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Extension on Regularity Condition in DEA Models

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Abstract

Data envelopment analysis (DEA) is a method to estimate the relative efficiency of decision making units(DMUs) performing similar tasks in a production system that consumes multiple inputs to produce multiple outputs. So far, a number of DEA models have been developed: The CCR model and the BCC model are well known as basic DEA models. The generic regularity condition for the CCR model is introduced so that each weakly efficient decision making unit in the CCR model of data envelopment analysis is also CCR-efficient. In this study, we give a general regularity condition under which each weakly efficient decision making unit in the BCC model and Additive model of data envelopment analysis is also BCC-efficient and Additive-efficient,respectively.

Keywords : Data Envelopment Analysis, Multi-objective Programming, Symmetric Model, General Position, Regularity Condition.

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1 Introduction

Data envelopment analysis (DEA) was suggested by Charnes, Cooper and Rhodes(CCR), and built on the idea of Farrell [7], which is concerned with the estimation of technical efficiency and efficient frontiers. The CCR model [4, 5] generalized the single output/single input ratio efficiency measure for each decision making unit (DMU) to multiple outputs/multiple inputs situations by forming the ratio of a weighted sum of outputs to a weighted sum of inputs. DEA is a method for measuring the relative efficiency of DMUs performing similar tasks in a production system that consumes multiple inputs to produce multiple outputs. The main characteristics of DEA are (a) it can be applied to analyze multiple outputs and multiple inputs without preassigned weights, (b) it can be used for measuring the relative efficiency based on the observed data without having information

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on the production function, and (c) decision makers' preferences can be incorporated in DEA models.
The CCR ratio model for data envelopment analysis was proposed by Charnes, Cooper, and Rhodes in [4] in order to determine relative efficiencies for a finite number of decision-making units (DMUs). Each DMU processes a given set of inputs into a given set of outputs, where prices of inputs and outputs are assumed to be unknown. The relative efficiency of a given DMU is defined via a multi-objective programming problem that determines how well the DMU accomplishes the task of processing little amounts of inputs into large amounts of outputs.
Later , Banker, Charnes and Cooper (BCC) suggested a model for estimating technical efficiency and scale inefficiency in DEA. The BCC model [1] relaxed the constant returns to scale assumption of the CCR model and made it possible to investigate whether the performance of each DMU was conducted in region of increasing, constant or decreasing returns to scale in multiple outputs and multiple inputs situations.
The preceding models required us to distinguish between input-oriented and output-oriented models. Now, however, it combines both orientations in a single model , called the Additive model. The Additive model is a linear programming (LP)-based method proposed by Charnes et al. [2].
The remaining part of this article is organized as follows. In Section 2 we introduce some basic notation concerning the CCR ratio model and its reformulations in multiplier and envelopment forms, as well as the definitions of weak efficiency and CCR efficiency. Then, we give equivalent reformulations of the multiplier and envelopment models in a form which is symmetric in outputs and inputs. A regularity condition for the CCR model [9] in data envelopment analysis is also introduced in this section. We show why problems that satisfy this regularity condition are actually in general position, and that each of their weakly efficient DMUs is also CCR-efficient. In Section 3, we show the BCC model in multiplier and envelopment forms, as well as the definitions of weak efficiency and BCC efficiency. Then, we introduce equivalent reformulations of the multiplier and envelopment models in a symmetric form in outputs and inputs in this section . Next a regularity condition on the BCC model in data envelopment analysis is introduced and we show that under this condition each of their weakly efficient DMUs is also BCC-efficient. In Section 4 Additive model, as well as a regular condition for and a symmetric form of it are introduced, and finally the conclusion is provided in Section 5.

