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The Effect of Magnetic Field on 2-D Problem of Generalized Thermoelasticity with Energy Dissipation

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Abstract

In this paper, the propagation of electro-magneto-thermoelastic disturbances produced by thermal shock in a perfectly-conducting half-space is studied based on Green and Naghdi (G-N) theory. Normal mode analysis is used to obtain the exact expressions of temperature, displacement and stresses. Comparisons are made with the results predicted by G-N theory of type II and type III in the presence and absence of the magnetic field.

Keywords : Generalized thermoelasticity; Green-Naghdi theory; Types II; Type III; Normal mode analysis; Magnetic field.

1   Introduction

Much attention has been devoted for generalizing the equations of coupled thermo-elasticity due to Biot [1], mainly because the heat equation of this theory is parabolic, and hence automatically predicts an infinite speed of propagation for heat waves. Clearly, this contradicts physical observations that the maximum wave speed can not exceed that of light in vacuum. Two generalizations about the coupled theory were introduced. The first is due to Lord and Shulman [11], who introduced the theory of generalized thermoelasticity with one relaxation time that is based on a new law of heat conduction to replace the classical Fourier's law.

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This new law contains the heat flux vector as well as its time derivative. It also contains a new constant acting as a relaxation time. The heat equation of this theory is of the wave type, ensuring finite speeds of propagation for heat wave. The remaining governing equations for this theory, namely, the equation of motion and constitutive relations remain the same as those for the coupled and uncoupled theories. The second generalization to the coupled theory of thermoelasticity is what is known as the theory of thermoelasticity with two relaxation times or the theory of temperature-rate-dependent thermoelasticity. [12], in a review of the thermodynamics of thermoelastic solids, proposed an entropy production inequality with the help of which he considered restrictions on a class of constitutive equations.

A generalization about this inequality was proposed by Green and Laws [3]. Green and Lindsay [4] obtained another version of the constitutive equations. These equations were also obtained independently and more explicitly by Suhubi [22]. This theory contains two constants that act as relaxation times and modify all the equations of the coupled theory, not only the heat equation. The Classical Fourier's law of heat conduction is not violated if the medium under consideration has a center of symmetry.

Investigation of the interaction between magnetic field and stress and strain in a thermoelastic solid is very important due to its many applications in the field of geophysics, plasma physics and related topics, especially in the nuclear field, where the extremely high temperature and temperature gradients, as well as the magnetic fields originating inside nuclear reactors, influence their design and operations.The theory of magneto-thermoelasticity is concerned with the influence of the magnetic field on the elastic and thermoelastic deformations of a solid body. This theory has aroused much interest in recent years, because of its application in various branches of science and technology. Othman et al. [13] formulated the magneto-thermoelastic coupled two-dimensional problem of thermally and perfect conducting half-space solid in the presence of moving internal heat source. In the context of the Green and Lindsay generalized thermoelasticity with two relaxation times, the problem of the propagation of electro-magneto-thermoelastic disturbances produced by a thermal shock in a perfectly conducting elastic half-space was discussed by Othman [14]. The development of the interaction of the electro-magnetic field, the thermal field and the elastic field are available in many works [15]-[21]. In the 1990's Green and Naghdi [5]-[10], proposed three new thermoelastic theories based on entropy balance rather than the usual entropy inequality. The constitutive assumptions for the heat flux vector are different in each theory. Thus, they obtained three theories which are called thermoelasticity of type I, of type II and of type III. When the type I theory is linearized we obtain the classical system of thermoelasticity. The type II theory (is limiting case of type III) does not admit energy dissipation. In the recent papers of Chandrasekharaiah and Srinath [2], Othman and Song [17], the theory proposed by Green and Naghdi [6], [7], is considered as an alternative way for the formulation of the propagation of heat. Othman et al. [18], studied the effect of rotation on the generalized magneto-thermo-viscoelastic plane waves without energy dissipation. This theory is developed in a relative way to produce a fully consistent theory, which is able to incorporate thermal pulse transmission in a very logical manner.

