consider undesirable output as an output and try to incorporate them into the production possibility set under a new axiom. Thus, they used the weak disposability assumption, which had been introduced by Shephard [27], between good (desirable) and bad (undesirable) outputs. Later, the study in this direction was extended by scholars such as Fare et al. [23], Zhou et al. [15], Kuosmanen [29], Kousmanen and Podinovski [30], Khoshandam et al. [13], and Lozano [28].

On the other hand, congestion is an economic phenomenon which points out to a situation where inputs are overinvested. A typical example of congestion is the case where an excessively high number of workers in an underground coal mine may lead to the reduction in the output of coal.

The subject of congestion was first defined and extended by Fare and Svensson [18] in 1980. Then, it was examined by Fare and Grosskopf [19] within the data envelopment analysis framework. They imposed the axioms of weak and strong disposability on the production possibility set to identify the evidence of congestion. Besides, their approach suffers from a number of deficiencies [33] dealing with congestion.

Afterwards, a slacks-based measure was presented by Cooper et al. [36] in 1996. It can identify the congested inputs and the amount of congestion in each input. This technique can well avoid the weaknesses of the previous approaches. Nevertheless, it is unable to discriminate the congestion effect from pure technical efficiency.

In addition, Jahanshahloo and Khodabakhshi [3] developed an input relaxation model for improving outputs, and accordingly, calculated the input congestion based upon the proposed model. Wei and Yan [17] in another work evaluated congestion by measuring technical efficiency and pure technical efficiency. This approach does not have the drawbacks of the two models above. Also, it is a simple method and easy to be understood.

Indeed, the investigations into congestion within the DEA framework have received considerable attention in the last few decades. Some of the other investigations in this area include Tone and Sahoo [11], Sueyoshi and Sekitani [31], Noura et al. [2], Zare Haghighi et al. [7, 8] just to name a few.

According to Cooper et al. [35], the common understanding of congestion is that a decrease (increase) in one or more inputs results in an increase (decrease) in one or more outputs. From this point of view, only desirable outputs are considered since all outputs are expected to rise. However, an important issue is that in actual applications, unavoidably undesirable outputs (like pollutants or wastes) are generated along with desirable outputs. Therefore, there should exist an alternative scheme for measuring congestion in the simultaneous presence of both desirable and undesirable outputs.

In this paper, the Kuosmanen [29] technology is chosen in order to refer to the undesirable outputs. The reasons will be described in Section 2 in detail. Then, the extended bounded adjusted measure [6] is used to assess the environmental efficiency in the presence of both good (desirable) and bad (undesirable) outputs. Therefore, following the Wei and Yan idea [17] for measuring congestion, an alternative definition and approach is introduced to treat congestion in the simultaneous presence of desirable and undesirable outputs.

The rest of this paper is organized as follows: In Section 2, the Kuosmanen [29] technology for dealing with undesirable outputs, the approach of Wei and Yan [17] for congestion measurement, and the extended bounded adjusted measure of Zare Haghighi and Rostamy-Malkhalifeh [6] are respectively reviewed within three subsections. Section 3 concentrates on congestion in the simultaneous presence of both desirable and undesirable outputs and offers a plain definition to clarify it. Furthermore, an approach is proposed to identify this type of congestion. In Section 4, the results of the mentioned models are supplied and interpreted, considering an empirical application corresponding to 31 administrative regions of China. The summary and conclusions of the study are provided in Section 5.

2 Preliminaries

At the beginning of our study, n observed and comparable DMUs are considered. The jth DMU, $j \in \{1, ..., n\}$ consumes a column vector of inputs (x_j) in order to generate not only a column vector of desirable (good) outputs (g_j) but also a column vector of undesirable (bad) outputs (b_j). Here, $x_j = (x_{1j}, x_{2j}, ..., x_{mj})^t$, $x_j \geq 0$, $x_j \neq 0$, and $g_j = (g_{1j}, g_{2j}, ..., g_{sj})^t$, $g_j \geq 0$, $g_j \neq 0$, and $b_j = (b_{1j}, b_{2j}, ..., b_{hj})^t$, $b_j \geq 0$, $b_j \neq 0$, where the superscript "t" indicates a vector transpose. Following this, the weakly disposable technology of Kuosmanen [29] for addressing undesirable outputs, and Wei and Yan [17] approach for congestion evaluation are delineated.

