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The Ritz-Galerkin method for MHD Couette flow of non-Newtonian fluid

Z. Barikbin *, R. Ellahi †, S. Abbasbandy ‡§

Abstract

In this paper, the Ritz-Galerkin method in Bernstein polynomial basis is applied for solving the nonlinear problem of the magnetohydrodynamic (MHD) flow of third grade fluid between the two plates. The properties of the Bernstein polynomials together with the Ritz-Galerkin method are used to reduce the solution of the MHD Couette flow of non-Newtonian fluid in a porous medium to the solution of algebraic equations.

Keywords : Ritz-Galerkin method; Bernstein polynomials; Satisfier function; MHD.

1 Literature review

Non-Newtonian fluid dynamics is one of the most important subjects in modern applied mathematics. This is due to the fact that non-Newtonian fluids are of considerable interest in many industrial and technological applications. In nature there are different kinds of non-Newtonian fluids. All such fluid in terms of their diverse characteristics cannot be described by one constitutive equation. Hence different fluid models have been proposed. Amongst these a subclass of differential type fluids third grade can predict the shear thinning/shear thickening effects even in steady unidirectional flow over rigid boundary. MHD consideration of non-Newtonian fluids is important in the metallurgical process. Further, the non-Newtonian fluids in porous media are encountered in fields like ceramics production, certain separation processes, filtration and oil recovery, petroleum production, food processing, polymer engineering etc. Having such importance of non-Newtonian fluids in mind, many researchers are engaged to examine various physical aspects of such fluids in different flow figurations (see for instance the studies [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] and many references therein).

Recently, the effects of nonlinear partial slip on the walls for steady flow and heat transfer of an incompressible, thermodynamically compatible third grade fluid in a channel is investigated in [11]. In that paper, the space between two plates is not a porous media and the upper plate is moving with uniform velocity.

The Ritz method is a variational method. Variational methods are approximate methods for the solutions of boundary value problems (BVP). These methods include the Collocation, Galerkin (with its various versions [12]), least squares methods and the moment methods; amongst others. In each variational algorithm, we obtain some Coefficients of some given assumed approx-

*Department of Mathematics, Imam Khomeini International University, Ghazvin, 34149, Iran.
†Department of Mathematics & Statistics, Faculty of Basic and Applied Sciences, International Islamic University, Islamabad, Pakistan.
‡Corresponding author. abbasbandy@yahoo.com
§Department of Mathematics, Imam Khomeini International University, Ghazvin, 34149, Iran.

imate solutions to the BVP that would satisfy Some boundary given conditions. In this paper, the Ritz-Galerkin method in Bernstein polynomial basis is used. The approximation of the problem is based on the modified Bernstein polynomial basis. The properties of Bernstein polynomial basis. The properties of Bernstein polynomials are first presented. The Bernstein polynomial basis vanish except the first polynomial at $x = 0$, which is equal to 1 and the last polynomial at $x = R$, which is also equal to 1 over the interval $[0, R]$. This provides greater flexibility in which to impose boundary conditions at the end points of the interval.

In this work, Couette flow of an incompressible third grade fluid is considered. The fluid is electrical conducted in the presence of uniform applied magnetic field B_0 in the y-direction. Induced magnetic field is not taken into account in view of small magnetic Reynolds number. The fluid saturates the porous space between two infinite plates. The lower plate at $y = 0$ is stationary while the upper plate at $y = h$ is suddenly jerked with the velocity U_0. Slip effects in terms of shear stress is considered. For unidirectional steady flow, the velocity field $V = (u(y), 0, 0))$ satisfies the incompressibility conditions and equation of motion in absence of axial pressure gradient gives

$$\mu\frac{d^2u}{dy^2} + 2\beta\frac{d}{dy}\left(\frac{du}{dy}\right)^3 - \tag{1.1}$$

$$\frac{\phi}{K}\left[\mu + 2\beta\left(\frac{du}{dy}\right)^2\right]u - \sigma B_0^2 u = 0,$$

with the subjected boundary conditions

$$u(0) - \gamma\left[\frac{du}{dy} + 2\beta\left(\frac{du}{dy}\right)^3\right]_{y=0} = 0, \tag{1.2}$$

$$u(2h) + \gamma\left[\frac{du}{dy} + 2\beta\left(\frac{du}{dy}\right)^3\right]_{y=2h} = U_0. \tag{1.3}$$

In above equations μ is the dynamic viscosity, β is the material parameter, ϕ and k are the porosity and permeability of porous medium, σ is the electrical conductivity and γ is the dimensional slip parameter.