2 CCR(Charnes-Cooper-Rhodes) Model

The CCR model is a linear programming (LP)-based method proposed by Charnes et al. [4]. In the CCR model, the efficiency of the entity evaluated is obtained as a ratio of the weighted output to the weighted input subject to the condition that the ratio for every entity is not larger than 1. Mathematically, it is described as follows:

$$
\begin{aligned}
FP_o: \quad & max \frac{u^T Y_o}{v^T X_o} \\
s.t. \quad & \frac{u^T Y_j}{v^T X_j} \leq 1 \quad j = 1, ..., n, \\
& u, v \geq 0
\end{aligned}
\tag{2.1}
$$

Here, the evaluated entities (DMUs) form a reference set and n is the number of DMUs. $Y_j = [y_{j1}, ..., y_{js}]^t$ and $X_j = [x_{j1}, ..., x_{jm}]^t$ in (2.1) are the given positive output and input vectors of the jth DMU, respectively, s and m are the number of outputs and inputs of the DMUs, respectively, u and v are the coefficient vectors of Y_j and X_j, respectively, and the index o indicates the evaluated DMU. $u \geq 0$ represents the vector whose elements are not less than zero and at least one element is positive, whereas $u > 0$ represents the vector with positive elements.

Let $P_j = \begin{pmatrix} Y_j \\ -X_j \end{pmatrix}$, FP_o can equivalently be written as (for a proof, cf., e.g., [6])

$$
\begin{aligned}
LP_o: \quad & max u^T Y_o \\
s.t. \quad & v^T X_o = 1 \\
& u^T Y_j \leq v^T X_j \quad j = 1, ..., n, \\
& u, v \geq 0
\end{aligned}
\tag{2.2}
$$

It can be seen from (2.2) that the essence of the CCR model is that the DMU evaluated tries to find out its weight vector to maximize its weighted output with the constraint that its weighted input is fixed as unity and the weighted output is not larger than the weighted input for all DMUs. In other words, each DMU seeks its favorite weight vector to its own advantage.

By introducing the vector $w = \begin{pmatrix} u \\ v \end{pmatrix} \in \mathbb{R}^{m+s}$ and the matrices

$X = (X_1, ..., X_n)$,
$Y = (Y_1, ..., Y_n)$,
$P = (P_1, ..., P_n)$,
$A = \begin{pmatrix} Y & -I & 0 \\ -X & 0 & -I \end{pmatrix} = (P| - I)$,

where I and 0 in the definition of A denote identity and zero matrices of appropriate dimensions, respectively, we have another form of LP_o as follows:

$$
\begin{aligned}
LP_o: \quad & max\ w^T \begin{pmatrix} Y_o \\ 0 \end{pmatrix} \\
s.t. \quad & w^T \begin{pmatrix} 0 \\ X_o \end{pmatrix} = 1 \\
& w^T A \leq 0
\end{aligned}
\tag{2.3}
$$

The dual of LP_o is given by:

$$
\begin{array}{lll}
DLP_o: & min\ \theta & \\
s.t. & A\begin{pmatrix}\lambda \\ s^+ \\ s^-\end{pmatrix} + \theta\begin{pmatrix}0 \\ X_o\end{pmatrix} = \begin{pmatrix}Y_o \\ 0\end{pmatrix} & \\
 & \lambda, s^+, s^- \geq 0 &
\end{array} \tag{2.4}
$$

where $\lambda \in \mathbb{R}^n$, $s^+ \in \mathbb{R}^s$, $s^- \in \mathbb{R}^m$ and $\theta \in \mathbb{R}$. Note that LP_o is called the "multiplier model", whereas DLP_o is referred to as the "envelopment model".

Definition 2.1. *(a) DMU_o is called Weakly Efficient if the optimal value of DLP_o equals 1.*

(b) DMU_o is called CCR-Efficient if it is weakly efficient, and if all solution points of DLP_o possess vanishing slack variables s^+ and s^-.

(c) DMU_o is called Inefficient if it is not weakly efficient.

Note that weak efficiency is also known as radial, technical or Farrell efficiency, whereas CCR-efficiency is also called Pareto-Koopmans efficiency(for more details, see [2]).

Theorem 2.1. *(a) DMU_o is called Weakly Efficient if and only if the optimal value of LP_o is 1.*

(b) DMU_o is called CCR-Efficient if and only if it is weakly efficient , and if there exists a positive solution $w^* > 0$ of LP_o.

For details of the proof, see [6].