2.1 Weakly Disposable Technology

Here, the weakly disposable technology for incorporating undesirable outputs into the production technology is described. The production technology can be described by the set $T = \{(x, g, b) \,|\, x \text{ can produce } (g, b)\}$. Let us consider the following axioms which have been introduced in the DEA literature [20, 22, 29] for including undesirable factors into the production technology:

(A1) *The observed points* (x_j, g_j, b_j), $(j = 1, ..., n)$ *belong to* T.

(A2) *Strong (free) disposability of inputs and good outputs.* If $(x, g, b) \in T$, $0 \leq g' \leq g$ and $x' \geq x$, then $(x', g', b) \in T$.

(A3) *Weak disposability of good and bad outputs.* If $(x, g, b) \in T$, $0 \leq \theta \leq 1$, then $(x, \theta g, \theta b) \in T$.

(A4) *T is convex.*

In DEA, the production technology is determined by the intersection of all subsets of $R^{m+s+h}_{\geq 0}$ that comply with the putative postulates. This is referred to as the *minimum extrapolation principle* [25] and characterizes the smallest possible production set that is consistent with all the mentioned axioms.

Postulate (A3), which was first introduced by Shephard [27], says that proportional reductions of desirable and undesirable outputs are feasible. Furthermore, it specifies the relation between good (desirable) and bad (undesirable) outputs because the pollutants and wastes can be decreased in proportion to the reduction of desirable outputs. The multiplier θ used in this axiom was referred to as the *abatement factor* by Kuosmanen [29].

Fare et al. [24] stated that when undesirable outputs are included in the DEA technology, the corresponding technology can be referred to as environmental DEA technology. The common procedure to modeling weak disposability in the environmental DEA technologies usually implements a single abatement factor [27] for all observed DMUs. Following this technique, Fare and Grosskopf [20] presented the following environmental technology (T_{FG}):

$$
\begin{aligned}
T_{\text{FG}} = \{(x, g, b) |\ & \sum_{j=1}^{n} \lambda_j \, x_j \leq x, \\
& \sum_{j=1}^{n} \theta \lambda_j \, g_j \geq g, \\
& \sum_{j=1}^{n} \theta \lambda_j \, b_j = b, \\
& \sum_{j=1}^{n} \lambda_j \ = \ 1, \\
& \lambda_j \geq 0, \ (j = 1, ..., n), \\
& 0 \leq \theta \leq 1 \ \},
\end{aligned} \tag{2.1}
$$

where λ_j is an unknown variable which is referred to as the intensity weight corresponding to the jth DMU. Associated with Axiom (A3), the multiplier θ allows the proportional reduction of desirable and undesirable outputs.

Later, Kuosmanen [29] explained that there is no theoretical reason why a similar abatement factor should be applied for all DMUs. He also illustrated that the correct execution of weak disposability postulate necessitates the use of n distinctive abatement factors; one for each observed DMU. Therefore, he employed different abatement factors θ_j corresponding to each observed DMU$_j$ $j = 1, ..., n$, and developed the following environmental technology (T_{K}):

$$T_{\mathrm{K}} = \{(x, g, b)| \begin{array}{l} \sum_{j=1}^{n} \lambda_j \ x_j \leq x, \\ \sum_{j=1}^{n} \theta_j \lambda_j \ g_j \geq g, \\ \sum_{j=1}^{n} \theta_j \lambda_j \ b_j = b, \\ \sum_{j=1}^{n} \lambda_j \ = \ 1, \\ \lambda_j \geq 0, \ (j = 1, ..., n), \\ 0 \leq \theta_j \leq 1, (j = 1, ..., n). \} \end{array} \tag{2.2}$$

Subsequently, Fare and Grosskopf [21] supported the method of Shephard's implementation and claimed that a single abatement factor is sufficient for modeling weak disposability assumption and that the Kuosmanen (T_{K}) technology that uses multiple abatement factors is larger than necessary.

However, Kuosmanen and Podinovski [30] showed by a numerical example that a single abatement factor does not produce all feasible production points, and moreover, it results in the violation of the convexity principle.

Furthermore, they [30] demonstrated that T_{K} is the exact technology that is consistent with the minimum extrapolation principle of DEA under the given axioms of (A1), (A2), (A3) and (A4).