At this point, we introduce the following dimensionless variables

$$u = \frac{u^*}{U_0},\ y = \frac{y^*}{h},\ \gamma = \frac{\gamma^*}{h},\ \frac{1}{K} = \frac{h^2\phi}{k},$$

$$M = \frac{\sigma B_0^2 h^2}{\mu},\ \beta^* = \frac{\beta U_0^2}{\mu h^2}.$$

Now dimensionless forms of Eqs. (1.1)-(1.3) are reduced as follows

$$\frac{d^2u}{dy^2} + 2\beta\frac{d}{dy}\left(\frac{du}{dy}\right)^3 - \tag{1.4}$$

$$\frac{1}{K}\left[1 + 2\beta\left(\frac{du}{dy}\right)^2\right]u - Mu = 0,$$

with conditions

$$u(0) - \gamma\left[\frac{du}{dy} + 2\beta\left(\frac{du}{dy}\right)^3\right]_{y=0} = 0, \tag{1.5}$$

$$u(2) + \gamma\left[\frac{du}{dy} + 2\beta\left(\frac{du}{dy}\right)^3\right]_{y=2} = 1, \tag{1.6}$$

where asterisks have been suppressed for simplicity.

2 Properties of Bernstein polynomials

The general form of the B-polynomials of mth-degree are defined on the interval $[0, 1]$ as [13]

$$B_{i,m}(x) = \frac{m!}{i!\,(m-i)!}x^i(1-x)^{m-i}, \quad 0 \le i \le m.$$

A recursive definition also can be used to generate the B-polynomials over $[0, 1]$ so that the ith mth degree B-polynomial can be written

$$B_{i,m}(x) = (1-x)B_{i,n-1}(x) + xB_{i-1,n-1}(x).$$

It can be readily shown that each of the B-polynomials is positive and also the sum of all the B-polynomials is unity for all real x belonging to the interval $[0, 1]$, that is, $\sum_{i=0}^{m} B_{i,m}(x) = 1$. It can be easily shown that any given polynomial of degree m can be expanded in terms of linear combination of the basis functions

$$P(x) = \sum_{i=0}^{m} C_i B_{i,m}(x), \quad m \ge 1.$$

Moreover the Bernstein basis polynomials have the following properties:

i) $B_{i,m}(0) = \delta_{i,0}$and $B_{i,m}(1) = \delta_{i,m}$ where δ is the Kronecker delta function.

ii) $B_{i,m}(x)$ has a root with multiplicity i at point $x = 0$ (note if i is 0 there is no root at 0).

iii) $B_{i,m}(x)$ has a root with multiplicity $m-i$ at point $x = 1$ (note if $m = i$ there is no root at 1).

Now suppose that $H = L^2[0,1]$ and$\{B_{0,m}, B_{1,m}, \dots, B_{m,m}\} \subset H$, be the set of Bernstein polynomials of mth degree and:

$$Y = Span\{B_{0,m}, B_{1,m}, \dots, B_{m,m}\}$$

and f be an arbitrary element in H. Since Y is a finite dimensional vector space, f has the unique best approximation out of Y such as $y_0 \in Y$, i.e.,

$$\exists y_0 \in Y; \forall y \in Y \parallel f - y_0 \parallel \leq \parallel f - y \parallel .$$

Since $y_0 \in Y$, there exist the unique coefficients $c_0, c_1, \dots, c_m$ such that:

$$f \cong y_0 = \sum_{i=0}^{m} c_i B_{i,m} = c^T \phi,$$

where $\phi^T = [B_{0,m}, B_{1,m}, \dots, B_{m,m}]$ and $c^T = [c_0, c_1, \dots, c_m]$, and c^T can be obtained by

$$c^T < \phi, \phi >=< f, \phi >,$$

in which

$$< f, \phi >= \int_0^1 f(x)\phi(x)^T dx = [< f, B_{0,m} >,$$

$$, < f, B_{1,m} >, \dots, < f, B_{m,m} >],$$

and $< \phi, \phi >$ is a $(m+1) \times (m+1)$ matrix and is said dual matrix of ϕ. Let

$$Q = [Q_{(i+1),(i+1)}] =< \phi, \phi >= \int_0^1 \phi(x)\phi(x)^T dx,$$

then

$$c^T = (\int_0^1 f(x)\phi(x)^T dx)Q^{-1}.$$

In the following lemma we present an upper bound to estimate the error.

