Science and Research Branch (IAU)

# The embedding method to obtain the solution of fuzzy linear systems

T. Allahviranloo [*][†], A. Hashemi [‡]

**Abstract**

In this paper,we study (investigates) general fuzzy linear system.The main aim of this paper is based on embedding approach.we find that it is necessary and sufficient conditions for existence of fuzzy solution.Finally,Numerical examples are presented to illustrate the proposed model.

*Keywords* : Fuzzy linear system; Embedding method; Fuzzy numbers.

## 1 Literature review

System of linear equation play a major role in various areas of science.In many problems at various areas of science, can be solved by solving a system of linear equation. since some of the systems are parametric and measurements are vague or imprecise, are represented by fuzzy numbers.Development of mathematical models and numerical procedures that would appropriately treat general fuzzy linear systems and solve them, is important. The system of linear equations, $AX = b$ , is called fuzzy system of linear equations(FSLE), in which coefficients matrix $(n \times n)$ $A$ is crisp and $b$ is a column matrix and a fuzzy number vector. The fuzzy linear equations has been studied by many authors.Friedman [17] proposed a general model for solving such fuzzy linear systems by using the embedding approach where replace the original system by $(2n) \times (2n)$ representation. In following Friedman et al. [17], Allahviranloo et al.in [3, 4, 5, 6, 7, 8] and other authors such as Abbasbandy et al. [1, 2], Asady

et al. [9], Dehghan et al. [15], Wang et al. [20], Zheng et al. [21], Ezzati et al. [16] designed some numerical methods for calculating the solution of fuzzy linear system. In this paper we propose a general model for solving an (FSLE). We use the embedding method and replace original $n \times n$ (FSLE) by two $n \times n$ crisp linear systems. In the Section 2, we introduce preliminary and (FSLE). In Section 3rd the proposed model for solving (FSLE) is discussed. The proposed model is illustrated by solving some example in section 4th.

## 2 Preliminaries

**Definition 2.1** *[13] For Arbitrary fuzzy number $\widetilde{u}$ in parametric form, is represented by an ordered pair of functions $(\underline{u}(r), \overline{u}(r)), 0 \le r \le 1$ which satisfy the following requirements:*

*(i) $\underline{u}(r)$ is a bounded left-continuous non-decreasing function over $[0, 1]$*

*(ii) $\overline{u}(r)$ is a bounded left-continuous non-increasing function over$[0, 1]$*

*(iii) $\underline{u}(r) \le \overline{u}(r), 0 \le r \le 1$ .*

*The set of all these fuzzy numbers is denoted by E.*

---
[*]Corresponding author. allahviranloo@yahoo.com

[†]Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran.

[‡]Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran.

**Definition 2.2** *[17]For arbitrary fuzzy numbers* $\widetilde{x} = (\underline{x}(r), \overline{x}(r))$, $\widetilde{y} = (\underline{y}(r), \overline{y}(r))$ *and real number* $k$ *we define equality* $\widetilde{x} = \widetilde{y}$, *addition* $\widetilde{x} + \widetilde{y}$ *and multiplication as follows:*

*(i)* $\widetilde{x} = \widetilde{y}$ *if and only if* $\underline{x}(r) = \underline{y}(r)$ *and* $\overline{x}(r) = \overline{y}(r)$.

*(ii)* $\widetilde{x} + \widetilde{y} = (\underline{x}(r) + \underline{y}(r), \overline{x}(r) + \overline{y}(r))$.

*(iii)* $k\widetilde{x} = \begin{cases} (k\underline{x}, k\overline{x}), & k \geq 0 \\ \\ (k\overline{x}, k\underline{x}), & k \leq 0 \end{cases}$

**Definition 2.3** *[17] The* $n \times n$ *linear system of equations*

$$\begin{cases} a_{11}\widetilde{x}_1 + a_{12}\widetilde{x}_2 + \cdots + a_{1n}\widetilde{x}_n = \widetilde{b}_1, \\ a_{21}\widetilde{x}_1 + a_{22}\widetilde{x}_2 + \cdots + a_{2n}\widetilde{x}_n = \widetilde{b}_2, \\ \vdots \\ a_{n1}\widetilde{x}_1 + a_{n2}\widetilde{x}_2 + \cdots + a_{nn}\widetilde{x}_n = \widetilde{b}_n, \end{cases} \quad (2.1)$$