In the present paper, we formulate the normal mode analysis of a two-dimensional problem of electro-magneto-thermoelasicity under Green-Naghdi theory in a perfectly conducting medium. The exact expressions for temperature distribution, thermal stress and displacement components are obtained, and represented graphically in the presence and

absence of the magnetic field for the different types of Green-Naghdi theory.

2   Formulation of the problem

We consider the problem of a thermoelastic half-space ($x \geq 0$). A magnetic field with constant intensity $\mathbf{H} = (0,0,H_0)$, acting parallel to the boundary plane (taken as the direction of the z-axis). The surface of the half-space is subjected to a thermal shock which is a function of y and t . Thus, all the quantities considered, will be functions of the time variable t, and of the coordinates x and y. We begin our consideration with linearized equations of slowly moving medium

$$\mathbf{J} = \mathrm{curl}\ \mathrm{h}\ - \varepsilon_0 \dot{\mathbf{E}}, \tag{2.1}$$

$$\mathrm{curl}\ \mathbf{E} = -\ \mu_0 \dot{\mathbf{h}}, \tag{2.2}$$

$$\mathbf{E} = -\ \mu_0\ (\ \dot{\mathbf{u}} \times \mathbf{H}\ ), \tag{2.3}$$

$$\nabla . \mathbf{h} = 0. \tag{2.4}$$

These equations are supplemented by the displacement equations of the theory of elasticity, taking into consideration the Lorentz force

$$\rho\ \ddot{\mathrm{u}}_{\mathrm{i}}\ = \sigma_{\mathrm{ij,j}} + \mu_0(\ \mathbf{J} \times \mathbf{H}\ )_{\mathrm{i}}, \tag{2.5}$$

$$\sigma_{\mathrm{ij}} =\ \ 2\ \mu\ \mathrm{e}_{\mathrm{ij}}\ +\ \lambda\ \mathrm{e}\,\delta_{\mathrm{ij}} -\ \gamma\ (\ \mathrm{T} - \mathrm{T}_0\ )\ \delta_{\mathrm{ij}}, \tag{2.6}$$

Where μ_0 is magnetic permeability, ε_0 is electric permeability and $\dot{\mathbf{u}}$ is the particle velocity of the medium, and the small effect of temperature gradient on $\mathbf{J}$ is also ignored, ρ is the density, σ_{ij} are the components of stress tensor, and λ, μ are Lam's constants, e is the dilatation, T is the temperature above reference temperature T_0 , $\gamma = (\,3\,\lambda + 2\,\mu\,)\,\alpha_{\mathrm{T}}$, in which α_{T} is the coefficient of linear thermal expansion and δ_{ij} is the Kronecker delta. The dynamic displacement vector is actually measured from a steady-state deformed position and the deformation is supposed to be small. Due to the application of the initial magnetic field $\mathbf{H}$, there results an induced magnetic field $\mathbf{h} = (0,0,\mathrm{h})$and an induced electric field $\mathbf{E}$, as well as, the simplified equations of electro-dynamics of a slowly moving medium for a homogeneous, thermal and electrically conducting elastic solid. Strain-displacement constitutive relations are:

$$\mathrm{e}_{\mathrm{xy}} = \frac{1}{2}\ (\ \frac{\partial\,\mathrm{u}}{\partial\,\mathrm{y}} + \frac{\partial\,\mathrm{v}}{\partial\,\mathrm{x}}\ )\ ,\quad \mathrm{e}_{\mathrm{xx}} = \frac{\partial\,\mathrm{u}}{\partial\,\mathrm{x}}\ \ ,\ \ \mathrm{e}_{\mathrm{yy}} = \frac{\partial\,\mathrm{v}}{\partial\,\mathrm{y}}\ ,\quad \mathrm{e}_{\mathrm{xz}} = \mathrm{e}_{\mathrm{yx}} = 0. \tag{2.7}$$

and e, the dilatation, is given by

$$\mathrm{e} = \frac{\partial\,\mathrm{u}}{\partial\,\mathrm{x}} + \frac{\partial\,\mathrm{v}}{\partial\,\mathrm{y}}. \tag{2.8}$$