Lemma 2.1 *Suppose that the function* $g : [t_0, t_f] \to R$ *is* $m+1$ *times continuously differentiable,* $g \in C^{m+1}[t_0, t_f]$ *and* $Y = Span\{B_{0,m}, B_{1,m}, \dots, B_{m,m}\}$. *If* $c^T\phi$ *be the best approximation of* g *out of* Y *then the mean error bounded is presented as follows*

$$\parallel g - c^T\phi \parallel \leq \frac{M(t_f - t_0)^{(2m+3)/2}}{(m+1)! \sqrt{2m+3}},$$

where $M = \max_{x\in[t_0,t_f]}|g^{(m+1)}(x)|$.

Proof. We consider the Taylor polynomial

$y_1(x) =$

$$g(t_0) + g'(t_0)(x - t_0) + ... + g^{(m)}(t_0)\frac{(x - t_0)^m}{m!},$$

which we know

$$|g(x) - y_1(x)| \leq |g^{(m+1)}(\eta)| \frac{(x - t_0)^{m+1}}{(m+1)!}, \quad (2.7)$$

where $\eta \in (t_0, t_f)$. Since $c^T\phi$ is the best approximation g out of Y, $y1 \in Y$ and using Eq. (2.7) we have

$$\parallel g - c^T\phi \parallel_2^2 \leq \parallel g - y_1 \parallel_2^2 = \int_{t_0}^{t_f} |g(x) - y_1(x)|^2 dx$$

$$\leq \int_{t_0}^{t_f} [g^{(m+1)}(\eta)\frac{(x - t_0)^{m+1}}{(m+1)!}]^2 dx$$

$$\leq \frac{M^2}{(m+1)!^2} \int_{t_0}^{t_f} (x - t_0)^{2m+2} dx =$$

$$\frac{M^2(t_f - t_0)^{2m+3}}{[(m+1)!^2](2m+3)},$$

and by taking square roots we have the above bound.

3 The Ritz-Galerkin Method

Consider the vector space of real functions whose domain is the closed interval $[a, b]$. We define the inner product of two functions $f(x)$ and $g(x)$ as follows

$$< f, g >= \int_a^b f(x)g(x)dx.$$

In this method, we have the following assumptions:

1) All the functions we use are assumed to be square-integrable.

2) If $< f, w >= \int_a^b f(x)w(x)dx = 0$ for any function $w(x)$, then $f(x) = 0$.

3) A complete basis for a vector space V of functions is a set of linear independent functions $S = \{\varphi_i(x)\}_{i=0}^{i=\infty}$ which has the property that any $f(x) \in$ V can be written uniquely as a linear combination

$$f(x) = \sum_{j=0}^{\infty} c_j\varphi_j(x).$$

If $f(x) \in$ V and $< \varphi_i, f >= 0$ for all $\varphi_i \in S$ then $f(x) \equiv 0$. A weighted residual method uses a finite number of functions $\{\varphi_i(x)\}_{i=0}^{i=n}$. Consider the differential equation

$$L[y(x)] + f(x) = 0, \quad (3.8)$$

over the interval $a \le x \le b$. The term $L[y(x)]$ denotes a linear differential operator.
Multiplying Eq. (3.8) by any arbitrary weight function $w(x)$ and integrating over the interval $[a, b]$ one obtains

$$\int_a^b w(x)(L[y(x)] + f(x))dx = 0, \quad (3.9)$$

for any arbitrary $w(x)$. The Eqs. (3.8) and (3.9) are equivalent, because $w(x)$ is any arbitrary function.