Additionally, it should be noted that T_{FG} and T_{K} are both nonlinear, since θ and θ_j are multiplied by λ_j. Nevertheless, Kuosmanen [29] expressed that T_{K} can be linearized by writing λ_j as:

$$\lambda_j = \underbrace{\theta_j \lambda_j}_{\eta_j} + \underbrace{(1 - \theta_j)\lambda_j}_{\mu_j} \ , \ (j = 1, ..., n),$$

where the column vectors $\eta = (\eta_1, ..., \eta_n)$ and $\mu = (\mu_1, ..., \mu_n)$ can be explained as structural weights: vector η denotes structural weights of inputs actively used in production, whereas vector μ represents the weights of inputs that are held inactive. In fact, the output vector $(g\,\eta, b\,\eta)$ is diminished using the scale-down property of weak disposability principle. Therefore, T_{K} can be linearized as follows:

$$T_{\mathrm{K}} = \{(x, g, b)| \begin{array}{l} \sum_{j=1}^{n} (\eta_j + \mu_j) \ x_j \leq x, \\ \sum_{j=1}^{n} \eta_j \ g_j \geq g, \\ \sum_{j=1}^{n} \eta_j \ b_j = b, \\ \sum_{j=1}^{n} (\eta_j + \mu_j) \ = \ 1, \\ \eta_j \geq 0 \ , \ \mu_j \geq 0 \ , \\ (j = 1, ..., n). \ \} \end{array} \tag{2.3}$$

However, Kuosmanen and Podinovski [29] stressed that T_{FG} cannot be linearized by the above method because it is not convex.

2.2 Congestion evaluation

As previously noted, the Wei and Yan [17] approach for congestion evaluation is used in this paper for two reasons. First, it is very simple and easy to be understood and calculated. Second, it maintains the current properties of the production and therefore can be easily combined with the Kuosmanen technology (T_{K}) for measuring congestion with both desirable and undesirable outputs.

To describe this method, first consider the production possibility set (PPS) of variable returns to scale (T_v) as follows:

$$T_v = \{(x, g) \mid x \geq \sum_{j=1}^{n} \lambda_j x_j, \ g \leq \sum_{j=1}^{n} \lambda_j g_j, \\ \sum_{j=1}^{n} \lambda_j = 1, \ \lambda_j \geq 0; \ j = 1, ..., n\} \tag{2.4}$$

Here, λ_j $(j = 1, ..., n)$ is the jth structural variable associated with the jth DMU. Through the above PPS, the following BCC output additive model [17] can be built:

$$
\begin{aligned}
\max \quad & z_{BCC} = \sum_{r=1}^{s} s_r^+ \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j\, x_{ij} \le x_{ik} && (i = 1, ..., m), \\
& \sum_{j=1}^{n} \lambda_j\, g_{rj} - s_r^+ = g_{rk} && (r = 1, ..., s), \\
& \sum_{j=1}^{n} \lambda_j \;=\; 1\;,\; \lambda_j \ge 0 && (j = 1, ..., n), \\
& s_r^+ \ge 0 && (r = 1, ..., s).
\end{aligned} \tag{2.5}
$$

In model (2.5), the k index refers to the DMU under evaluation and s_r^+ is the slack variable for the increase in the rth output.

Now we return to congestion that is defined as a situation in which an increase in one or more inputs causes the worsening of one or more outputs. In order to potentially deal with such a phenomenon, the production possibility set needs to be modified as follows:

$$
T_{\text{NEW}} = \{(x, g) \,|\, x = \sum_{j=1}^{n} \lambda_j x_j,\; g \le \sum_{j=1}^{n} \lambda_j g_j,\; \sum_{j=1}^{n} \lambda_j = 1,\; \lambda_j \ge 0;\; j = 1, ..., n\}. \tag{2.6}
$$

This technology is obtained by assuming weak input disposability which implies that it is not possible to dispose of any positive input slacks. In this way, the input equality format is adopted and the nonzero slack cannot be associated with any input [17]. This technology usually applies in congestion evaluation because the input disposability principle is not consistent in a congested firm [35]. Based on this technology, the NEW output additive model can be built [17] as follows:

$$
\begin{aligned}
\max \quad & z_{\text{NEW}} = \sum_{r=1}^{s} s_r^+ \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j\, x_{ij} = x_{ik} && (i = 1, ..., m), \\
& \sum_{j=1}^{n} \lambda_j\, g_{rj} - s_r^+ = g_{rk} && (r = 1, ..., s), \\
& \sum_{j=1}^{n} \lambda_j \;=\; 1\;,\; \lambda_j \ge 0 && (j = 1, ..., n), \\
& s_r^+ \ge 0 && (r = 1, ..., s).
\end{aligned} \tag{2.7}
$$

The above model computes the maximally possible outputs that can be augmented to the outputs of DMU$_k$ in order to remain in T_{NEW}. If the optimal amount of the objective function were zero, then, DMU$_k$ is called NEW output efficient and also lies on the frontier of T_{NEW}. The following definition -which was offered by Wei and Yan [17]- recognizes congestion of NEW output efficient DMUs.