In the above equations a dot denotes differentiation with respect to time, and a comma followed by a subscript denotes partial differentiation with respect to the corresponding coordinates. The summation notation is used. We shall consider only the simplest case of the two-dimensional problem. We assume that all cases producing the wave propagation are independent of the variable z, and that waves are propagated only in the xy-plane. Thus, all quantities appearing in Eqs (2.1)-(2.8) are independent of the variable z. Thus,

the displacement vector has components$(u(x,y,t), v(x,y,t), 0)$. Substituting for σ_{ij} into Eq. (2.5) and using Eq. (2.8), one obtains

$$\rho\,\ddot{u}_i \;=\; (\lambda + \mu)\,u_{j,ji} + \mu\,u_{i,jj} - \gamma\,T_{,i} + \mu_0(\mathbf{J}\times\mathbf{H})_i, \tag{2.9}$$

Expressing the components of the vector $\mathbf{J} = (J_1, J_2, J_3)$ in terms of displacement, where, $J_1 = H_0(-\frac{\partial e}{\partial y} + \mu_0\varepsilon_0\dot{v})$, $J_3 = 0$, $J_2 = H_0(\frac{\partial e}{\partial x} - \mu_0\varepsilon_0\dot{u})$, by eliminating the quantities $\mathbf{h}$ and E from Eq. (2.1) and introducing them into the displacement Eq. (2.9), we arrive at

$$\rho\,\ddot{u} \;=\; (\lambda + \mu)\frac{\partial e}{\partial x} + \mu\nabla^2 u - \gamma\frac{\partial T}{\partial x} - \mu_0 H_0\frac{\partial h}{\partial x} - \mu_0^2 H_0^2\,\varepsilon_0\ddot{u}, \tag{2.10}$$

$$\rho\,\ddot{v} \;=\; (\lambda + \mu)\frac{\partial e}{\partial y} + \mu\nabla^2 v - \gamma\frac{\partial T}{\partial y} - \mu_0 H_0\frac{\partial h}{\partial y} - \mu_0^2 H_0^2\,\varepsilon_0\ddot{v}, \tag{2.11}$$

we introduce the displacement potentials φ, ψ and by the relations

$$u = \varphi_{,x} + \psi_{,y}, \qquad v = \varphi_{,y} - \psi_{,x}, \tag{2.12}$$

from Eqs. (2.1)-(2.4) and (2.8), we can obtain

$$h = -H_0\nabla^2\varphi. \tag{2.13}$$

The heat conduction equation in the absence of heat sources under G-N III theory is

$$K^*\,T_{,ii} + K\,\dot{T}_{,ii} = \rho\,C_E\ddot{T} + \gamma\,T_o\ddot{u}_{i,i}. \tag{2.14}$$

Where C_E is the specific heat at constant strain, K^* and K are respectively the thermal conductivity and material constant characteristic of the theory. When $K \to 0$, Eq. (2.14) reduces to the heat conduction equation based on G-N theory of type II. Eq. (2.14) together with Eq. (2.5) and (2.6) constitutes the complete system of generalized magneto-thermo-elasticity based on G-N theory of type III. For convenience, the following non-dimensional variables are used:

$$\bar{x}_i = x_i/c_1\omega_1, \quad \bar{u}_i = u_i/c_1\omega_1, \quad \bar{\varphi} = \varphi\big/(c_1\omega_1)^2, \quad \bar{\psi} = \psi\big/(c_1\omega_1)^2,$$
$$\bar{t} = t/\omega_1, \qquad \bar{\sigma}_{ij} = \sigma_{ij}/\mu, \qquad \bar{h} = h/H_0, \qquad \bar{T} = \gamma\,T/\rho\,c_1^2. \tag{2.15}$$

where $c_1^2 = (\lambda + 2\mu)/\rho$, $\omega_1 = K/\rho\,C_E c_1^2$.

