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Effect of Hall Current and Wall Conductance on Hydromagnetic Natural Convective Flow Between Vertical Walls

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Abstract

This paper has examined the analytical solution of steady fully developed natural convective flow of a viscous incompressible and electrically conducting fluid between vertical channel by taking the Hall current and induced magnetic field into account. We have obtained the non-dimensional simultaneous ordinary differential equations using the suitable non-dimensional variables and parameters in the governing equations of the model. By obtaining the analytical solution of the simultaneous ordinary differential equations, the effects of the Hall current and the Hartmann number on the primary and secondary components of the velocity, induced magnetic field and induced current density are presented by the graphs. The influence of the Hall current is to increase both the primary and secondary components of the velocity and induced magnetic field profiles but decrease the components of induced current density. It is found that the effect of Hartmann number is to decrease both the primary and secondary components of velocity and induced current density but increase the components of the induced magnetic field.

Keywords : Hall current; Induced magnetic field; Natural convection; Hydromagnetic; Induced current density.

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1 Introduction

The problem of hydromagnetic flow of an electrically conducting fluid permeated through a magnetic field by taking heat and mass transfer has considerable interest in the theoretical and experimental investigations. This type of problems has wide range of its applications in the field of engineering, astrophysics and manufacturing process in industry. Also it has attracted applications in the solar energy collector, plasma study, MHD power generator, high speed aircrafts and their atmospheric re-entry and cooling of nuclear reactor. Faraday first observed that when an electrically conducting fluid moves in a magnetic field then the electric currents are induced in the fluid which produce their own magnetic field and thus improve the original magnetic field and that field is called induced magnetic field. Generally, in the study of magnetohydrodynamic flows, the effect of induced magnetic field has been neglected in order to make easy the mathematical analysis of the problem. By neglecting the induced magnetic field Poots [18], Sparrow and Cess [32], Riley [20], Geograntopoulos and Nanousis [5], Raptis and Singh [19], Chamkha [3], Ghosh and

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Nandi [6], Singh and Singh [27], Sarkar et al [21], Hamza et al [10], Sheikholeslami and Ellahi [25], Mamourian et al [17] and Ellahi et al [4] have studied the effect of magnetic field for different kinds of flow formations.

By considering into account the induced magnetic field, Globe [8], Arora and Gupta [2], Guria et al [9], Singh et al [28] and Kwanza and Balakiyema [16] have performed the study of magnetohydrodynamic flows. Later on, the effect of induced magnetic field on natural convection in the vertical concentric annuli has been investigated by Singh and Singh [29]. Kumar and Singh [12] have performed the analytical study of unsteady magnetohydrodynamic free convective flow past a semi-infinite vertical wall with induced magnetic field. Jha and Sani [11] have studied the magnetohydrodynamic free convective flow of an electrically conducting and viscous incompressible fluid in a vertical channel due to symmetric heating with induced magnetic field. Further, Kumar and Singh [13] have discussed the effect of the induced magnetic field on free convection between concentric cylinders when one of the cylinders is heated /cooled asymmetrically. Akbar et al [1] have observed the interaction of nano particles in the peristaltic flow having asymmetric channel by taking induced magnetic field into account. Currently, Sarveshanand and Singh [22] have obtained an exact solution of magnetohydrodynamic free convective flow between vertical walls taking into account the induced magnetic field. Kumar and Singh [14] have studied the effect of induced magnetic field on natural convection with the Newtonian heating/cooling in vertical concentric annuli by obtaining the exact solution of the problem. Further, Kumar and Singh [15] have analyzed the exact study of induced magnetic field and heat source/sink on hydromagnetic flow between vertical concentric cylinders.

Application of the magnetic field perpendicular to the electric field results in an electromagnetic force, perpendicular to both the electric field and magnetic field, which causes the charged particles to move in its own direction and thus give rise to an electrical current density known as the Hall current. Generally, the effect of Hall current is ignored in applying the Ohms law for small and moderate values of the magnetic field. If the strong magnetic field is applied or the density of the fluid is low then the strong magnetic field electromagnetic force is noticeable and the effect of Hall current cannot be neglected. Singh [30] has performed an analytical study of effect of Hall current on MHD free convection flow in the Stokes problem for a vertical porous plate for small magnetic Reynolds number. Also, Singh [31] has extended this problem when the plate is moving with accelerated velocity. An analytical solution of hydromagnetic flow in a rotating channel with the induced magnetic field and heat transfer has studied by Ghosh et al [7]. Seth and Hussain [23] have obtained an exact solution of steady hydromagnetic Couette flow of class-II of a viscous, incompressible and electrically conducting fluid with non-conducting walls in a rotating system in the presence of an inclined magnetic field with the Hall Effect. Siddiqa et al [26] have performed the numerical study of magnetohydrodynamic natural convective flow with the Hall current and strong cross magnetic field. Vafai et al [33] have analyzed the study of peristaltic motion of third grade fluid under the effects of Hall current and heat transfer. Seth and Singh [24] have obtained an analytical solution of mixed convection hydromagnetic flow in a rotating channel with Hall and wall conductance effects.