We introduce a trial solution $u(x)$ to Eq. (3.8) of the form

$$u(x) = \varphi_0(x) + \sum_{j=1}^{n} c_j\varphi_j(x),$$

and replace $y(x)$ with $u(x)$ on the left side of the Eq. (3.8). The residual is defined as follows

$$r(x) = L[u(x)] + f(x).$$

The goal is to construct $u(x)$ so that the integral of the residual will be zero for some choices of weight functions. That is, $u(x)$ will partially satisfy (3.9) in the sense that

$$\int_a^b w(x)(L[y(x)] + f(x))dx = 0.$$

for some choices of $w(x)$. One of the most important weighted residual methods was introduced by the Russian mathematician, Boris Grigoryevich Galerkin (February 20, 1871 - July 12, 1945). Galerkin's method selects the weight functions in a special way: they are chosen from the basis functions, i.e. $w(x) \in \{\varphi_i(x)\}_{i=1}^{n}$. It is required that the following n equations hold true

$$\int_a^b \varphi_i(x)(L[y(x)] + f(x))dx = 0,$$

for $i = 1, 2, \ldots, n$. To apply the method, we solve these n equations for the coefficients $\{c_j\}_{j=1}^{\infty}$. Suppose we wish to solve a boundary value problem over the interval $[a, b]$ with the above method, we select $\varphi_i(x)$, $i = 1, 2, \ldots, m$ so that satisfy the homogeneous form of the specified essential boundary conditions and φ_0 must satisfy the specified essential boundary conditions.

4 Solution procedure

By the change of variable

$$y = 2x,$$

we get $\frac{du}{dy} = (1/2)\frac{du}{dx}$ and $\frac{d^2u}{dy^2} = (1/4)\frac{d^2u}{dx^2}$ then the Eqs. (1.4)-(1.6) can be written as the differential equation with boundary conditions on interval $[0, 1]$, i.e.

$$2\frac{d^2u}{dx^2} + \beta\frac{d}{dx}\left(\frac{du}{dx}\right)^3 -$$

$$\frac{1}{K}\left[8 + 4\beta\left(\frac{du}{dx}\right)^2\right]u - 8Mu = 0,$$

with conditions

$$u(0) - \gamma\left[(1/2)\frac{du}{dx} + (1/4)\beta\left(\frac{du}{dx}\right)^3\right]_{x=0} = 0, \quad (4.10)$$

$$u(1) + \gamma\left[(1/2)\frac{du}{dx} + (1/4)\beta\left(\frac{du}{dx}\right)^3\right]_{x=1} = 1. \quad (4.11)$$

As y goes from 0 to 2 in the original problem continuously, x goes from 0 to 1 as well in the above differential equation, continuously. Now, having boundary conditions at interval $[0, 1]$, we apply Bernstein Ritz-Galerkin method to handle the above problem as follows.

4.1 Satisfier function and Ritz-Galerkin method

Let

$$y(x) = \sum_{i=0}^{n} c_i\phi_i(x),$$

where for arbitrary constant n, $\phi_i(x)$ are following modified Bernstein polynomials:

$$\phi_i(x) = B_{i,n}, \quad i = 1, 2, \ldots, n-1,$$

$$\phi_0(x) = xB_{0,n}, \quad \phi_n(x) = (1-x)B_{n,n}.$$

With these choices of polynomials the function $y(x)$ has the following properties:

$$y(0) = y(1) = 0, \quad (4.12)$$

$$y'(0) = c_0 + nc_1, \quad (4.13)$$

$$y'(1) = -nc_{n-1} - c_n. \tag{4.14}$$

Let us define the satisfier function $\phi(x)$ which helps $y(x)$ satisfies conditions (4.10)-(4.11).

Consider the truncated series

$$\bar{u}^{(n)}(x) = \sum_{i=0}^{n} c_i\phi_i(x) + \phi(x) = y(x) + \phi(x).$$

The goal is to construct $\phi(x)$ so that $\bar{u}^{(n)}(x)$ satisfies the conditions (4.10)-(4.11).

In general, $\phi(x)$ is not unique. The simplest choice of $\phi(x)$ is

$$\phi(x) = a + (3b - 3a)x^2 + (2a - 2b)x^3,$$

such that

$$a = \gamma((1/2)(c_0 + nc_1) + (1/4)\beta(c_0 + nc_1)^3),$$

$$b =$$

$$1-\gamma((1/2)(-nc_{n-1}-c_n)+(1/4)\beta(-nc_{n-1}-c_n)^3).$$

It is worth pointing out that for $\phi(x)$ we have

$$\phi'(0) = \phi'(1) = 0,$$

$$\phi(0) = \gamma((1/2)(c_0 + nc_1) + (1/4)\beta(c_0 + nc_1)^3),$$

$$\phi(1) =$$

$$1-\gamma((1/2)(-nc_{n-1}-c_n)+(1/4)\beta(-nc_{n-1}-c_n)^3).$$

Therefore, by using (4.12)-(4.14), it's easy to see that $\bar{u}^{(m)}(x)$ satisfies the conditions (4.10)-(4.11). Now let