Definition 2.1 *Let* DMU$_k$ *be NEW output efficient. If there exists* $(\hat{x}, \hat{g}) \in T_{\text{NEW}}$, *such that* $\hat{x} \le x_k, \hat{x} \ne x_k$ *and* $\hat{g} \ge g_k, \hat{g} \ne g_k$, *then* DMU$_k$ *is said to evidence weak congestion [17].*

Based on the above definition, the following theorem was demonstrated by Wei and Yan [17]:

Theorem 2.1 *Let* DMU$_k$ *be NEW output efficient. Then,* DMU$_k$ *evidences weak congestion if and only if* DMU$_k$ *is not BCC output efficient.*

Finally, in order to test for occurrence of congestion in the performance of DMU$_k$, the following procedure was suggested by Wei and Yan [17]:

1. Solve the NEW output additive model, i.e., model (2.7). Let (λ^*, s^{+*}) be the optimal solution of this model and $\hat{g}_k = g_k + s^{+*}$. Obviously, $(x_k, \hat{g}_k)$ is NEW output efficient.

2. Solve the BCC output additive model, i.e., model (2.5), for evaluating $(x_k, \hat{g}_k)$.

3. If $z^*_{BCC} > 0$, then, DMU$_k$ evidences congestion.

2.3 The bounded adjusted measure for undesirable outputs

The BAM model (bounded adjusted measure) is a slacks-based efficiency measure which was presented by Cooper et al. [34]. It is an additive DEA model, and can capture all the inefficiencies that the model can identify. Recently, Zare Haghighi and Rostamy-Malkhalifeh [6] extended the BAM model for evaluating the environmental efficiency in the presence of both desirable and undesirable outputs. This extended measure is employed in the next section of this study for evaluating congestion. This model is used for two reasons. First, it is a new model and was proposed recently in 2017 by Zare Haghighi and Rostamy-Malkhalifeh [6]. Second, it is a non-radial efficiency measure, and therefore, when calculating the efficiency score all the inefficiencies that the model can identify are accounted for in the efficiency calculations. Hence, it can easily combine desirable (good) and undesirable (bad) outputs in a unified manner. The extended BAM model of Zare Haghighi and Rostamy-Malkhalifeh [6] is as follows:

$$
\begin{aligned}
& \max \sum_{i=1}^{m} B_i^x s_i^x + \sum_{r=1}^{s} B_r^g s_r^g + \sum_{f=1}^{h} B_f^b s_f^b \\
& s.t. \sum_{j=1}^{n} (\eta_j + \mu_j) x_{ij} + s_i^x = x_{ik} (i = 1, ..., m), \\
& \sum_{j=1}^{n} \eta_j \ g_{rj} \ - s_r^g \ = g_{rk} \ (r = 1, ..., s), \\
& \sum_{j=1}^{n} \eta_j \ b_{fj} \ + s_f^b \ = b_{fk} \ (f = 1, ..., h), \\
& \sum_{j=1}^{n} (\eta_j + \mu_j) \ = \ 1 \ , \\
& \eta_j \geq 0 \ , \ \mu_j \geq 0 \ (j = 1, ..., n), \\
& s_i^x \geq \ \ (i = 1, ..., m), \\
& s_r^g \geq 0 \ \ (r = 1, ..., s), \\
& s_f^b \geq 0 \ \ (f = 1, ..., h).
\end{aligned} \tag{2.8}
$$

Here, s_i^x $(i = 1, ..., m)$, s_r^g $(r = 1, ..., s)$, and s_f^b $(f = 1, ..., h)$ are respectively the slack variables associated with inputs, desirable outputs, and undesirable outputs. The ranges included in model (2.8) are determined as follows:

$$B_i^x = \frac{1}{(m+s+h)(x_{ik} - \underline{x}_i)} \quad (\forall i),$$

$$B_r^g = \frac{1}{(m+s+h)(\overline{g}_r - g_{rk})} \quad (\forall r), \ \text{and}$$

$$B_f^b = \frac{1}{(m+s+h)(b_{fk} - \underline{b}_f)} \quad (\forall f),$$

where

$$\underline{x}_i = \min\{x_{ij} | j = 1, ..., n\},$$

$$\overline{g}_r = \max\{g_{rj} | j = 1, ..., n\},$$

and

$$\underline{b}_f = \min\{b_{fj} | j = 1, ..., n\}.$$

According to the convention of Zare Haghighi and Rostamy-Malkhalifeh [6], if for DMU$_k$, input i satisfies $x_{ik} = \underline{x}_i$, there is no capacity for improvement of the ith input, i.e., $s_i^{-*} = 0$, and then B_i^x is considered zero. Similarly, if desirable output r and undesirable output f respectively satisfy $g_{rk} = \overline{g}_r$ and $b_{fk} = \underline{b}_f$, then B_r^g and B_f^b are taken as zero.