Stimulated by above study, in this paper, we analyze the natural convective flow of an electrically conducting and viscous incompressible fluid between two infinite long vertical walls with the effects of Hall current and induced magnetic field. We have obtained the analytical solution for the velocity, induced magnetic field and temperature field by solving the non-dimensional governing linear simultaneous ordinary differential equations using the non-dimensional boundary condition. Further, we have obtained the expressions of primary and secondary components of the velocity, induced magnetic field and temperature field. Also, we have calculated the skin-friction and mass flow rate in the primary and secondary flow directions. Finally, we have focused on the effects of the Hall current parameter and the Hartmann number on the primary and secondary components of the velocity, induced magnetic, induced current density using the graphs and the numerical values of the primary and secondary components of the skin-friction and mass flow rate are presented in the tabular form.

2 Mathematical Formulation

The governing equations together with the Maxwells electromagnetic equations in the case of steady state flow are as follows:

Continuity equation

$$\nabla.\mathbf{V} = \mathbf{0}, \tag{2.1}$$

Momentum equation

$$\begin{aligned}\rho(\mathbf{V}.\nabla)\mathbf{V} = -\nabla P + \mu(\nabla^2\mathbf{V}) \\ +\mu_e(\mathbf{J}\times\mathbf{H}) + \rho\mathbf{g},\end{aligned} \tag{2.2}$$

Energy equation

$$(\mathbf{V}.\nabla)T = \frac{\kappa}{\rho C_p}(\nabla^2 T), \tag{2.3}$$

Ohms law with Hall current

$$\mathbf{J} + \frac{\omega_e\tau_e}{H_0'}(\mathbf{J}\times\mathbf{H}) = \sigma(\mathbf{E} + {}^{-}{}_{\mathbf{e}}\mathbf{V}\times\mathbf{H}), \tag{2.4}$$

Maxwell's equations

$$\begin{aligned}\nabla.\mathbf{E} = \frac{\text{æ}}{\text{„}}, \nabla.\mathbf{H} = \mathbf{0}, \nabla\times\mathbf{E} = \mathbf{0}, \\ \nabla\times\mathbf{H} = \mathbf{J}.\end{aligned} \tag{2.5}$$

In above equations, $\mathbf{V}$, g, $\mathbf{H}$, $\mathbf{J}$, $\mathbf{E}$ μ, ν,

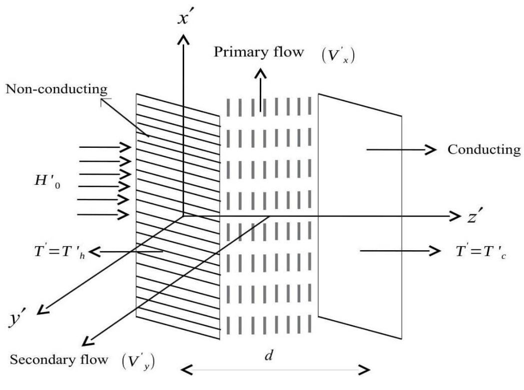

Figure 1: Physical Model

μ_e, ρ, ϵ, κ, σ and $m = \omega_e\tau_e$ are velocity vector, acceleration due to gravity, magnetic field vector, current density vector, electric field vector, coefficient of viscosity, kinematic viscosity of the fluid, magnetic permeability, density of the fluid, permittivity of the fluid, coefficient of thermal expansion, thermal conductivity, electrical conductivity of the fluid and Hall current parameter respectively.