$$F(u) = 2\frac{d^2u}{dx^2} + \beta\frac{d}{dx}\left(\frac{du}{dx}\right)^3 - \tag{4.15}$$

$$\frac{1}{K}\left[8 + 4\beta\left(\frac{du}{dx}\right)^2\right]u - 8Mu.$$

For solving (4.15), we approximate $u(x)$ as

$$u(x) \approx \bar{u}^{(n)}(x) = \sum_{i=0}^{n} c_i\phi_i(x) + \phi(x). \tag{4.16}$$

Now the expansion coefficients c_i are determined by the Galerkin equations

$$< F(\bar{u}^{(n)}), B_{i,n} >= 0, \quad i = 1, 2, \ldots, n, \tag{4.17}$$

where $< . >$ denotes the inner product defined by

$$< F(\bar{u}^{(n)}), B_{i,n} >= \int_0^1 F(\bar{u}^{(n)})(x)B_{i,n}(x)dx.$$

Eq. (4.17) gives a system of nonlinear equations which can be solved for the elements of c_i using Newton's iterative method.

It is important to not that, we made satisfier function $\phi(x)$ so that sum of that and truncated series $\sum_{i=0}^{n} c_i\phi_i(x)$ is satisfied boundary conditions. This is different from other Ritz method, where the satisfier function satisfies the each boundary condition.

4.2 Collocation method

By the change of variable

$$y = x + 1,$$

we get $\frac{du}{dy} = \frac{du}{dx}$ then the problem (1.4)-(1.6) can be written as the differential equation with boundary conditions on interval $[-1, 1]$, i.e.,

$$\frac{d^2u}{dx^2} + 2\beta\frac{d}{dx}\left(\frac{du}{dx}\right)^3 - \tag{4.18}$$

$$\frac{1}{K}\left[1 + 2\beta\left(\frac{du}{dx}\right)^2\right]u - Mu = 0,$$

with conditions

$$u(-1) - \gamma\left[\frac{du}{dx} + 2\beta\left(\frac{du}{dx}\right)^3\right]_{x=-1} = 0, \tag{4.19}$$

$$u(1) + \gamma\left[\frac{du}{dx} + 2\beta\left(\frac{du}{dx}\right)^3\right]_{x=1} = 1. \tag{4.20}$$

Now having boundary conditions at interval $[-1, 1]$, we apply collocation method to handle the above problem as follows.

The unknown function $u(x)$ is approximated as a truncated series of Chebyshev polynomials

$$u(x) = \sum_{i=0}^{n} u_iT_i(x),$$

where $T_i(x)$ is the ith Chebyshev polynomial and u_i are the Chebyshev coefficients. For the numerical solution of (4.18) using collocation method with collocation points

$$c_0 = -1, \quad c_1, \ldots, c_{n-1} \in (-1, 1), \quad c_n = 1,$$

we substitute $u(x)$ in (4.18)-(4.20) and require that the ODE (4.18) hold at the collocation points $c_1, \ldots, c_{n-1}$, and (4.19) and (4.20) hold at the collocation points c_0 and c_n respectively.

4.3 Approximate solutions of the model

By using presented method with $n = 5$ and from (4.16), we approximate $u(x)$ by a finite linear combination of the form

$$u(x) = \sum_{j=0}^{5} c_j\phi_j + \phi(x)$$

with

$\phi_0 =$

$(1-x)^5 x, \quad \phi_1 = 5(1-x)^4 x, \quad \phi_2 = 10(1-x)^3 x^2,$

$\phi_3 =$

$10(1-x)^2 x^3, \quad \phi_4 = 5(1-x)x^4, \quad \phi_5 = (1-x)x^4,$

and

$\phi(x) = \gamma((1/2)(c0 + 5c1) + (1/4)\beta(c0 + 5c1)^3)+$

$(1 - \gamma((1/2)(-5c4 - c5) + (1/4)\beta(-5c4 - c5)^3)-$

$\gamma((1/2)(c0+5c1)+(1/4)\beta(c0+5c1)^3))(-2x^3+3x^2).$

Function $u(x)$ for different values of β, γ, K, M are plotted in the Figures 1-4, respectively. Also Tables (1)-(4) show the obtained value of $u(0)$ for different values of parameters with two methods described in this manuscript. These tables show that our computations are more accurate.