Applying Eq. (2.15) on Eqs. (2.6), (2.10), (2.11), (2.13) and (2.14) we get (dropping the bar for convenience).

$$\sigma_{xx} = \beta^2\nabla^2\varphi - 2\varphi_{,yy} + 2\psi_{,xy} - \beta^2\theta, \tag{2.16}$$

$$\sigma_{yy} = \beta^2\nabla^2\varphi - 2\varphi_{,xx} - 2\psi_{,xy} - \beta^2\theta, \tag{2.17}$$

$$\sigma_{xy} = 2\varphi_{,xy} + \psi_{,yy} - \psi_{,xx}, \tag{2.18}$$

$$\beta_0^2\,u_{,xx} + (\beta_0^2 - 1)\,v_{,xy} + u_{,yy} - \beta^2\theta_{,x} = \alpha_0\,u_{,tt}, \tag{2.19}$$

$$\beta_0^2\,v_{,yy} + (\beta_0^2 - 1)\,u_{,xy} + v_{,xx} - \beta^2\theta_{,y} = \alpha_0\,v_{,tt}, \tag{2.20}$$

$$\varepsilon_2\theta_{,ii} + \varepsilon_3\dot{\theta}_{,ii} - \ddot{\theta} = \varepsilon_1\nabla^2\ddot{\phi}, \tag{2.21}$$

$$h = -\nabla^2\varphi. \tag{2.22}$$

ε_1 is usually the thermoelastic coupling factor, ε_2 is the characteristic parameter of the G-N theory (of type II) and ε_3 is the characteristic parameter of the G-N theory (of type III), where

$$\mathrm{a}_0^2 = \mu_0 \mathrm{H}_0^2/\rho, \qquad \mathrm{c}_0^2 = \mathrm{c}_1^2 + \mathrm{a}_0^2, \qquad \mathrm{c}_2^2 = \frac{\mu}{\rho},$$

$$\beta^2 = \mathrm{c}_1^2/\mathrm{c}_2^2, \qquad \beta_0^2 = \mathrm{c}_0^2/\mathrm{c}_2^2, \qquad c^2 = 1/\mu_0 \varepsilon_0,$$

$$\theta = \mathrm{T} + \mathrm{T}_0, \qquad \alpha = 1 + \mathrm{a}_0^2/c^2, \qquad \alpha_0 = \alpha \, \beta^2,$$

$$\varepsilon_1 = \gamma^2 \mathrm{T}_0/\rho^2 \mathrm{c}_1^2 \mathrm{C_E}, \quad \varepsilon_2 = \mathrm{K}^*/\rho \, \mathrm{c}_1^2 \mathrm{C_E}, \quad \varepsilon_3 = \mathrm{K}/\rho \, \mathrm{c}_1^2 \mathrm{C_E} \omega_1$$

Introducing the potential functions defined by Eq. (2.12) in Eqs. (2.19) and (2.20), we obtain

$$(\nabla^2 - \alpha_1 \frac{\partial^2}{\partial \mathrm{t}^2}) \, \varphi - \beta_1^2 \theta = 0, \tag{2.23}$$

$$(\nabla^2 - \alpha_0 \frac{\partial^2}{\partial \mathrm{t}^2}) \, \psi = 0, \tag{2.24}$$

where $\alpha_1 = \alpha/\beta_0^2, \; \beta_1^2 = \beta^2/\beta_0^2.$

3 Normal Mode Analysis

The solution of the considered physical variable can be decomposed in terms of normal modes as follows:

$$\left[\theta, \varphi, \psi, \sigma_{\mathrm{ij}} \right] \, (\mathrm{x}, \mathrm{y}, \mathrm{t}) = \left[\theta^*, \varphi^*, \psi^*, \sigma_{\mathrm{ij}}^* \right] \, (\mathrm{x}) \, \exp(\omega \, \mathrm{t} + \mathrm{i} \, \mathrm{a} \, \mathrm{y}), \tag{3.25}$$

where ω is the (complex) time constant, and a is the wave number in the y-direction. By using Eq. (3.25), we can obtain the following equations from Eqs. (2.23), (2.24) and (2.21) respectively.