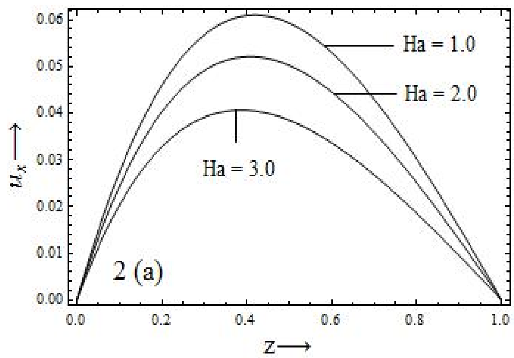

Figure 2: Primary velocity profiles for different Ha at m=1.0.

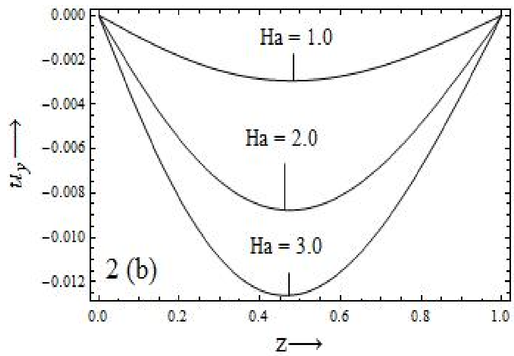

Figure 3: Secondary velocity profiles for different Ha at for m=1.0

In the present paper, we have considered the steady magnetohydrodynamic natural convection flow of an electrically conducting and viscous incompressible fluid between two infinite long vertical plates by taking into account the induced magnetic field and the Hall current. The distance between two plates is taken as d and we have introduced a Cartesian coordinate system with x'-axis in the vertical upward direction along one of the walls and z'- axis perpendicular to both the walls. A uniform magnetic field H_0' is applied perpendicular to both walls. The wall at is non-conducting while the wall at is conducting. Further, the walls at $z' = 0$ and at $z' = d$ are kept at the temperature T_h' and T_c' such that $T_h' > T_c'$ .The physical configuration considered here is shown in Figure-1. As walls are of infinite extent in x' and y' - directions and fluid flow is steady and fully developed, all the variables describing the flow formation will depend only on the co-ordinate z' except the density in gravitational force.

Hence, in this case the velocity $\mathbf{V}$, induced mag-

Table 1: Numerical values of the primary and secondary skin-friction and mass flow rate.

m	Ha	$(\tau_x)_{z=0}$	$(\tau_y)_{z=0}$	$(\tau_x)_{z=1}$	$(\tau_y)_{z=1}$	Q_x	Q_y
1.0	1.0	0.322270	-0.0101056	0.156992	-0.0087493	0.0395930	-0.00188315
1.0	2.0	0.291169	-0.0307132	0.130034	-0.0256604	0.0337928	-0.00560600
1.0	3.0	0.249963	-0.0459923	0.095555	-0.0358757	0.0262611	-0.00806823
2.0	1.0	0.328653	-0.0085476	0.162550	-0.0074471	0.0407864	-0.00159868
2.0	2.0	0.312600	-0.0301027	0.148273	-0.0258288	0.0377474	-0.00558037
2.0	3.0	0.283843	-0.0538892	0.122717	-0.0447559	0.0323033	-0.00980376
3.0	1.0	0.330947	-0.0065360	0.164564	-0.0057067	0.0412174	-0.00122397
3.0	2.0	0.322096	-0.0244426	0.156623	-0.0211746	0.0395332	-0.00455646
3.0	3.0	0.303678	-0.0481831	0.139908	-0.0410236	0.0360037	-0.00889262

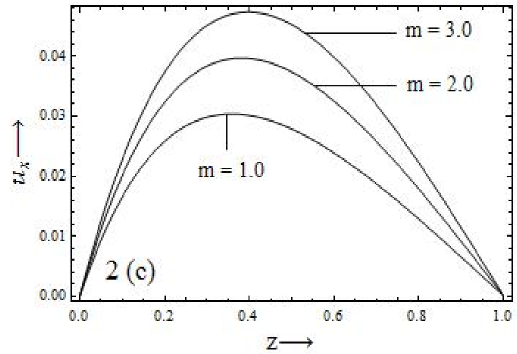

Figure 4: Primary velocity profiles for different m at Ha=4.0.