The environmental efficiency measure (Γ_E) of DMU$_k$ is computed on the optimality of model (2.8) as follows:

$$\Gamma_E = $$

$$1 - \left(\sum_{i=1}^{m} \ B_i^x s_i^{x*} + \sum_{r=1}^{s} \ B_r^g s_r^{g*} + \sum_{f=1}^{h} \ B_f^b s_f^{b*} \right)$$

The output-oriented version of the above model can be constructed by neglecting the reduction of inputs, i.e., by deleting s_i^x, in the objective function of model (2.8). Therefore, the following model is achieved:

$$
\begin{aligned}
& \alpha^* = \max \ \sum_{r=1}^{s} \ B_r^g s_r^g + \sum_{f=1}^{h} \ B_f^b s_f^b \\
& s.t. \sum_{j=1}^{n} (\eta_j + \mu_j) \ x_{ij} \leq x_{ik} \ (i = 1, ..., m), \\
& \sum_{j=1}^{n} \ \eta_j \ g_{rj} - s_r^g = g_{rk} \ (r = 1, ..., s), \\
& \sum_{j=1}^{n} \ \eta_j \ b_{fj} + s_f^b = b_{fk} \ (f = 1, ..., h), \\
& \sum_{j=1}^{n} (\eta_j + \mu_j) \ = \ 1 \ , \\
& \eta_j \geq 0 \ , \ \mu_j \geq 0 \ \ (j = 1, ..., n), \\
& s_i^x \geq \ \ (i = 1, ..., m), \\
& s_r^g \geq 0 \ \ (r = 1, ..., s), \\
& s_f^b \geq 0 \ \ (f = 1, ..., h).
\end{aligned} \tag{2.9}
$$

3 Congestion with undesirable outputs

As described before, in traditional congestion, only desirable outputs are considered. Nevertheless, in the real world, unavoidably undesirable outputs (like pollutants or wastes) are produced along with desirable outputs. In this Section, we attempt to introduce an alternative framework for measuring congestion in the simultaneous presence of both desirable and undesirable outputs. To this end, Wei and Yan approach [17] which was reviewed in Section 2, is applied. Also, the Kuosmanen technology is used to address the undesirable outputs. Accordingly, the NEW technology corresponding to Kuosmanen [29] technology ($T_{K_{\text{NEW}}}$), which is acquired by changing the input inequalities of T_{K} into input equalities, is displayed as follows:

$$T_{K_{\text{NEW}}} = \{(x,g,b)| \begin{array}{l} \sum_{j=1}^{n}(\eta_j+\mu_j)\ x_j = x, \\ \sum_{j=1}^{n}\eta_j\ g_j \geq g, \\ \sum_{j=1}^{n}\eta_j\ b_j = b, \\ \sum_{j=1}^{n}(\eta_j+\mu_j) \ = \ 1, \\ \eta_j \geq 0\ ,\ \mu_j \geq 0\ , \\ (j=1,...,n).\ \} \end{array} \tag{3.10}$$

The definition we present is as follows:

Definition 3.1 *Let* $\text{DMU}_k = (x_k, g_k, b_k)$ *be on the frontier of* $T_{K_{\text{NEW}}}$. *Then, it evidences congestion if there exist* $(\hat{x},\hat{g},\hat{b}) \in T_{K_{\text{NEW}}}$ *such that* $\hat{x} \leq x_k$, $\hat{x} \neq x_k$, $\hat{g} \geq g_k$, $\hat{b} \leq b_k$ *and* $(\hat{g},\hat{b}) \neq (g_k, b_k)$.