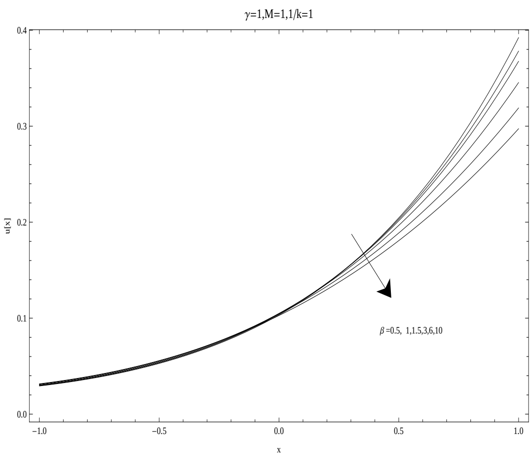

Figure 1: Variation of β on u

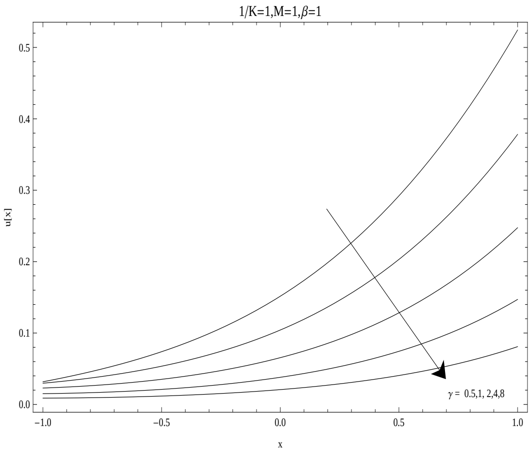

Figure 2: Variation of γ on u

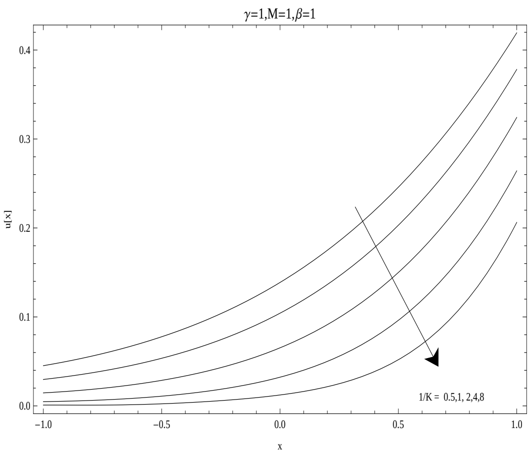

Figure 3: Variation of K on u

5 Numerical results

In this section, we have discussed the analytical and numerical results for Magnetohydrodynamic (MHD) Couette flow of third grade fluid between the two plates subject to nonlinear partial slip. The important finding in this paper is the combined effects of the nonlinear slip, MHD, porosity and the third grade fluid parameter on the velocity.Accurate numerical solutions are obtained using Ritz-Galerkin method in Bernstein polynomial basis.

To see the effects of emerging parameters on velocity figures 1 to 4 have been displayed. The effects of third grade parameter β are shown in Fig. 1. It is observed that by keeping all the other parameters fixed, an increase in the non-Newtonian parameter results in the decreased velocity. The Fig. 2 has been prepared to explain the effects of slip parameter γ. The partial slip is controlled by a dimensionless slip factor, which can vary from

Table 1: $u(0)$ for $\gamma = 1$, $M = 1$ and $\beta = 1$ for Ritz-Galerkin method (RGM) and Collocation method (CM) with $n = 7$.

$1/K$	RGM	CM
0	0	0
0.5	0.0452078	0.0452073
1	0.029734126	0.029701378
2	0.0146237	0.0146159
4	0.00474728	0.00475515
8	0.000910552	0.000909626

Table 2: $u(0)$ for $\gamma = 1$, $M = 1$ and $\frac{1}{K} = 1$ for Ritz-Galerkin method (RGM) and Collocation method (CM) with $n = 7$.