*where the coefficients matrix,* $A = [a_{ij}]_{i,j=1}^n$ *is a crisp* $n \times n$ *matrix and* $\widetilde{b}_i$ *are fuzzy numbers, is called a fuzzy linear system. The matrix form of system (2.1 ) is as follows:*

$$AX = b, \quad (2.2)$$

*where* $X = (\widetilde{x}_1, \widetilde{x}_2, \ldots, \widetilde{x}_n)^T$, $b = (\widetilde{b}_1, \widetilde{b}_2, \ldots, \widetilde{b}_n)^T$ *are the fuzzy number vectors.*

**Definition 2.4** *[17]A fuzzy number vector* $X = (\widetilde{x}_1, \widetilde{x}_2, \ldots, \widetilde{x}_n)^T$ *that has been given by* $\widetilde{x}_j = (\underline{x}_j(r), \overline{x}_j(r))$ , $1 \leq j \leq n$ , $0 \leq r \leq 1$ *is called a solution of (2.1) if*

$$\underline{\sum_{j=1}^n a_{ij}\widetilde{x}_j} = \sum_{j=1}^n \underline{a_{ij}\widetilde{x}_j} = \underline{b}_i$$

$$\overline{\sum_{j=1}^n a_{ij}\widetilde{x}_j} = \sum_{j=1}^n \overline{a_{ij}\widetilde{x}_j} = \overline{b}_i$$

$$i = 1, 2, \ldots, n. \quad (2.3)$$

## 3　The proposed model

In this section at first we are going to define an embedding map to form a new crisp system.

**Definition 3.1** *For an arbitrary fuzzy number* $\widetilde{x}$ *in parametric form the embedding* $\pi : \mathbb{R}^2 \to \mathbb{R}^2$ *is defined as follows*

$$\pi(\underline{x}(r), \overline{x}(r)) = (\overline{x}(r) - \underline{x}(r), \overline{x}(r) + \underline{x}(r)). \quad (3.4)$$

**Lemma 3.1** *Let* $\widetilde{x} = (\underline{x}(r), \overline{x}(r))$, $\widetilde{y} = (\underline{y}(r), \overline{y}(r))$ *are arbitrary fuzzy numbers and let* $k$ *is real number.Then*

*(i)* $\widetilde{x} = \widetilde{y}$ *if and only if* $\pi(\widetilde{x}) = \pi(\widetilde{y})$

*(ii)* $\pi(\widetilde{x} + \widetilde{y}) = \pi(\widetilde{x}) + \pi(\widetilde{x})$

*(iii)* $\pi(k\,\widetilde{x}) = \pi(k(\underline{x}(r), \overline{x}(r))) = (|k|(\overline{x}(r) - \underline{x}(r)), k(\overline{x}(r) + \underline{x}(r)))$

**Proof.** *The properties (i) and (ii) are clear. We just prove the item (iii): Let* $k \geq 0$, *since* $k\widetilde{x} = (k\underline{x}, k\overline{x})$. *Then* $\pi(k\,\widetilde{x}) = (k(\overline{x}(r) - \underline{x}(r)), k(\overline{x}(r) + \underline{x}(r))) = (|k|(\overline{x}(r) - \underline{x}(r)), k(\overline{x}(r) + \underline{x}(r)))$.

*Let* $k \leq 0$ *since* $k\widetilde{x} = (k\overline{x}, k\underline{x})$. *Then* $\pi(k\,\widetilde{x}) = (-k(\overline{x}(r) - \underline{x}(r)), k(\overline{x}(r) + \underline{x}(r))) = (|k|(\overline{x}(r) - \underline{x}(r)), k(\overline{x}(r) + \underline{x}(r)))$.

By the previous lemma (3.1) , Eq. (2.1) can be replaced by the following parametric system:

$$\pi\left(\sum_{j=1}^n \left(a_{ij}(\underline{x}_j(r), \overline{x}_j(r))\right)\right) = \pi\left(\underline{b}_i(r), \overline{b}_i(r)\right),$$
$$(3.5)$$
$$i = 1, 2, \ldots, n.$$

$$\sum_{j=1}^n \left(\pi\left(a_{ij}(\underline{x}_j(r), \overline{x}_j(r))\right)\right) =$$
$$\left(\overline{b}_i(r) - \underline{b}_i(r), \overline{b}_i(r) + \underline{b}_i(r)\right), i = 1, 2, \ldots, n.$$
$$(3.6)$$