In fact, congestion prevails at the performance of DMU_k whenever a reduction in one or more inputs can increase one or more desirable outputs, or decrease one or more undesirable output, without worsening any other input, desirable output, or undesirable output. Now, we try to recognize this type of congestion and determine its sources and amounts by expanding Wei and Yan [17] approach. First, it is needed to verify whether DMU_k is on the boundary of $T_{K_{\text{NEW}}}$ or not. Through the (3.10) PPS, the following NEW output-oriented version of the BAM model is presented as:

$$\begin{array}{l} \beta^* = \ \max \ \sum_{r=1}^{s} B_r^g s_r^g + \sum_{f=1}^{h} B_f^b s_f^b \\ s.t.\ \sum_{j=1}^{n}(\eta_j+\mu_j)\ x_{ij} = x_{ik}\ (i=1,...,m), \\ \sum_{j=1}^{n} \eta_j\ g_{rj} - s_r^g \ = g_{rk}\ (r=1,...,s), \\ \sum_{j=1}^{n} \eta_j\ b_{fj} + s_f^b \ = b_{fk}\ (f=1,...,h), \\ \sum_{j=1}^{n}(\eta_j+\mu_j) \ = \ 1\ , \\ \eta_j \geq 0\ ,\ \mu_j \geq 0\ \ (j=1,...,n), \\ s_i^x \geq \ \ (i=1,...,m), \\ s_r^g \geq 0\ \ (r=1,...,s), \\ s_f^b \geq 0\ \ (f=1,...,h). \end{array} \tag{3.11}$$

If the optimal amount of the objective function of the above model is zero, then, DMU_k is on the frontier of $T_{K_{\text{NEW}}}$. Therefore, in order to examine the occurrence of congestion in the performance of DMU_k, it is assessed via model (2.9). If it was inefficient under model (2.9), then, it evidences congestion. The following procedure explains a brief and comprehensive overview of our method:

1. Solve the NEW output-oriented version of the extended BAM model, i.e., model (3.11). If the optimal amount of the objective function were zero, i.e., $\beta^* = 0$, then, DMU_k is on the boundary of $T_{K_{\text{NEW}}}$. Therefore, go to step 2.

2. Solve the output-oriented BAM measure, i.e., model (2.9), for evaluating the DMUs which are on the frontier of $T_{K_{\text{NEW}}}$.

3. If $\alpha^* > 0$, then DMU_k evidences congestion.

It should be noticed that the above algorithm estimates the status of congestion for the DMUs which are on the frontier of $T_{K_{\text{NEW}}}$. The other DMUs which do not exist on the boundary of $T_{K_{\text{NEW}}}$, should be first projected on the frontier via solving model (3.11). Afterwards, their congestion can be verified through the above algorithm. Finally, in order to capture the input congestion sources and amounts, the following model is employed:

$$\begin{aligned}
&\min \quad \sum_{i=1}^{m} c_i \\
&s.t. \ \sum_{j=1}^{n}(\eta_j + \mu_j)\ x_{ij} = x_{ik} - c_i \ (i = 1, ..., m), \\
&\quad \sum_{j=1}^{n} \ \eta_j \ g_{rj} \ = g_{rk} + \ s_r^{g*} \ (r = 1, ..., s), \\
&\quad \sum_{j=1}^{n} \ \eta_j \ b_{fj} \ = b_{fk} - \ s_f^{b*} \ (f = 1, ..., h), \\
&\quad \sum_{j=1}^{n}(\eta_j + \mu_j) \ = \ 1 \ , \\
&\quad \eta_j \geq 0 \ , \ \mu_j \geq 0 \ \ (j = 1, ..., n), \\
&\quad c_i \geq \ \ (i = 1, ..., m).
\end{aligned} \tag{3.12}$$

Here (s^{g*}, s^{b*}) are the optimal amounts which are calculated under model (2.9). Fixing the outputs on these obtained optimal amounts of outputs, model (3.12) computes the minimum amount (c_i) that can be reduced from the ith input of DMU$_k$ in order to acquire these optimal amounts of outputs, and also reach $T_{K_{\mathrm{NEW}}}$. The value of $c_i > 0$ shows the amount of congestion in the ith input of DMU$_k$.

4 Numerical example

In this section, the proposed models are used on a data set consisting of 31 administrative regions of China. See the data set in Table 1, which is selected from Wu et al.'s article [9]. These data consist of two inputs: the total investment in fixed assets of industry (TIFA) and the electricity consumption by industry (EC), one desirable output: the gross industrial output value (GIOV), and two undesirable outputs: the total volume of industrial waste gas emission (TWGE) and the total volume of waste water discharge (TWWD). For ease of comparison, the industries are labeled D1 to D31, exhibited in the second column of Table 1.