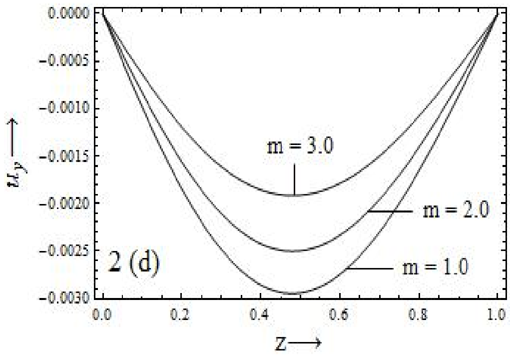

Figure 5: Secondary velocity profiles for different m at Ha=1.0

netic field $\mathbf{H}$, induced current density $\mathbf{J}$ and electric current $\mathbf{E}$ are defined as:

$$\mathbf{V} = (V'_{x'}, V'_{y'}, 0), \mathbf{H} = (H'_{x'}, H'_{y'}, H'_0), \\ \mathbf{J} = (J'_{x'}, J'_{y'}, 0), \mathbf{E} = (E'_{x'}, E'_{y'}, 0). \tag{2.6}$$

In case of steady state, Faradays law (i.e. $\nabla \times \mathbf{E}$) results as:

$$\frac{dE'_{x'}}{dz'} = 0, \frac{dE'_{y'}}{dz'} = 0, \tag{2.7}$$

Above equation, clearly shows that $E'_{x'}$ and $E'_{y'}$ are considered as constant within the fluid and on the interfaces of the walls.

Thus, under the usual Boussinesq approximation, the governing equations for the steady flow of a viscous, incompressible and electrically conducting fluid, taking into consideration the Hall current and induced magnetic field, are obtained as follows [Seth and Singh (2016)]:

$$\nu \frac{d^2V'_{x'}}{dz'^2} + \frac{\mu_e H'_0}{\rho}\frac{dH'_{x'}}{dz'} + g\beta(T' - T'_c) = 0, \tag{2.8}$$

$$\nu \frac{d^2V'_{y'}}{dz'^2} + \frac{\mu_e H'_0}{\rho}\frac{dH'_{y'}}{dz'} = 0, \tag{2.9}$$

$$m\frac{dH'_{x'}}{dz'} - \frac{dH'_{y'}}{dz'} = \sigma(E'_{x'} + \mu_e H'_0 V'_{y'}), \tag{2.10}$$

$$m\frac{dH'_{y'}}{dz'} + \frac{dH'_{x'}}{dz'} = \sigma(E'_{y'} - \mu_e H'_0 V'_{x'}), \tag{2.11}$$

$$\frac{d^2T'}{dz'^2} = 0, \tag{2.12}$$

For the considered model, the boundary conditions for the velocity, induced magnetic field and temperature field are as follows:

$$V'_{x'} = V'_{y'} = 0, H'_{x'} = H'_{y'} = 0, \\ T' = T'_h at z' = 0, \tag{2.13}$$

$$V'_{x'} = V'_{y'} = 0, \frac{dH'_{x'}}{dz'} = \frac{dH'_{y'}}{dz'} = 0, \\ T' = T'_c at z' = d. \tag{2.14}$$

By differentiating Eqs. (2.10) and (2.11) with respect to z' and then using $\nabla \times \mathbf{E}$) , we have

$$m\frac{d^2H'_{x'}}{dz'^2} - \frac{d^2H'_{y'}}{dz'^2} = \sigma\mu_e H'_0 \frac{dV'_{y'}}{dz'}, \tag{2.15}$$

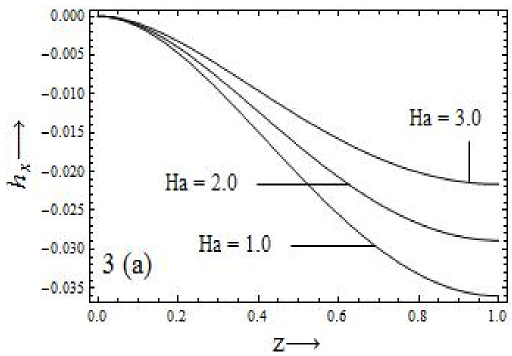

Figure 6: Primary induced magnetic field profile for different Ha at m=0.25.

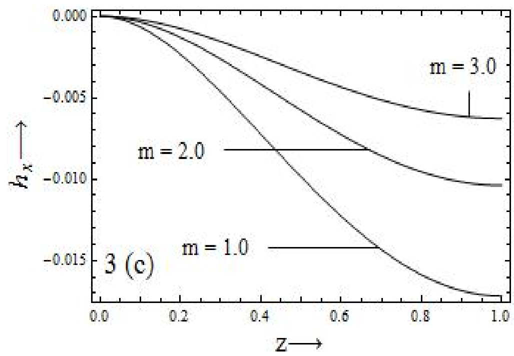

Figure 8: Primary induced magnetic field profiles for different m at Ha=3.0.