$$\left[\mathrm{D}^2 - \mathrm{a}^2 - \alpha_1 \, \omega^2 \right] \, \phi^*(\mathrm{x}) - \beta_1^2 \, \theta^*(\mathrm{x}) = 0, \tag{3.26}$$

$$\left[\mathrm{D}^2 - \mathrm{a}^2 - \alpha_0 \, \omega^2 \right] \, \psi^*(\mathrm{x}) = 0, \tag{3.27}$$

$$\left[\varepsilon_2(\mathrm{D}^2 - \mathrm{a}^2) + \varepsilon_3 \omega \, (\mathrm{D}^2 - \mathrm{a}^2) - \omega^2 \right] \theta^*(\mathrm{x}) = \varepsilon_1 \omega^2 \, (\mathrm{D}^2 - \mathrm{a}^2) \, \phi^*(\mathrm{x}). \tag{3.28}$$

Here, $\mathrm{D} = \mathrm{d}/\mathrm{dx}$. On decomposing Eqs. (3.26) and (3.28), we obtain

$$\left[\mathrm{D}^4 - \mathrm{A} \, \mathrm{D}^2 + \mathrm{B} \right] \, [\phi^*(\mathrm{x}), \theta^*(\mathrm{x})] = 0. \tag{3.29}$$

Where A and B are defined in Appendex I.
Equation (3.29) can be factorized as

$$(\mathrm{D}^2 - \mathrm{k}_1^2) \, (\mathrm{D}^2 - \mathrm{k}_2^2) \, [\phi^*(\mathrm{x}), \theta^*(\mathrm{x})] = 0. \tag{3.30}$$

where

$$\mathrm{k}_{1,2}^2 = \frac{1}{2}(\mathrm{A} \pm \sqrt{ \mathrm{A}^2 - 4\mathrm{B} }), \tag{3.31}$$

are the roots of the following characteristic equation

$$\mathrm{k}^4 - \mathrm{A} \, \mathrm{k}^2 + \mathrm{B} = 0. \tag{3.32}$$

The solution of (3.30) which are bounded for x > 0, are given by:

$$\phi^*(x) = \sum_{j=1}^{2} R_j e^{-k_j x}, \tag{3.33}$$

$$\theta^*(x) = \sum_{j=1}^{2} R_j' e^{-k_j x}, \tag{3.34}$$

where R_j and R_j' are some parameters depending on a and ω. Substituting Eqs. (3.33) and (3.34) into Eq. (3.26), we obtain the following relation

$$R_j' = [(k_j^2 - a^2 - \alpha_1 \omega^2) / \beta_1^2] R_j, \qquad j = 1, 2, \tag{3.35}$$

then, Eq. (3.34) becomes

$$\theta^*(x) = \sum_{j=1}^{2} [(k_j^2 - a^2 - \alpha_1 \omega^2) / \beta_1^2] R_j e^{-k_j x}, \tag{3.36}$$

The solution of Eq. (3.27), bounded as $x \to \infty$, is given by

$$\psi^*(x) = C\, e^{-m x}, \tag{3.37}$$

where $C(a, \omega)$ is some parameter depending on a and ω, and

$$m = \sqrt{a^2 + \alpha_0 \omega^2}. \tag{3.38}$$

Normal mode analysis of the stresses yields the following:

$$\sigma_{xx}^*(x) = \sum_{j=1}^{2} \{ \beta^2 (k_j^2 - a^2) + 2a^2 - [\beta^2 (k_j^2 - a^2 - \alpha_1 \omega^2)/\beta_1^2] \} R_j\, e^{-k_j x} - 2\, i\, a\, mC\, e^{-m x}, \tag{3.39}$$

$$\sigma_{yy}^*(x) = \sum_{j=1}^{2} \{ \beta^2 (k_j^2 - a^2) - 2 k_j^2 - [\beta^2 (k_j^2 - a^2 - \alpha_1 \omega^2)/\beta_1^2] \} R_j e^{-k_j x} + 2\, i\, a\, m\, C e^{-m x}, \tag{3.40}$$

$$\sigma_{xy}^*(x) = -\sum_{j=1}^{2} 2\, i\, a\, k_j R_j\, e^{-k_j x} - (a^2 + m^2)\, C e^{-m x}. \tag{3.41}$$