Table 2, displays the results of the presented procedure for congestion measurement. The first column of Table 2 shows the results which have been computed under model (2.8). D2, D6, D16, D23, D24, D27, and D28 are identified as the environmental efficient industries and attain $\Gamma_E = 1$. Thus, it is deduced that these 7 industries pay attention to the reduction of their pollutants along with focusing on their commercial targets.

It should be noted that some industries (such as D7, D8, D15, D20, D21, D25, D29, and D30) have obtained a very low environmental efficiency score. For instance, the pure environmental efficiency of D20 is 0.31293. This low level of Γ_E reveals the industry's incapability in controlling pollutants and wastes. As a result, these industries should be seriously concerned about their pollutants and wastes. Especially, they should pay more attention to the environmental protection rules to prevent the current issues of global warming and climate change. The notable point is that the consumers, who are environmentally conscious, gradually avoid these dirty industries. Moreover, the government may ordain that any industry violating of environmental protection regulations should pay a fine. Therefore, these industries will lose consumers and their revenue will decrease unless they make significant efforts by utilizing new technology or new management.

The second column of Table 2 exhibits the results of the NEW output-oriented version of the extended BAM model which is computed by model (3.11). The eleven industries that acquired $\beta^* = 0$ are on the boundary of $T_{K_{\mathrm{NEW}}}$. In the next stage, these eleven industries are evaluated via model (2.9) to verify whether they evidence congestion. These results are given in the third column of Table 2. Four industries (D1, D18, D19, and D21) which have $\alpha^* > 0$ in this column, show congestion.

The amount of congestion in the inputs, each of which are calculated by model (3.12), are represented in the two last columns of Table 2. The 7 above mentioned industries, which performed efficiently, do not show congestion in any input. D1, D18, D19, and D21 evidence congestion in one or both of their inputs. For instance, congestion exists in the second input of D21. Therefore, D5 can reduce its electricity consumption in amounts of $c_2 = 337.78765$, and accordingly, its desirable output increases and its undesirable outputs decrease. Or, D19 show congestion in both of its inputs. Therefore, it can reduce its investment in amounts of $c_1 = 7390.18017$, and its electricity consumption in amounts of $c_2 = 116.46835$, and accordingly, its desirable output increases and its undesirable outputs decrease.

Table 1: Data set of industry of China in 2010.

District	DMU	Input1 TIFA	Input2 EC	des-output GIOV	Und-out1 TWGE	Und-out2 TWWD
Anhui	D1	9,121.829	1077.91	18,732	17,849	70,971
Beijing	D2	4,554.356	809.9	13,699.84	4,750	8,198
Chongqing	D3	5,049.258	626.44	9,143.55	10,943	45,180
Fujian	D4	6,534.803	1315.09	21,901.23	13,507	124,168
Gansu	D5	2,274.305	804.43	4,882.68	6,252	15,352
Guangdong	D6	11,903.36	4060.13	85,824.64	24,092	187,031
Guangxi	D7	5,166.135	993.24	9,644.13	14,520	165,211
Guizhou	D8	2,483.012	835.38	4,206.37	10,192	14,130
Hainan	D9	903.8264	159.02	1,381.25	1,360	5,782
Hebei	D10	11,737.07	2691.52	31,143.29	56,324	114,232
Heilongjiang	D11	5,019.085	747.84	9,535.15	10,111	38,921
Henan	D12	12,868.24	2353.96	34,995.53	22,709	150,406
Hubei	D13	7,276.638	1330.44	21,623.12	13,865	94,593
Hunan	D14	7,374.157	1171.91	19,008.83	14,673	95,605
Inner Mongolia	D15	6,831.416	1536.83	13,406.11	27,488	39,536
Jiangsu	D16	18,977.92	3864.37	92,056.48	31,213	263,760
Jiangxi	D17	6,696.149	700.51	13,883.06	9,812	72,526
Jilin	D18	6,313.748	576.98	13,098.35	8.240	38,656
Liaoning	D19	12,480.94	1715.26	36,219.42	26,955	71,521
Ningxia	D20	1,193.702	546.77	1,924.39	16,324	21,977
Qinghai	D21	789.5051	465.18	1,481.99	3,952	9,031
Shaanxi	D22	5,462.784	859.22	11,199.84	13,510	45,487
Shandong	D23	17,664.34	3298.46	83,851.4	43,837	208,257
Shanghai	D24	4,252.32	1295.87	30,114.41	12,969	36,696
Shanxi	D25	4,702.091	1460	12,471.33	35,190	49,881
Sichuan	D26	9,790.274	1549.03	23,147.38	20,107	93,444
Tianjin	D27	4,571.888	645.74	16,751.82	7,686	19,680
Tibet	D28	306.567	20.41	62.22	16	736
Xinjiang	D29	2,749.838	661.96	5,341.9	9,310	25,413
Yunnan	D30	4,024.972	1004.07	6,464.63	10,978	30,926
Zhejiang	D31	10,246.41	2820.93	51,394.2	20,434	217,426

5 Conclusion

Nowadays, global warming, climate change, an increased emission of CO_2 in the air, and water pollution are major problems all over the world. These worldwide problems reveal the importance of incorporating undesirable factors into performance assessment.