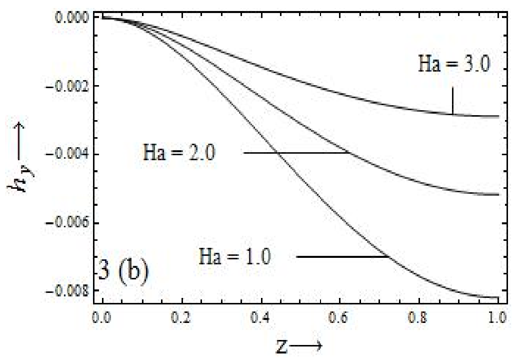

Figure 7: Secondary induced magnetic field profile for different Ha at m=0.25.

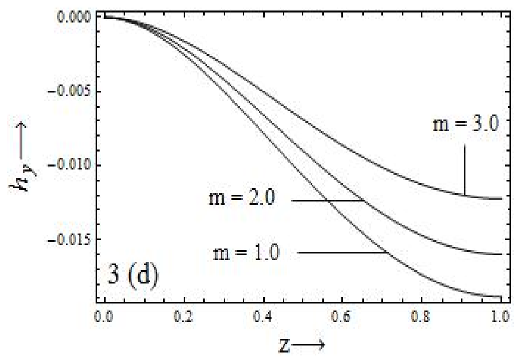

Figure 9: Secondary induced magnetic field profiles for different m at Ha=1.0.

$$m\frac{d^2H'_{y'}}{dz'^2}+\frac{d^2H'_{x'}}{dz'^2}=-\sigma\mu_e H'_0\frac{dV'_{x'}}{dz'}. \quad (2.16)$$

Using the relations $V'=V'_{x'}+iV'_{y'}$, $H'=H'_{x'}+iH'_{y'}$, $J'=J'_{x'}+iJ'_{y'}$, $E'=E'_{x'}+iE'_{y'}$, we can combine Eq. (2.8) with (2.9) and Eq. (2.15) with (2.16). By doing so, we get

$$\nu\frac{d^2V'}{dz'^2}+\frac{\mu_e H'_0}{\rho}\frac{dH'}{dz'}+g\beta(T'-T'_c)=0, \quad (2.17)$$

$$\frac{d^2H'}{dz'^2}+\frac{\sigma\mu_e H'_0}{(1-im)}\frac{dV'}{dz'}=0. \quad (2.18)$$

The boundary conditions for the velocity, induced magnetic field and temperature field in compact form are as follows:

$$V'=0, H'=0, T'=T'_h at z'=0, \quad (2.19)$$

$$V'=0, \frac{dH'}{dz'}=0, T'=T'_c at z'=d. \quad (2.20)$$

In order to represent Eqs. (2.17), (2.18) and (2.12) in non-dimensional form, we introduce the following non-dimensional variables given as follows:

$$u=\frac{V'}{U}, z=\frac{z'}{d}, h=\frac{H'}{\sigma\mu_e H'_0 U d},$$
$$T=\frac{(T'-T'_c)}{(T'_h-T'_c)}, U=\frac{g\beta d^2}{\nu}(T'_h-T'_c), \quad (2.21)$$
$$Ha=\mu_e H'_0 d\sqrt{\frac{\sigma}{\mu}}.$$

Using Eqn. (2.21), Eqs.(2.17), (2.18) and (2.12) in non-dimensional form are obtained as:

$$\frac{d^2u}{dz^2}+Ha^2\frac{dh}{dz}+T=0, \quad (2.22)$$

$$\frac{d^2h}{dz^2}+\frac{1}{(1-im)}\frac{du}{dz}=0, \quad (2.23)$$

$$\frac{d^2T}{dz^2}=0. \quad (2.24)$$

The boundary conditions (2.19) and (2.20) for the velocity, induced magnetic field and temperature field, in dimensionless form, are given as:

$$u=0, h=0, T=1 at z=0, \quad (2.25)$$

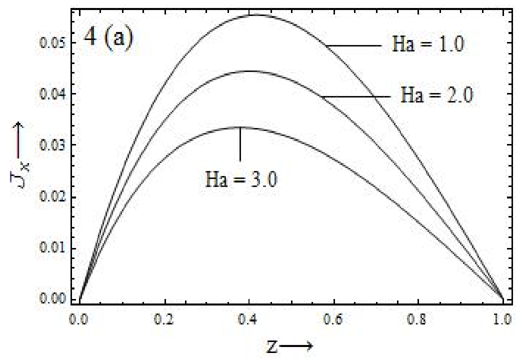

Figure 10: Primary induced current density field profiles for different Ha at m=0.25.