2.1 Symmetric CCR model

Problems LP_o and DLP_o are not symmetric in the inputs X_o and outputs Y_o,("Symmetric" is just supposed to mean that units(inputs and outputs) may be exchanged in the model without changing the model . In the previous models this is not the case, so in this sense they are "asymmetric".). The symmetric form of LP_o is defined as follows:

$$
\begin{array}{ll}
SLP_o: & max\ w^T P_o \\
s.t. & w^T e = 1, \\
 & w^T A \leq 0
\end{array} \tag{2.5}
$$

It is called the *symmetric multiplier model*, and its dual

$$
\begin{array}{ll}
DSLP_o: & min\ \theta \\
s.t. & A\begin{pmatrix}\lambda \\ s^+ \\ s^-\end{pmatrix} + \theta e = P_o \\
 & \lambda, s^+, s^- \geq 0
\end{array} \tag{2.6}
$$

the *symmetric envelopment model*. Here, e denotes an all-one vector of appropriate dimension.
For details, see [9].

Theorem 2.2. *(a) DMU_o is called Weakly Efficient if and only if the optimal value of SLP_o is 0.*

(b) DMU_o is called CCR-Efficient if and only if it is weakly efficient , and if there exists a positive solution $w^* > 0$ of SLP_o.

For details of the proof see [9].

Remark 2.1. *(a) DMU_o is called Weakly Efficient if and only if the optimal value of $DSLP_o$ is 0.*

(b) DMU_o is called CCR-Efficient if and only if it is weakly efficient , and if all solution points of $DSLP_o$ possess vanishing slack variables s^+ and s^-.

2.2 A regular condition for the CCR model

In this section , it is shown that if the data which define the DMUs are in "general position " , each weakly efficient DMU will also be CCR-efficient.

Definition 2.2. *$DMU_1, ..., DMU_n$ are said to be in general position if the following regularity condition (RC) holds: each $(s+m) \times (s+m)$-submatrix of the $(s+m) \times (s+m+n)$-matrix*

$$A = \begin{pmatrix} Y & -I & 0 \\ -X & 0 & -I \end{pmatrix} = (P| - I),$$

is non-singular.

Theorem 2.3. *Let $DMU_1, ..., DMU_n$ be in general position. Then a DMU_o is weakly efficient if and only if it is CCR-efficient.*

For more details of the proof see [9].

3 BCC(Banker-Charnes-Cooper) Model

In this section , we introduce the BCC model in multiplier and envelopment forms , then we define weak efficiency and BCC-efficiency.
Banker, Charnes and Cooper [1] introduced, the BCC model, whose production possibility set P_B is defined as:

$$P_B = \{(x, y)| x \geq X\lambda, y \leq Y\lambda, e\lambda = 1, \lambda \geq 0\} \tag{3.7}$$

where $X = (x_j) \in \mathbb{R}^{m \times n}$ and $Y = (y_j) \in \mathbb{R}^{s \times n}$ are a given data set, $\lambda \in \mathbb{R}^n$ and e is a row vector with all elements equal to 1. The BCC model differs from the CCR model only in the adjunction of the condition $e\lambda = 1$.
The BCC model evaluates the efficiency of $DMU_o(o = 1, ..., n)$ by solving the following (envelopment form) linear program:

$$\begin{array}{ll} min & \theta \\ s.t. & X\lambda \leq \theta x_o \\ & Y\lambda \geq y_o \\ & e\lambda = 1 \\ & \lambda \geq 0 \end{array} \tag{3.8}$$

where θ is a scalar.
The dual multiplier form of this linear program is expressed as:

$$\begin{array}{lll} max & z = u^T Y_o + u_0 & \\ s.t. & v^T X_o = 1 & \\ & u^T Y_j - v^T X_j + u_0 e \leq 0 & j = 1, ..., n, \\ & u \geq 0, v \geq 0, u_0 \ free \ in \ sign. & \end{array} \tag{3.9}$$

where z and u_0 are scalars and the latter, being "free in sign" may be positive or negative (or zero).
Since u_0 is free in sign, it could be written as the difference of two positive variables in a subtraction. So, by introducing u_0^+ and u_0^- , we will have $u_0 = u_0^+ - u_0^-$.
Consequently, by substitution in model (3.9), we have model the following:

$$\begin{array}{lll} max & z = u^T Y_o + u_0^+ - u_0^- & \\ s.t. & v^T X_o = 1 & \\ & u^T Y_j - v^T X_j + u_0^+ - u_0^- \leq 0 & j = 1, ..., n, \\ & u \geq 0, v \geq 0, u_0^+ \geq 0, u_0^- \geq 0 & \end{array} \tag{3.10}$$

