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Production Planning Using Fixed Levels of Inputs: A DEA-based Approach

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Abstract

The traditional production planning based on linear programming has been studied extensively in the literature. In this paper, a DEA-based production planning idea is proposed. It is assumed that supplies for the inputs of the new operational unit can be forecasted in the next production season. Moreover, the input-output levels of the existing units remain unchanged. With these forecasted inputs, the paper develops a DEA-based production planning approach to determine the most favorable production plan for new operational unit.

Keywords : DEA; Efficiency; Production planning.

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1 Introduction

Data Envelopment Analysis (DEA), is currently a popular technique for analyzing technical efficiency of a set of comparable decision making units (DMUs) with multiple inputs and outputs and it has had a number of applications. In DEA, we assume that the assessed units are homogeneous, i.e. they perform the same tasks with similar objectives and consume similar inputs and produce similar outputs. Moreover, they operate in similar operational environments. During the recent years, many applications of DEA have been studied. An important application of this technique is in solving the problem of production planning. Many authors have studied the production planning from various perspectives. Chazal et al. (2008) studied the production planning and inventory management problem based on the assumption that the firm under discussion acts in continuous time on a finite period in order to dynamically maximize its instantaneous profit. Using DEA in production planning or similar studies is not new. Golany (1988) was the first that used DEA

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in production planning. He presented an interactive multi-objective linear programming procedure to generate a set of alternative efficient points for DMU to consider. Du et al.(2010) looked at the production planning problem from the productivity and efficiency perspective using data envelopment analysis. They proposed two planning ideas in a centralized decision making environment when demand changes can be forecasted. One was optimizing the average or overall production performance of the entire organization, and the other one was simultaneously maximizing total outputs produced and minimizing total inputs consumed by all units. In the current study, we present a production planning approach for a new operational unit by using a fixed level of inputs. We construct a new operational unit and supplies for the inputs of this new operational unit can be forecasted in the next production season. Moreover, it is assumed that the input-output levels of the existing units remain unchanged. With these forecasted resources for new operational unit, the paper develops a DEA-based production planning approach to determine the most favorable production plan. A principal idea behind the procedure is to use the empirical production function defined by the observed inputs and outputs to make the new operational unit as most productive scale size (MPSS). The paper is structured as follows: A brief background of DEA is presented in section 2. The proposed production planning approach is given in section 3. Section 4 gives a simple numerical example. Conclusions appear in section 5.

2 Preliminaries

The technology set T is extrapolated from the observed data on input-output pairs $\{(x_j, y_j) \mid j = 1, \ldots, n\}$ in which $x_j = (x_{1j}, x_{2j}, \ldots, x_{mj}) \geq 0$ and $y_j = (y_{1j}, y_{2j}, \ldots, y_{sj}) \geq 0$ are the nonzero input and output vectors, respectively, corresponding to DMU_j. Charnes et al. (1978) constructed the technology set T_c as

$$T_c = \left\{ (x, y) \mid x \geq \sum_{j=1}^{n} \lambda_j x_j, \quad y \geq \sum_{j=1}^{n} \lambda_j y_j, \quad \lambda_j \geq 0, \quad j = 1, \ldots, n \right\}$$

The classical DEA radial input measure of efficiency is calculated as

$$Min\ \{\theta \mid (\theta x_o, y_o) \in T_c\}$$

the above formulation can be restated as

$$\begin{array}{lll} Min & \theta & \\ s.t. & \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io}, & i = 1, ..., m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \leq y_{ro}, & r = 1, ..., s \\ & \lambda_j \geq 0 & for\ all\ j \end{array} \tag{2.1}$$

Banker et al. (1984) took the nature of returns to scale into account and constructed the technology set T_ν as

$$T_\nu = \left\{ (x, y) \mid x \geq \sum_{j=1}^{n} \lambda_j x_j, \quad y \geq \sum_{j=1}^{n} \lambda_j y_j, \quad \sum_{j=1}^{n} \lambda_j = 1, \quad j = 1, \ldots, n \right\}$$

Definition 2.1. *A surface*

$$H = \{(x, y) \mid u^t y - v^t x = 0,\ u \geq 0,\ v \geq 0\} \cap T_c$$

is called an efficient supporting surface of T_c if for each extreme efficient observation j, $u^t y_j - v^t x_j \leq 0$.

Banker (1984) introduced the concept of MPSS as follows:

Definition 2.2. *A production possibility $(x, y) \in T_\nu$ is Banker's MPSS if for all $(\beta x, \alpha y) \in T_\nu$, we have $\frac{\alpha}{\beta} \leq 1$.*

This definition shows that Banker's MPSS must be associated with boundary points and maximizes, average productivity, $\frac{\alpha}{\beta}$, for the $DMU(x, y)$. Cooper, Thompson and Thrall (1996) proposed the following linear fractional programming problem to determine the MPSS in T_ν:

$$\begin{array}{lll} Max & \frac{\alpha}{\beta} & \\ s.t. & \alpha y_o \leq \sum_{j=1}^{n} \lambda_j y_{rj}, & r = 1, ..., s \\ & \beta x_o \leq \sum_{j=1}^{n} \lambda_j x_{ij}, & i = 1, ..., m \\ & \sum_{j=1}^{n} \lambda_j = 1, & \\ & \alpha, \beta, \lambda_j \geq 0 & for\ all\ j \end{array} \tag{2.2}$$

They stated that DMU_o is MPSS if the following two conditions are satisfied:

(i) $\frac{\alpha^*}{\beta^*}$

(ii) All slack variables must be zero in any optimal solution.