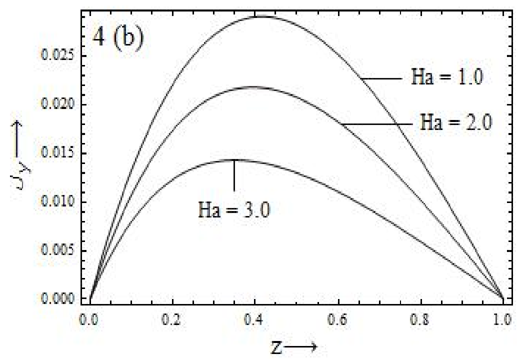

Figure 11: Secondary induced current density field profile for different Ha at m=1.0.

$$u = 0, \frac{dh}{dz} = 0, T = 0 at z = 1. \tag{2.26}$$

Thus, Eqs.(2.22), (2.23) and (2.24) with boundary conditions (2.25) and (2.26) represent the mathematical formulation of the model in the form of the boundary value problem.

3 Analytical solution

First, we obtain the analytical solution of the boundary value problem and then by separating those into real and imaginary parts, the components of the velocity and induced magnetic field can be obtained as:

$$u(z) = u_x(z) + iu_y(z) = E_1 cosh(\lambda z) + E_2 sinh(\lambda z) + (1-z)\frac{1}{\lambda^2}, \tag{3.27}$$

$$h(z) = h_x(z) + ih_y(z) = E_3 - \{E_1 \lambda sinh(\lambda z) + E_2 \lambda cosh(\lambda z) + (z - \frac{z^2}{2})\}\frac{1}{Ha^2}, \tag{3.28}$$

$$T(z) = 1 - z. \tag{3.29}$$

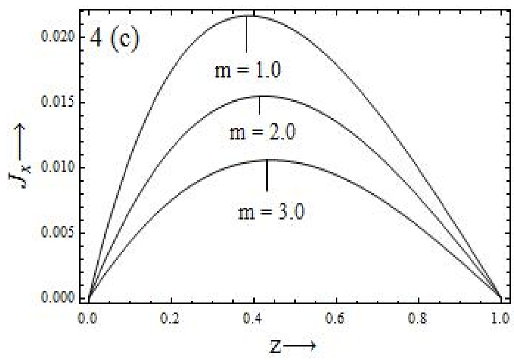

Figure 12: Primary induced current density field profiles for different m at Ha=4.0.

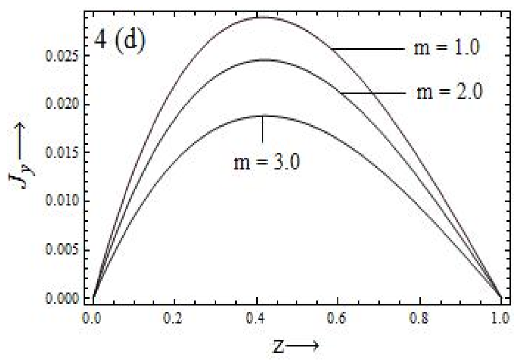

Figure 13: Secondary induced current density field profile for different m at Ha=1.0.

Separating, Eqs. (2.23) and (2.24) into real and imaginary parts, we get

$$u_x(z) = \{E_4 cosh(\alpha z) + E_5 sinh(\alpha z)\}cos(\beta z) + \{E_6 sinh(\alpha z) + E_7 cosh(\alpha z)\}sin(\beta z) + \frac{(1-z)}{Ha^2}, \tag{3.30}$$

$$u_y(z) = \{E_4 sinh(\alpha z) + E_5 cosh(\alpha z)\}sin(\beta z) - \{E_6 cosh(\alpha z) + E_7 sinh(\alpha z)\}cos(\beta z) - \frac{m(1-z)}{Ha^2}, \tag{3.31}$$

$$h_x(z) = \{E_8 cosh(\alpha z) + E_9 sinh(\alpha z)\}sin(\beta z) - \{E_{10} sinh(\alpha z) + E_{11} cosh(\alpha z)\}cos(\beta z) + E_{11} - \frac{(2z - z^2)}{2Ha^2}, \tag{3.32}$$

$$h_y(z) = -\{E_8 sinh(\alpha z) + E_9 cosh(\alpha z)\}cos(\beta z) -\{E_{10} cosh(\alpha z) + E_{11} sinh(\alpha z)\}sin(\beta z) +E_9. \quad (3.33)$$