β	RGM	CM
0	0	0
0.5	0.0293163	0.02931436
1	0.029734126	0.029701378
1.5	0.03000307655120	0.02998722999818
3	0.0305548224868	0.0305542409344
6	0.0311526024483	0.031155063902
10	0.0315581316154	0.0315582

Table 3: $u(0)$ for $\gamma = 1$, $\beta = 1$ and $\frac{1}{K} = 1$ for Ritz-Galerkin method (RGM) and Collocation method (CM) with $n = 7$.

M	RM	CM
0	0	0
0.5	0.0435755	0.0435111
1	0.029734126	0.029701378
2	0.0154279	0.0153686
4	0.00528567	0.00527870
8	0.00106387	0.00106134

Table 4: $u(0)$ for $M = 1$, $\beta = 1$ and $\frac{1}{K} = 1$ for Ritz-Galerkin method (RGM) and Collocation method (CM) with $n = 7$.

γ	RGM	CM
0.5	0.0317452	0.0317472
1	0.029734126	0.029701378
2	0.0229654	0.0229626
4	0.015086	0.0151021
8	0.00883166	0.00883382

total adhesion to infinity. It is found that the velocity decreases by an increase in γ. The porosity analysis is carried out in Fig. 3. It is seen that with increase in porosity, velocity field decreases. Variation of MHD parameter is displayed in Fig. 2. This graph elucidate that the velocity decrease by increasing the MHD parameter.

The second set of results not only displays the effects of sundry parameters but also provide a comparison between Ritz-Galerkin and Collocation methods shown in Tables 1 to 4. The proposed method offers a superior intrinsic accuracy for differential operator. The numerical results indicate the usefulness of the Ritz-Galerkin method in Bernstein polynomial and Collocation method in obtaining accurate solu-

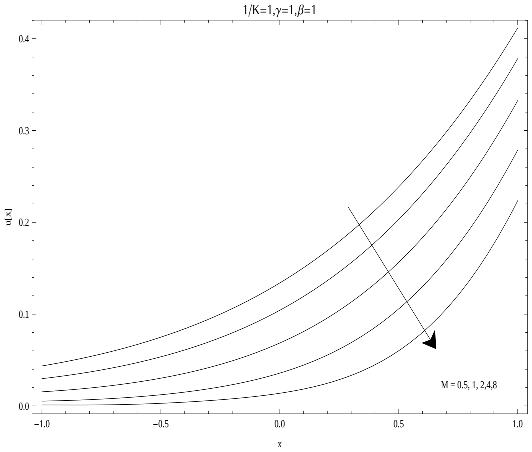

Figure 4: Variation of M on u

tions to the nonlinear problems arising in non-Newtonian fluid mechanics. As compared to other numerical techniques, such as finite differences, the nonlinearity is not a major complication for Ritz-Galerkin method in Bernstein polynomial method.

To the best of our knowledge, this is the first attempt to apply this robust and highly effective analytical technique as well as highly accurate method to study the non-Newtonian flows in this geometry. The results presented in this paper will now be available for experimental verification to give confidence for the well-posedness of this non-linear boundary value problem.

6 Conclusions

The properties of the Bernstein polynomials together with the Ritz-Galerkin method are used to reduce the solution of the MHD Couette flow of non-Newtonian fluid in a porous medium to the solution of algebraic equations. The choice of basis and $\phi(x)$ provides greater flexibility in which to impose boundary conditions. Numerical results are included to demonstrate the validity and applicability of the new technique.

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Zahra Barikbin is an Assistant professor in the department of mathematics, Imam Khomeini International University, Ghazvin. She was graduated in Shahid Beheshti University of Tehran and her main research interests include numerical solution of Partial Differential Equations, Integral Equations and computational mathematics.

Ramat Ellahi is founder Chairperson of the Department of Mathematics and Statistics at IIUI, Pakistan. He has several awards and honors on his credit: Fulbright Fellow University of California USA, Productive Scientist of Pakistan, Best University Teacher Award by higher education commission (HEC) Pakistan, Best Book Award, and Valued Reviewer Award by Elsevier. He is also an author of six books and editor of five international journals.

Saeid Abbasbandy has got PhD degree from Kharazmi University in 1996 and now he is the full professor in Imam Khomeini International University, Ghazvin, Iran. He has published more than 200 papers in international journals and conferences. Now, he is working on numerical analysis and fuzzy numerical analysis.