In order to determine the parameters $R_j (j = 1, 2)$ and C, we need to consider the boundary conditions at x = 0 as follows: Thermal boundary condition: The surface of the half-space is subjected to a thermal shock:

$$\theta(0, y, t) = n(y, t). \tag{3.42}$$

Mechanical boundary condition: The surface of the half-space is traction free

$$\sigma_{xx}(0, y, t) = \sigma_{xy}(0, y, t) = 0. \tag{3.43}$$

Using Eq. (3.25) and substituting the expressions of considered variables into the above boundary conditions, we can obtain the following equations satisfied by the parameters :

$$\sum_{j=1}^{2} [(k_j^2 - a^2 - \alpha_1 \omega^2)/\beta_1^2] \, R_j = n^*, \tag{3.44}$$

$$\sum_{j=1}^{2} \{ \beta^2 (k_j^2 - a^2) + 2a^2 - [\beta^2 (k_j^2 - a^2 - \alpha_1 \omega^2)/\beta_1^2] \} \, R_j \, e^{-k_j x} - 2 \, i \, a \, mC \, e^{-mx} = 0, \tag{3.45}$$

$$\sum_{j=1}^{2} 2 \, i \, a \, k_j R_j + (a^2 + m^2) \, C = 0. \tag{3.46}$$

By solving Eqs. (3.44)-(3.46) we get the parameters R_j($j = 1, 2$) and C as defined in Appendix II.

4   Numerical results

Aiming to illustrate the problem, we will present some numerical results. The material chosen for the purpose of numerical computation is copper, the physical data for which in SI units is given by:

$T_0 = 293$ K, $\rho = 8954$ kg/m^3, $C_E = 383.1$ J/(kg.K), $\alpha_T = 1.78 \times 10^{-5}$K^{-1},

$\varepsilon_0 = 0.3$,   K $= 386$ W/(m.K), $\lambda = 7.76 \times 10^{11}$kg/(m.s^2), $\mu = 3.86 \times 10^{11}$kg/(m.s^2),

$\mu_0 = 1.7$

Since we have $\omega = \omega_0 + i \zeta$, the real parts of the functions θ (x, y, t),u (x, y, t), σ_{xx} (x, y, t) and σ_{yy} (x, y, t) are calculated by numerical techniques under different conditions on the plane y = 0 at t = 0.1, where

$$L = 2, \quad \omega_0 = 2, \quad \zeta = 1, \quad n^* = 1, \quad a = 1.2$$

The results are shown in Figs. 1(a, b, c, d)-4(a, b,c, d). Figures 1(a, b, c, d) depict the influence of the magnetic field on the temperature θ, the horizontal component of displacement u and the stress components σ_{xx},σ_{yy} based on G-N theory of type II. Here, $\varepsilon_1 = 0.0168$, $\varepsilon_2 = 0.4$,$\varepsilon_3 = 0$, when $\alpha = 1$,(i.e. $H_0 = 0$) has been shown by solid line, $\alpha = 1.4$,(i.e. $H_0 = 60$) as shown by dashed and dot line, $\alpha = 1.8$, (i.e. $H_0 = 90$) as shown by dot line. We see from Fig. 1(a) that the magnetic field has increasing effect on the temperature for x > 1.6 and converges to zero with increasing the distance x for x > 1.6. Fig. 1(b) shows that the magnetic field has decreasing effect on the horizontal component of displacement for x < 1.3 and converges to zero with increasing the distance x for x > 1.3. Fig. 1(c) depicts that the magnetic field has decreasing effect on the stress component σ_{xx} for x > 1.7 and converges to zero with increasing the distance x for x > 1.7. It is evident from Fig.1(d) that the magnetic field has decreasing effect on the stress component σ_{yy} for x < 1.6 and converges to zero with increasing the distance x for x > 1.6. Clearly, the magnetic field plays an important role in the field quantities.
Figures 2(a, b, c, d) show the influence of the magnetic field on the temperature θ, the