The induced current density is given by

$$J = J_x(z) + iJ_y(z) = -\frac{dh}{dz} = \{E_1\lambda^2 cosh(\lambda z) + E_2\lambda^2 sinh(\lambda z) +(1-z)\}\frac{1}{Ha^2}. \quad (3.34)$$

Separating the real and imaginary parts of the above equation, we get

$$J_x(z) = \{E_{12} cosh(\alpha z) + E_{13} sinh(\alpha z)\}cos(\beta z) -\{E_{14} sinh(\alpha z) + E_{15} cosh(\alpha z)\}sin(\beta z) +\frac{(1-z)}{Ha^2}, \quad (3.35)$$

$$J_y(z) = \{E_{12} sinh(\alpha z) + E_{13} cosh(\alpha z)\}sin(\beta z) +\{E_{14} cosh(\alpha z) + E_{15} sinh(\alpha z)\}cos(\beta z). \quad (3.36)$$

In the absence of Hall current (m=0), we get $u_y(z) = h_y(z) = J_y(z) = 0$ and $u_x(z) = u(z), h_x(z) = h(z), J_x(z) = J(z)$ are same as obtained by Sarveshanand and Singh (2015).

Further, the expressions for the skin friction in non-dimensional form on the two walls at and at, due to the primary and secondary flows respectively, are given by

$$(\tau)_{z=0} = \{\tau_x(z) + i\tau_y(z)\}_{z=0} = \{\frac{du}{dz}\}_{z=0} = E_2\lambda - \frac{1}{\lambda^2}, \quad (3.37)$$

$$(\tau)_{z=1} = -\{\tau_x(z) + i\tau_y(z)\}_{z=1} = -\{\frac{du}{dz}\}_{z=1} = -\{E_1\lambda sinh(\lambda) +E_2\lambda cos(\lambda) - \frac{1}{\lambda^2}\}. \quad (3.38)$$

The mass flow rates Q_x and Q_y, in non-dimensional form along primary and secondary flow directions respectively, are given as:

$$Q = Q_x(z) + iQ_y(z) = \int_{z=0}^{z=1} u(z)dz = [E_1 sinh(\lambda) + E_2\{cosh(\lambda) - 1\}]\frac{1}{\lambda} +\frac{1}{2\lambda^2}. \quad (3.39)$$

The constants $\lambda = \alpha + i\beta$, $\alpha, \beta = Ha[\frac{(\sqrt{1+m^2}+1)}{(2(1+m^2))}]^{\frac{1}{2}}$, E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8, E_9, E_{10}, E_{11}, E_{12}, E_{13}, E_{14} and E_{15} appearing in above equations are defined in appendix.

4 Results and Discussion

In the present study of magnetohydrodynamic free convective flow of an electrically conducting fluid between two vertical parallel walls, the effect of the Hall current and the induced magnetic field has been considered. Here, we have focused our attention on the effects of the Hall current parameter and the Hartmann number on the primary and secondary components of the fluid velocity, induced magnetic fields and induced current density through the graphical representations and the numerical values of the skin-friction and mass flow rate are presented in the tabular form. The effect of the Hartmann number on the primary and secondary fluid velocity is demonstrated in Figs. 2-3. It is observed that with increasing values of the Hartmann number the primary and secondary components of the velocity profiles decrease. This shows that the Hartmann number tends to slow down the fluid flow in both the primary and secondary direction due to presence of the electromagnetic force which opposes the fluid flow. Figures 4-5 shows the influence of the Hall current parameter on the primary and secondary components of the fluid velocity. It is seen that both components of the velocity increase on increasing the Hall current parameter. This implies that the Hall current parameter tends to enhance the fluid flow in both the primary and secondary directions because the electrical conductivity of the fluid decreases with increasing Hall current which ultimately reduces the magnetic force and hence both components of the velocity have increased considerably.