We introduce the vector $w = \begin{pmatrix} u \\ v \\ u_0^+ \\ u_0^- \end{pmatrix} \in \mathbb{R}^{m+s+2}$ and the matrices

$X = (X_1, ..., X_n)$,
$Y = (Y_1, ..., Y_n)$,
$P = (P_1, ..., P_n)$,
$A = \begin{pmatrix} Y & -I & 0 & 0 & 0 \\ -X & 0 & -I & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 & -1 \end{pmatrix}$
where I and 0 in the definition of A denote identity and zero matrices of appropriate dimensions, respectively. Then, we have another form of model (3.10) as follows:

$$\begin{array}{ll} BLP_o: & max \ w^T \begin{pmatrix} Y_o \\ 0 \\ 1 \\ -1 \end{pmatrix} \\ s.t. & w^T \begin{pmatrix} 0 \\ X_o \\ 0 \\ 0 \end{pmatrix} = 1 \\ & w^T A \leq 0 \end{array} \tag{3.11}$$

The dual of BLP_o is given by:

$$
\begin{aligned}
DBLP_o: \quad & min\ \theta \\
s.t. \quad & A \begin{pmatrix} \lambda \\ s^+ \\ s^- \\ t^+ \\ t^- \end{pmatrix} + \theta \begin{pmatrix} 0 \\ X_o \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} Y_o \\ 0 \\ 1 \\ -1 \end{pmatrix} \\
& \lambda, s^+, s^-, t^+, t^- \geq 0
\end{aligned} \tag{3.12}
$$

where $\lambda \in \mathbb{R}^n$, $s^+ \in \mathbb{R}^s$, $s^- \in \mathbb{R}^m$ and $\theta \in \mathbb{R}$. Note that BLP_o is called the "multiplier model", whereas $DBLP_o$ is referred to as the "envelopment model".

Definition 3.1. *(a) DMU_o is called Weakly Efficient if the optimal value of $DBLP_o$ equals 1.*

(b) DMU_o is called BCC-Efficient if it is weakly efficient , and if all solution points of $DBLP_o$ possess vanishing slack variables s^+ , s^-, t^+ and t^-.

(c) DMU_o is called Inefficient if it is not weakly efficient.

Example 3.1. *Consider the six DMUs with one input and one output in Table 1. The input and output data of the DMUs are depicted in Fig. 1. Straightforward calculations show that A, B, C, D are BCC-efficient, F is inefficient, and E is weakly efficient , but not BCC-efficient.*

Table 1
Input and Output of DMUs

DMU	*A*	*B*	*C*	*D*	*E*	*F*
Input	*1*	*2*	*3*	*4*	*5*	*3.5*
Output	*1*	*2*	*3*	*4*	*4*	*2.5*

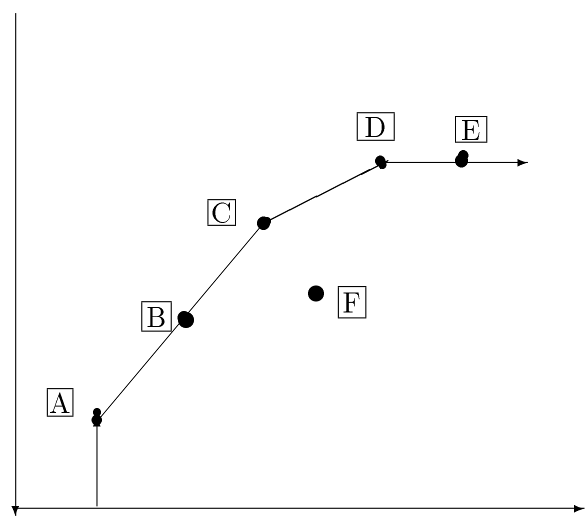

Fig. 1.

Theorem 3.1. *(a) DMU_o is called Weakly Efficient if and only if the optimal value of BLP_o is 1.*

(b) *DMU_o is called BCC-Efficient if and only if it is weakly efficient , and if there exists a positive solution $w^* > 0$ of BLP_o.*

Proof: Part (a) follows immediately form the strong duality theorem of linear programming, and part (b) is due to the strong complementarity theorem.