The concept of congestion, which is mainly applied in economics, points to a situation where inputs are excessively used. Recently, many researchers have studied this subject by means of data envelopment analysis (DEA). However, the previous endeavors only consider the framework of desirable outputs. Indeed, firms in actual applications, unavoidably generate undesirable outputs (like pollutants or wastes) along with desirable outputs. Consequently, there should exist an alternative scheme for measuring congestion in the simultaneous presence of both desirable and undesirable outputs.

In this paper, we briefly introduced the two technologies which is available in the DEA literature for modeling environmental performance [20, 29] under weak disposability postulate of desirable and undesirable outputs.

Afterwards, we used the extended bounded adjusted measure (extended BAM) of Zare Haghighi and Rostamy-Malkhalifeh [6] for evaluating the environmental performance in the presence of both desirable and undesirable outputs. Based on the this model, an alternative definition and approach were introduced to deal with congestion in the simultaneous presence of desirable and undesirable outputs.

Then, the presented method was applied to

Table 2: Results of the proposed models.

District	DMU	Γ_E	β^*	α^*	c_1	c_2
Anhui	D1	0.55893	0	0.27499	4054.73170	41.24088
Beijing	D2	1	0	0	0	0
Chongqing	D3	0.49693	0.29104			
Fujian	D4	0.61708	0.24357			
Gansu	D5	0.45605	0.28117			
Guangdong	D6	1	0	0	0	0
Guangxi	D7	0.40627	0.34799			
Guizhou	D8	0.39520	0.32682			
Hainan	D9	0.46869	0.32753			
Hebei	D10	0.48583	0.25977			
Heilongjiang	D11	0.50210	0.30879			
Henan	D12	0.59197	0.18007			
Hubei	D13	0.60994	0.21731			
Hunan	D14	0.56908	0.23071			
Inner Mongolia	D15	0.45106	0.31093			
Jiangsu	D16	1	0	0	0	0
Jiangxi	D17	0.59667	0.13726			
Jilin	D18	0.67358	0	0.19617	2487.40397	0
Liaoning	D19	0.71822	0	0.14663	7390.18017	116.46835
Ningxia	D20	0.31293	0.37877			
Qinghai	D21	0.36605	0	0.37107	0	337.78765
Shaanxi	D22	0.49898	0.31276			
Shandong	D23	1	0	0	0	0
Shanghai	D24	1	0	0	0	0
Shanxi	D25	0.43703	0.31907			
Sichuan	D26	0.55057	0.19825			
Tianjin	D27	1	0	0	0	0
Tibet	D28	1	0	0	0	0
Xinjiang	D29	0.42378	0.34055			
Yunnan	D30	0.39904	0.33865			
Zhejiang	D31	0.75634	0.15893			

study the pollutants (waste gas emission ad waste water discharge) of 31 administrative regions of China. From this application, it is deduced that 7 industries pay attention to the reduction of their pollutants accompanying with along with advancing their commercial goals. Consequently, they do not exhibit congestion in any input. four industries, however, evidence congestion in one or both of their inputs. By reducing the amount of congestion in each input, these industries can increase their desirable output and decrease their undesirable outputs.

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Maryam Khodadadi has got MSc degree in Applied Mathematics from Kharazmi University in 2009 and PhD degree from Central Tehran Branch, Islamic Azad University in 2014. Now, she is an assistant professor in Department of Mathematics, Urmia Branch, Islamic Azad University, Urmia, Iran. Her main research interests include linear and nonlinear programming, data envelopment analysis, ordered weighted averaging (OWA) operators, multi criteria decision making (MCDM), and performance evaluation.

Habibe Zare Haghighi has got MSc degree in Applied Mathematics fromKharazmi University. Then, she has received PhD degree in 2014 fromScience and Research Branch, Islamic Azad University, Tehran, Iran. Now, she teaches mathematics in Islamic Azad University. Her mainresearch interests include operations research, data envelopmentanalysis, and environmental assessment.