Figures 6-7 show the influence of the Hartmann number on the primary and secondary components of the induced magnetic field. It is observed from the figures that both the components of the induced magnetic fields monotonically increase with increasing the Hartmann number. This shows that the effect of the Hartmann number is to enhance the strength of both the primary and secondary components of the induced magnetic fields in the region due to increase in the Lorentz force. Figures 8-9 illustrate the influence of the Hall current parameter on the primary and secondary components of the induced magnetic fields. It is seen that both the components of the induced magnetic fields have increasing tendency with increasing the Hall current parameter. This implies that the Hall current accelerates the fluid in the primary and secondary directions.

Figs. 10-11 exhibits the effect of the Hartmann number on the primary and secondary components of the induced current density. It is evident from these figures that both the primary and secondary components of the induced current density decrease with increasing the Hartmann number. Figs. 12-13 indicates the influence of the Hall current parameter on the primary and secondary components of the induced current density. It is observed that both components of the induced current density decrease with the Hall current parameter. Table 1 shows the effect of the Hartmann number and the Hall current parameter on the numerical values of the primary and secondary components of the skin-friction and mass flow rate at the surface of both plates i.e. at and at . It clearly shows that the effect of Hartmann number is to decrease both the components of the skin-friction at both the walls and . It is also clear that with increasing values of the Hall current parameter, both the primary and secondary components of the skin-friction increase at and at . It also reveals that the effect of the Hartmann number is to decrease both the components of the mass flow rate. Further, it is noticed that the values of the primary and secondary components of the mass flow rate increase with increase in the value of the Hall current parameter.

5 Conclusion

In the present analysis, we have obtained the analytical solution of the model to see the effects of Hall current and induced magnetic on hydromagnetic natural convective flow in the vertical channel. The influences of the physical parameters such as the Hartmann number and the Hall current parameter on the primary and secondary components of the velocity, induced magnetic field, induced current density, skin-friction, and mass flow rate have been investigated and the following conclusions have been drawn:

1. The influence of the Hartmann number is to reduce the primary and secondary components of the velocity and induced current density.

2. The impact of the Hartmann number is to enhance both the components of the induced magnetic fields.

3. The effect of the Hall current is to increase both the primary and secondary components of the velocity and induced magnetic field.

4. The primary and secondary components of the induced current density decrease with increasing the Hall current.

5. The effect of the Hartmann number is to decrease the numerical values of the skin friction at the surface of both plates.

6. The numerical values of the primary and secondary components of the skin-frictions increase with the Hall current number.

7. The effect of the Hartmann number is to decrease both components of the mass flow rate while both components of the mass flow rate have reverse effect on increasing the Hall current.

Appendix

$\lambda = \alpha + i\beta$, $\alpha, \beta = Ha[\frac{(\sqrt{1+m^2}+1)}{(2(1+m^2))}]$ 12,$\alpha_1 = [sinh(\alpha)cos(\beta) - mcosh(\alpha)sin(\beta)]$, $\beta_1 = [msinh(\alpha)cos(\beta) + cosh(\alpha)sin(\beta)]$, $E_1 = E_4 - iE_6$, $E_2 = E_5 - iE_7$, $E_3 = \frac{E_2\lambda}{Ha^2}$, $E_4 = \frac{-1}{Ha^2}$, $E_5 = \frac{(1+m^2)}{Ha^2({\alpha_1}^2+{\beta_1}^2)}[\alpha_1\{(cosh(\alpha)cos(\beta) - 1) + 1\} + \beta_1\{sinh(\alpha)sin(\beta)\}]$, $E_6 = \frac{-m}{Ha^2}$, $E_7 = \frac{(1+m^2)}{Ha^2({\alpha_1}^2+{\beta_1}^2)}[\beta_1\{(cosh(\alpha)cos(\beta) - 1) + 1\} - \alpha_1\{sinh(\alpha)sin(\beta)\}]$, $E_8 = \frac{(E_4\beta - E_6\alpha)}{Ha^2}$, $E_9 = \frac{(E_5\beta - E_7\alpha)}{Ha^2}$, $E_{10} = \frac{(E_4\alpha + E_6\beta)}{Ha^2}$, $E_{11} = \frac{(E_5\alpha + E_7\beta)}{Ha^2}$, $E_{12} = \frac{(E_4 + mE_6)}{1+m^2}$, $E_{13} = \frac{(E_5 + mE_7)}{1+m^2}$, $E_{14} = \frac{(mE_4 - E_6)}{1+m^2}$, $E_{15} = \frac{(mE_5 - E_7)}{1+m^2}$.

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