$$|a_{ij}|(\overline{x}_j(r) - \underline{x}_j(r)), a_{ij}(\overline{x}_j(r) + \underline{x}_j(r)) =$$
$$\sum_{j=1}^n \left(\left(\overline{b}_i(r) - \underline{b}_i(r), \overline{b}_i(r) + \underline{b}_i(r)\right), (3.7)\right.$$
$$i = 1, 2, \ldots, n.$$

$$|a_{ij}|(\overline{x}_j(r) - \underline{x}_j(r)), \sum_{j=1}^n a_{ij}(\overline{x}_j(r) + \underline{x}_j(r)) =$$
$$\left(\sum_{j=1}^n \left(\overline{b}_i(r) - \underline{b}_i(r), \overline{b}_i(r) + \underline{b}_i(r)\right), (3.8)\right.$$
$$i = 1, 2, \ldots, n.$$

Now we have the following equations :

$$\sum_{j=1}^n |a_{ij}|(\overline{x}_j(r) - \underline{x}_j(r)) = \overline{b}_i(r) - \underline{b}_i(r), \quad (3.9)$$
$$i = 1, 2, \ldots, n.$$

$$\sum_{j=1}^n a_{ij}(\overline{x}_j(r) + \underline{x}_j(r)) = \overline{b}_i(r) + \underline{b}_i(r), \quad (3.10)$$

$$i = 1, 2, \ldots, n.$$

Consequently in order to solve the system given by (2.1) we must solve two $(n \times n)$ crisp linear system of Eq. (3.9) and (3.10).
The matrix form of systems (3.9) and (3.10) is as follows:

$$BU = Z, AY = W \qquad (3.11)$$

where the coefficients matrix $B = [ \ |a_{ij}| \ ]_{i,j=1}^{n}$ and $A = [a_{ij}]_{i,j=1}^{n}$ are crisp $n \times n$ matrices and the right hand side columns are the vectors $Z = (\bar{b}_1(r) - \underline{b}_1(r), \ldots, \bar{b}_n(r) - \underline{b}_n(r))^T$ ,
$W = (\bar{b}_1(r) + \underline{b}_1(r), \ldots, \bar{b}_n(r) + \underline{b}_n(r))^T$.
$U = (\bar{x}_1(r) - \underline{x}_1(r), \ldots, \bar{x}_n(r) - \underline{x}_n(r))^T$ and $Y = (\bar{x}_1(r) + \underline{x}_1(r), \ldots, \bar{x}_n(r) + \underline{x}_n(r))$ are the solutions of the crisp linear systems of Eq. (3.11) .

**Theorem 3.1** *The fuzzy linear system (2.1) has a unique solution if and only if the matrices $A$ and $B$ are both nonsingular.*
**Proof.** *It is obvious.*

So the solution vector is unique but it is not still an appropriate fuzzy number vector.
The following theorems explain guaranteed conditions for recieving fuzzy number vector solution. In order to have an appropriate solution we use following theorems.

**Theorem 3.2** *[17] The unique solution X of Eq. (3.9) is nonnegative for arbitrary Z if and only if $B^{-1}$ is nonnegative.*

**Proof.** Let $B^{-1} = (t_{ij})$, $\qquad 1 \leq i, j \leq n$. Then $U = B^{-1}Z$ and $Z = (\bar{b}_1(r) - \underline{b}_1(r), \ldots, \bar{b}_n(r) - \underline{b}_n(r))^T$

$$u_i = \sum_{j=1}^{n} t_{ij} z_j \qquad (3.12)$$

Let $u_i \geq 0$ , $1 \leq i \leq n$ , since $z_j \geq 0$ , $1 \leq j \leq n$ , then $t_{ij} \geq 0$ , $1 \leq i, j \leq n$.
Because, if we consider $\exists i \exists j$ , $t_{ij} < 0$ and can choose $z_j = e_j$ consequently $u_i < 0$ and this is contradiction. Converse case is obvious .