3.1 Symmetric BCC models

Problems BLP_o and $DBLP_o$ which are used to ascertain the efficiency properties of DMU_o clearly are not symmetric in the inputs X_o and outputs Y_o. In this section, we introduce symmetric BCC model in two forms. We call

$$\begin{array}{ll} SBLP_o: & max\ w^T P_o \\ s.t. & w^T e = 1, \\ & w^T A \leq 0 \end{array} \tag{3.13}$$

the *symmetric multiplier model* , where $P_o = \begin{pmatrix} Y_o \\ -X_o \\ 1 \\ -1 \end{pmatrix}$, and its dual

$$\begin{array}{ll} DSBLP_o: & min\ \theta \\ s.t. & A \begin{pmatrix} \lambda \\ s^+ \\ s^- \\ t^+ \\ t^- \end{pmatrix} + \theta e = P_o \\ & \lambda, s^+, s^-, t^+, t^- \geq 0 \end{array} \tag{3.14}$$

the *symmetric envelopment model.* Here, e denotes an all-one vector of appropriate dimension. Setting $\lambda_o = 1$ and all remaining variables of $DSBLP_o$ to zero yields a feasible point, and the optimal value of $DSBLP_o$ does certainly not exceed zero. Furthermore, the constraints $-X\lambda - s^- + \theta e = -X_o, \lambda, s^- \geq 0$ together with $X \geq 0$ imply $\theta e \geq -X_o$, so that θ is bounded from below by $-min\{x_{io} : i = 1, ..., m\}$ on the feasible set of $SDBLP_o$. Note that, one can be verify which the BCC model have conformed from $\mathbb{R}^{m+s}$ space to regular CCR model in $\mathbb{R}^{m+s+2}$ space . Actually , in this case, space's dimension only is exchanged. Consequently, according to the model (3.14), all theorem, associated with the regular CCR model are proved.

Theorem 3.2. *(a) DMU_o is weakly efficient if and only if the optimal value of $SBLP_o$ is 0.*

(b) *DMU_o is BCC-efficient if and only if it is weakly efficient, and if there exists a positive solution $w^* > 0$ of $SBLP_o$.*

Corollary 3.1. *(a) DMU_o is weakly efficient if and only if the optimal value of $DSBLP_o$ is 0.*

(b) *DMU_o is BCC-efficient if and only if it is weakly efficient, and if all solution points of $DSBLP_o$ possess vanishing slack variables*

3.2 A regularity condition for the BCC model

In this section , it is shown that if the data which define the DMUs are in "general position " , each weakly efficient DMU will also be BCC-efficient.

Definition 3.2. *$DMU_1, ..., DMU_n$ are said to be in general position if the following regularity condition (RC) holds: each $(s+m+2) \times (s+m+2)$-submatrix of the $(s+m+2) \times (s+m+n+2)$-matrix*

$$A = \begin{pmatrix} Y & -I & 0 & 0 & 0 \\ -X & 0 & -I & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 & -1 \end{pmatrix}$$

is non-singular.

Theorem 3.3. *Let $DMU_1, ..., DMU_n$ be in general position. Then, a DMU_o is weakly efficient if and only if it is BCC-efficient.*

Note that the models introduced in this section and the next section (the Additive model) are reducible to symmetric CCR forms, so all properties and theorems from the symmetric CCR model also apply for these models.

4 The Additive model

The Additive model is a linear programming (LP)-based method proposed by Charnes et al. [2]. There are several types of Additive models in data envelopment analysis , among which we select the following:

$$\begin{aligned} ADD_o : min \quad & z = -es^- - es^+ \\ s.t. \quad & -X\lambda - s^- = -X_o \\ & Y\lambda - s^+ = Y_o \\ & \lambda \geq 0, s^-, s^+ \geq 0 \end{aligned} \tag{4.15}$$

The above model is called the *multiplier form of the Additive model* and the dual problem to the above can be expressed as follows :

$$\begin{aligned} DADD_o : max \quad & w = uY_o - vX_o \\ s.t. \quad & uY_j - vX_j \leq 0 \qquad j = 1, ..., n \\ & -u \leq -1, -v \leq -1 \end{aligned} \tag{4.16}$$

which is the *envelopment form of the Additive model.*

Let the optimal solutions of model (4.15) be $(\lambda^*, s^{-^*}, s^{+^*})$. The definition of efficiency is as follows for the Additive model.