3 Production planning model

We assume that there are n $DMUs$ indexed by $\{DMU_j \mid j = 1, \ldots, n\}$. The $i-$th input and $r-$th output of DMU_j are denoted by $\{x_{ij} \mid (i = 1, \ldots, m)\}$ and $\{y_{rj} \mid (r = 1, \ldots, s)\}$, respectively. Suppose that the supply for input $\{i \mid i = 1, \ldots, m\}$ in the next production season can be forecasted as $\{\overline{x}_i \mid i = 1, \ldots, m\}$. For these forecasted supplies, we will determine the most favorable output plans for the new operational unit. We first solve the following linear programming problem in order to find the most favorable efficient surface of the production possibility set T_c that the new operational unit DMU_{n+1} is projected to:

$$\begin{array}{lll} Min & \sum_{j=1}^{n} s_j & \\ s.t. & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + s_j = 0, & j = 1, ..., n \\ & \sum_{i=1}^{m} v_i \overline{x}_i = 1, & \\ & u_r, v_i \geq 0 & for\ all\ i, r \end{array} \tag{3.3}$$

Let u^*, v^* and s^* be an optimal solution to (3.3). It is easy to show that $s_j^* = 0$ for some $j \in \{1, 2, \ldots, n\}$. Moreover, the optimal solution to (3.3) gives a supporting surface of T_c as follows:

$$H = \{(x, y) \mid u^* y - v^* x = 0\} \cap T_c$$

$DMUs$ that do belong to H are clearly MPSS in T_ν.

Now, each production plan that lies on H, makes the new operational unit as MPSS. The set of all optimal production plans that make DMU_o as MPSS is defined as follows:

$$P_{n+1} = \left\{ (\overline{y}_1, \overline{y}_2, \ldots, \overline{y}_s) \mid \sum_{r=1}^{s} u_r^* \overline{y}_r = 1, \quad \overline{y}_r \geq l_r, \quad r = 1, \ldots, s \right\}$$

$l_r \geq 0$ is a user- defined constant to reflect a lower bound on the $r-$th output. One production plan, for example, can be determined as $\overline{y}_r = \frac{1}{s u_r^*}$, $r = 1, \ldots, s$. Clearly, $\sum_{r=1}^{s} u_r^* \overline{y}_r = 1$ and hence DMU_{n+1} is $CCR-$efficient.

A multi-objective linear programming (MOLP) model is developed as follows to simultaneously maximize all output productions:

$$\begin{array}{ll} Max & \{\overline{y}_1, \overline{y}_2, \ldots, \overline{y}_s\} \\ s.t. & \sum_{r=1}^{s} u_r^* \overline{y}_r = 1, \\ & \overline{y}_r \geq l_r, \qquad r = 1, \ldots, s \end{array} \tag{3.4}$$

Taking the priority of the outputs into account, we can develop the following linear programming model as equivalence to model (3.4):

$$\begin{array}{ll} Max & \rho \\ s.t. & \sum_{r=1}^{s} u_r^* \overline{y}_r = 1, \\ & \rho \leq \beta_r \overline{y}_r, \qquad r = 1, \ldots, s \\ & \overline{y}_r \geq l_r, \qquad r = 1, \ldots, s \end{array} \tag{3.5}$$

where β_rs represent positive values that reflect the importance of y_rs with $\sum_{r=1}^{s} \beta_r = 1$.

Theorem 3.1. *Planned by the foregoing planning procedure, the new operational unit DMU_{n+1} can become a MPSS when evaluated under the original production possibility set.*

Proof: Since $DMU_{n+1} \in H$, the proof is clear.

4 Numerical example

Suppose thet we have data on 10 $DMUs$ with two outputs and inputs as given in Table 1 and suppose that we are given a resource allocation vector $(x_1, x_2) = (6.5, 6.2)$ (This example is taken from Golany (1988)). Solving the program (3.3) numerically yields

$$u_1^* = 0.202808$$

$$u_2^* = 0.062402$$

$$u_3^* = 0.109204$$

$$u_4^* = 0.046802$$

This solution gives the following supporting surface of T_c:

$$H = \{(x_1, x_2, y_1, y_2) \mid 0.202808y_1 + 0.062402y_2 - 0.109204x_1 - 0.046802x_2 = 0\} \cap T_c$$

The set of all optimal production plans is determined as

$$P = \{(\overline{y}_1, \overline{y}_2) \mid 0.202808\overline{y}_1 + 0.062402\overline{y}_2 = 1, \quad \overline{y}_1, \overline{y}_2 \geq 0\}$$

Clearly,

$$y_1 = \frac{1}{2(0.202808)} = 2.465386, \quad y_2 = \frac{1}{2(0.062402)} = 8.012564$$

is an optimal production plan for DMU_{11} in the next production season and with this plan, DMU_{11} is a MPSS. If we incorporate the priority of outputs as $\beta_1 = 0.3$ and $\beta_2 = 0.7$ and apply the LP model (3.5), the optimal plan is determined as:

$$\overline{y}_1 = 0.4356317, \quad \overline{y}_2 = 1.866993$$

Clearly, DMU_{11} with $(\overline{y}_1, \overline{y}_2) = (0.4356317, 1.866993)$ is a MPSS.

5 Conclusion

In this paper, a DEA-based approach has been developed for making future production plans for a new operational unit. The procedure proposed in this paper, is aimed at assisting a new decision maker in setting the most favorable production goals. The principal idea behind the procedure is to use the empirical production function defined by the observed inputs and outputs to make the new operational unit as MPSS.

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