**Theorem 3.3** *[19] The inverse of a nonnegative matrix A is nonnegative if and only if A is a generalized permutation matrix.*

**Theorem 3.4** *The fuzzy linear system (2.1) has a fuzzy solution*
*If $B^{-1}$ , $B^{-1} - A^{-1}$ , $B^{-1} + A^{-1}$ are nonnegative matrices.*

**Proof.** Let $B^{-1} = (t_{ij})$ and $A^{-1} = (s_{ij})$ , $1 \leq i, j \leq n$ then

$$U = B^{-1}Z, Y = A^{-1}W \qquad (3.13)$$

$u_i = \bar{x}_i(r) - \underline{x}_i(r)$ and $y_i = \bar{x}_i(r) + \underline{x}_i(r)$ , $1 \leq i \leq n$ , respectively are the solutions of Eq.(3.9) and Eq.(3.10) . Thus we can write : $\bar{x}_i = \frac{1}{2}(y_i + u_i)$

$$\bar{x}_i = \frac{1}{2}\left( \sum_{j=1}^{n} s_{ij} w_j + \sum_{j=1}^{n} t_{ij} z_j \right) \qquad (3.14)$$

With replacement $z_j = (\bar{b}_j(r) - \underline{b}_j(r))$ and $w_j = (\bar{b}_j(r) + \underline{b}_j(r))$ in Eq. (3.14) , the result will be

$$\bar{x}_i = \frac{1}{2}\left( \sum_{j=1}^{n} (s_{ij} + t_{ij}) \bar{b}_j + \sum_{j=1}^{n} (s_{ij} - t_{ij}) \underline{b}_j \right) \qquad (3.15)$$

Since $\bar{b}_j$ is monotonically decreasing and $\underline{b}_j$ is monotonically increasing for all $j$ ,and according to assumptions of theorem, $\bar{x}_i$ to be monotonically decreasing. Similarly $\underline{x}_i = \frac{1}{2}(y_i - u_i)$ is monotonically increasing.

**Theorem 3.5** *With notation of theorem (3.4), the fuzzy linear system (2.1) has a fuzzy number solution, if and only if*

$$\begin{cases} u_i \geq 0 \\ |\frac{dy_i}{dr}| \leq -\frac{du_i}{dr} \end{cases} \qquad (3.16)$$

*where $U = B^{-1}Z$ and $Y = A^{-1}W$.*

**Proof.** Let (FSLE) (2.1) has a fuzzy number solution vector $X = (\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n)^T$ which $\tilde{x}_i = (\underline{x}_i(r), \bar{x}_i(r))$. Therefore, $u_i = \bar{x}_i(r) - \underline{x}_i(r) \geq 0$, $i = 1, \ldots, n$. Since $\underline{x}_i = \frac{1}{2}(y_i - u_i)$ is monotonically increasing and $\bar{x}_i = \frac{1}{2}(y_i + u_i)$ is monotonically decreasing, then $\frac{d\underline{x}_i}{dr} \geq 0$ and $\frac{d\bar{x}_i}{dr} \leq 0$. Thus $\frac{d(y_i - u_i)}{dr} \geq 0$, $\frac{d(y_i + u_i)}{dr} \leq 0$ i.e. $-\frac{du_i}{dr} \geq -\frac{dy_i}{dr}$ and $-\frac{du_i}{dr} \geq \frac{dy_i}{dr}$. Consequently, $|\frac{dy_i}{dr}| \leq -\frac{du_i}{dr}$. Conversely is obvious.

# 4 Numerical example

**Example 4.1** *[17] Consider the 2 × 2 fuzzy system*
$\tilde{x}_1 - \tilde{x}_2 = (r, 2 - r)$,
$\tilde{x}_1 + 3\tilde{x}_2 = (4 + r, 7 - 2r)$.

$det(A) = 4$ and $det(B) = 2$, therefore Eq. (3.9)

and Eq. (3.10) will have solutions as follow:

$$U = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} \overline{x}_1(r) - \underline{x}_1(r) \\ \overline{x}_2(r) - \underline{x}_2(r) \end{pmatrix} = B^{-1}Z =$$

$$\begin{pmatrix} 1.5 & -0.5 \\ -0.5 & 0.5 \end{pmatrix} \begin{pmatrix} 2 - 2r \\ 3 - 3r \end{pmatrix} = \begin{pmatrix} 1.5 - 1.5r \\ 0.5 - 0.5r \end{pmatrix}$$

$$Y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \overline{x}_1(r) + \underline{x}_1(r) \\ \overline{x}_2(r) + \underline{x}_2(r) \end{pmatrix} = A^{-1}W =$$

$$\begin{pmatrix} 0.75 & 0.25 \\ -0.25 & 0.25 \end{pmatrix} \begin{pmatrix} 2 \\ 11 - r \end{pmatrix} = \begin{pmatrix} 4.25 - 0.25r \\ 2.25 - 0.25r \end{pmatrix}$$