Definition 4.1.
DMU_o is *ADD-efficient* (Additive-efficient) if and only if $s^{-^*} = 0$, $s^{+^*} = 0$.

Theorem 4.1.
DMU_o is *ADD-efficient* if and only if it is BCC-efficient.

Let $u' = -u + 1$ and $v' = -v + 1$. So, the model (4.16) can be reformulated as follows:

$$\begin{array}{lll} DADD_o: max & w = -u'Y_o + v'X_o + (1Y_o - 1X_o) & \\ s.t. & -u'Y_j + v'X_j + (1Y_j - 1X_j) \leq 0 \quad j = 1, ..., n & \\ & u' \leq 0, v' \leq 0 & \end{array} \tag{4.17}$$

Let $1Y_j - 1X_j = k_j$ and $X_j^{new} = \begin{pmatrix} X_j \\ k_j \end{pmatrix}$.
By introducing the vector $w = \begin{pmatrix} v_{new} \\ u' \end{pmatrix} \in \mathbb{R}^{m+s+1}$, where $v_{new}^t = (v', 1)$, and the matrices
$X = (X_1, ..., X_n)$,
$Y = (Y_1, ..., Y_n)$,
$A = \begin{pmatrix} X^{new} & I_1 & 0 \\ -Y & 0 & I_2 \end{pmatrix}$,
where I_1, I_2 and 0 in the definition of A denote identity $((m + 1) \times (m + 1))$, identity $(s \times s)$ and zero matrices of appropriate dimensions, respectively, we have another form of $DADD_o$ as follows:

$$\begin{array}{ll} DADD_o: max & w^T \begin{pmatrix} X_o^{new} \\ -Y_o \end{pmatrix} \\ s.t. & w^T A \leq 0 \end{array} \tag{4.18}$$

Note that $DADD_o$ is referred to as the the "multiplier model". We can conveniently write the dual of $DADD_o$ which is called the "envelopment model".

4.1 Symmetric Additive model

With regard to the definition of "symmetry" in subsection 2.1 , we observe that problem $DADD_o$ and its dual are asymmetric. Hence, we define the symmetric $DADD_o$ model as follows:

$$\begin{array}{ll} SDADD_o: & max\ w^T P_o \\ s.t. & w^T e = 1, \\ & w^T A \leq 0 \end{array} \tag{4.19}$$

This is called the *symmetric multiplier model*, where $P_o = \begin{pmatrix} X_o \\ k_o \\ -Y_o \end{pmatrix}$.

Here, e denotes an all-one vector of appropriate dimension.

We can conveniently write the dual of $SDADD_o$, is called the " *symmetric envelopment model*".
Note that, like the symmetric BCC's models , space's dimension only is exchanged.

4.2 A regular condition for the Additive model

Definition 4.1. *$DMU_1, ..., DMU_n$ are said to be in general position if the following regularity condition (RC) holds: each $(s+m+1) \times (s+m+1)$-submatrix of the $(s+m+1) \times (s+m+n+1)$-matrix*
$$A = \begin{pmatrix} X^{new} & I_1 & 0 \\ -Y & 0 & I_2 \end{pmatrix}$$
is non-singular.

Theorem 4.1. *Let $DMU_1, ..., DMU_n$ be in general position. Then, a DMU_o is weakly efficient if and only if it is ADD-efficient.*

For details of the proof see [8, 9].

5 Conclusion

DEA evaluates the relative efficiency of a set of DMUs. The relative efficiency of a DMU is the result of comparing the inputs and outputs of the DMU and those of other DMUs in the PPS (Production Possibility Set). The generic regularity condition for the CCR model was introduced by Neralic and Stein [9]. In this article, we give a general regularity condition under which each weakly efficient decision making unit in the BCC model and Additive model of data envelopment analysis is also BCC-efficient and ADD-efficient, respectively. Note that, this approach can be adapted to other models in data envelopment analysis like the BCC additive model and FDH model. Also , we can investigate the regularity condition (RC) for these model with inaccurate (interval, fuzzy, ordinal, etc.) data.

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