$\forall r, 0 \le r \le 1$ , $u_1 = 1.5 - 1.5r$ and $u_2 = 0.5 - 0.5r$, both are nonnegative .
Also $\forall r, 0 \le r \le 1$, $|\frac{dy_i}{dr}| \le -\frac{du_i}{dr}$ , $i = 1, 2$. The result will be

$$\overline{x}_1 = \frac{1}{2}(y_1 + u_1) = 2.875 - 0.875r,$$

$$\underline{x}_1 = \frac{1}{2}(y_1 - u_1) = 1.375 + 0.625r$$

$$\overline{x}_2 = \frac{1}{2}(y_2 + u_2) = 1.375 - 0.375r,$$

$$\underline{x}_2 = \frac{1}{2}(y_2 - u_2) = 0.875 + 0.125r$$

Therefore the fuzzy number solutions are $\widetilde{x}_1 = (\underline{x}_1(r), \overline{x}_1(r))$, $\widetilde{x}_2 = (\underline{x}_2(r), \overline{x}_2(r))$

**Example 4.2** [17] Consider the $3 \times 3$ fuzzy system
$\widetilde{x}_1 + \widetilde{x}_2 - \widetilde{x}_3 = (r, 2 - r)$,

$\widetilde{x}_1 - 2\widetilde{x}_2 + \widetilde{x}_3 = (2 + r, 3)$,

$2\widetilde{x}_1 + \widetilde{x}_2 + 3\widetilde{x}_3 = (-2, -1 - r)$.

$det(A) = -13$ and $det(B) = 1$, therefore

Eq. (3.9) will has solution as follow:

$$U = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} \overline{x}_1(r) - \underline{x}_1(r) \\ \overline{x}_2(r) - \underline{x}_2(r) \\ \overline{x}_3(r) - \underline{x}_3(r) \end{pmatrix} = B^{-1}Z =$$

$$\begin{pmatrix} 5 & -2 & -1 \\ -1 & 1 & 0 \\ -3 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2 - 2r \\ 1 - r \\ 1 - r \end{pmatrix} = \begin{pmatrix} 7 - 7r \\ -1 + r \\ -4 + 4r \end{pmatrix}$$

$\forall r, 0 \le r \le 1$ , $u_1 = 7 - 7r \ge 0$ , $u_2 = -1 + r \le 0$ and $u_3 = -4 + 4r \le 0$, so this (FSLE) will not have fuzzy number solution.

**Example 4.3** Consider the $2 \times 2$ fuzzy system

$\widetilde{x}_1 - 2\widetilde{x}_2 = (r, 2 - r)$,

$\widetilde{x}_1 + 3\widetilde{x}_2 = (r, 2.9 - 1.9r)$.

$det(A) = 5$ and $det(B) = 1$, therefore Eq.

(3.9) and Eq. (3.10) will have solutions as follow:

$$U = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} \overline{x}_1(r) - \underline{x}_1(r) \\ \overline{x}_2(r) - \underline{x}_2(r) \end{pmatrix} = B^{-1}Z =$$

$$\begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 2 - 2r \\ 2.9 - 2.9r \end{pmatrix} = \begin{pmatrix} 0.2 - 0.2r \\ 0.9 - 0.9r \end{pmatrix}$$

$$Y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \overline{x}_1(r) + \underline{x}_1(r) \\ \overline{x}_2(r) + \underline{x}_2(r) \end{pmatrix} = A^{-1}W =$$

$$\begin{pmatrix} 0.6 & 0.4 \\ -0.2 & 0.2 \end{pmatrix} \begin{pmatrix} 2 \\ 2.9 - 0.9r \end{pmatrix} = \begin{pmatrix} 2.36 - 0.36r \\ 0.18 - 0.18r \end{pmatrix}$$

$\forall r, 0 \le r \le 1$ , $u_1 = 0.2 - 0.2r$ and $u_2 = 0.9 - 0.9r$, both are nonnegative .

Also $\forall r, 0 \le r \le 1$, $|\frac{dy_1}{dr}| \ge -\frac{du_1}{dr}$ , $|\frac{dy_2}{dr}| \le -\frac{du_2}{dr}$ so this (FSLE) can not have fuzzy number solution.

## 5    Conclusion

In this paper we propose a general model for solving a system $n \times n$ of (FSLE). The original system with a matrix $\tilde{A}$ is replaced by two crisp linear system with two matrix $A$ and $B$ which matrix $n \times n$, $B$ is formed with absolute matrix element $A$. Therefore, the matrix $B$ may be singular even if $A$ is nonsingular.

## References

[1] S. Abbasbandy, R. Ezzati, A. Jafarian, *LU decomposition method for solving fuzzy system of linear equations*, Applied Mathematics and Computation 172 (2006) 633-643.

[2] S. Abbasbandy, A. Jafarian, *Steepest descent method for system of fuzzy linear equations*, Applied Mathematics and Computation 175 (2006) 823-833.

[3] T. Allahviraloo, *Discussion: A comment on fuzzy linear systems*, Fuzzy Sets and Systems 140 (2003) 559-568.

[4] T. Allahviraloo, *Numerical methods for fuzzy system of linear equations*, Applied Mathematics and Computation 155 (2004) 493-502.

[5] T. Allahviraloo, *Successive over relaxation iteration iterative method for fuzzy system of linear equations*, Applied Mathematics and Computation 162 (2005a) 189-196.

[6] T. Allahviraloo, *The Adomian decomposition method for fuzzy system of linear equations*, Applied Mathematics and Computation 163 (2005b) 553-563.

[7] T. Allahviranloo, M. Afshar Kermani, *Solution of a fuzzy system of linear equation*, Applied Mathematics and Computation 175 (2006) 519-531.

[8] T. Allahviranloo, E. Ahmady, N. Ahmady, Kh. Shams Alketaby, *Block Jacobi two stage method with Gauss Sidel inner iterations for fuzzy systems of linear equations*, Applied Mathematics and Computation 175 (2006) 1217-1228.

[9] B. Asady, S. Abbasbandy, M. Alavi, *Fuzzy general linear systems*, Applied Mathematics and Computation 169 (2005) 34-40.

[10] J. J. Buckley, Y. Qu, *Solving linear and quadratic fuzzy equations*, Fuzzy Sets and Systems 38 (1990) 43-59.

[11] J. J. Buckley, Y. Qu, *Solving fuzzy equations: a new solution concept*, Fuzzy Sets and Systems 39 (1991a) 291-301.

[12] J. J. Buckley, Y. Qu, *Solving systems of linear fuzzy equatios*, Fuzzy Sets and Systems 43 (1991b) 33-43.

[13] Wu. Cong-Xin, Ma Ming, *Embedding problem of fuzzy number space: PartI*, Fuzzy Sets and Systems 44 (1991) 33-38.

[14] Wu. Cong-Xin, Ma Ming, *Embedding problem of fuzzy number space: PartI*, Fuzzy Sets and Systems 46 (1992) 281-286.

[15] M. Dehghan, B. Hashemi, *Iterative solution of fuzzy linear systems*, Applied Mathematics and Computation 175 (2006) 645-674.

[16] R. Ezzati, *Solving fuzzy linear systems*, Springer-Verlag 2010.

[17] M. Friedman, M. Ming, A. Kandel, *Fuzzy linear systems*, Fuzzy Sets and Systems 96 (1998) 201-209.

[18] M. Friedman, M. Ming, A. Kandel, *Discussion:Author's reply*, Fuzzy Sets and Systems 5 (2001) 140-561.

[19] H. Minc, *Nonnegative Matrices*, (Wiley, New York,1988).

[20] Ke. Wang, B. Zheng, *Inconsistent fuzzy linear systems*, Applied Mathematics and Computation 181 (2006) 973-981.

[21] B. Zheng, K. Wang, *General fuzzy linear systems*, Applied Mathematics and Computation 181 (2006) 1276-1286.

[22] H. J. Zimmermann, *Fuzzy set theory and applications*, Kluwer, Dorrecht 1985.

Tofigh Allahviranloo was born in the west Azarbayejan, khoy. He has got Phd degree in 2001 now he is a professor of applied mathematics. He has published more than 130 papers in international journals and he is editing more than 5 journals as an editor in chief and associated editor.



Alireza Hashemi he has born in Tehran-Iran in 1971. Got B.Sc degree in pure mathematics from University of Tabriz, and M.Sc degree in applied mathematics, numerical analysis field from Iran University of Science  Technology and student PHD degree in applied mathematics, here numerical analysis field in Science and Research Branch, Islamic Azad University. Main research interest include fuzzy system and fuzzy differential equations.