horizontal component of displacement u and the stress components σ_{xx}, σ_{yy} based on G-N theory of type III. Here, $\varepsilon_2 = 0.4, \varepsilon_3 = 0.6$, when $\alpha = 1$, has been shown by solid line, dashed and dot line when $\alpha = 1.4$ and dot line when $\alpha = 1.8$. We see from Fig. 2(a) that the magnetic field has increasing effect on the temperature for x < 2 and converges to zero with increasing the distance x for x > 2. Fig. 2(b)-2(d) show that the magnetic field has decreasing effect on the horizontal component of displacement and the stress components σ_{xx}, σ_{yy} for $0 < x < 2$ and converges to zero with increasing the distance x for x > 2. The variations of the temperature θ, the horizontal component of displacement u and the stress components σ_{xx}, σ_{yy} with x in the presence of magnetic field for different values of ε_2 are shown in Figs. 3(a, b, c, d). Here $\alpha = 1.4, \varepsilon_1 = 0.0168$, $\varepsilon_2 = 0.2$, 0.4, 0.6 and $\varepsilon_3 = 0$. Fig. 3(a) shows that the temperature decreases with increase ε_2 for $0 < x < 0.8$ but for $0.8 < x < 1.7$ the temperature increases with increase ε_2 and converges to zero with increasing the distance x for x > 1.7. Fig. 3(b) depicts that the horizontal component of displacement u increases with increasing ε_2 for $0 < x < 0.4$ but decreases for $0.4 < x < 1.5$ and converges to zero with increasing the distance x for x > 1.5. Fig. 3(c) shows that the stress component σ_{xx}, increases with increasing ε_2 for $0 < x < 0.9$ and decreases for $0.9 < x < 1.9$ and converges to zero with increasing the distance x for x > 1.9. Fig. 3(d) shows that the stress component σ_{yy}, increases with increasing ε_2 for $0 < x < 0.8$ and decreasing effect for $0.8 < x < 1.7$ and converges to zero with increasing the distance x for x > 1.7.
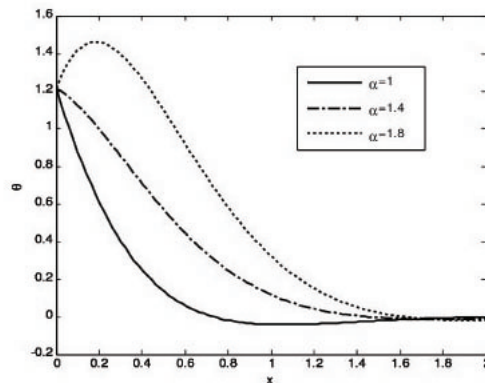
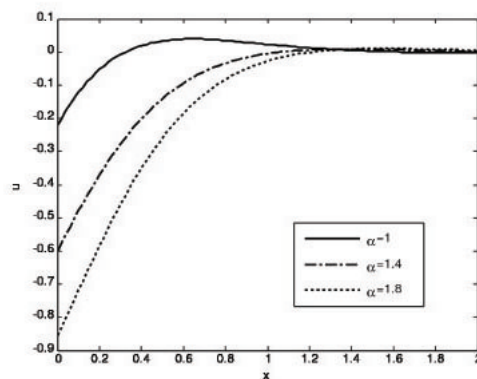Fig. 1. (a) Temperature distribution for $y = 0$, $\varepsilon_2 = 0.4$ and $\varepsilon_3 = 0$

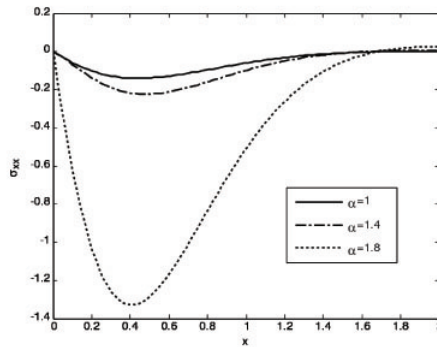Fig. 1. (b) Displacement distribution for $y = 0$, $\varepsilon_2 = 0.4$ and $\varepsilon_3 = 0$

Fig. 1. (c) Stress distribution σ_{xx} for $y = 0$, $\varepsilon_2 = 0.4$ and $\varepsilon_3 = 0$
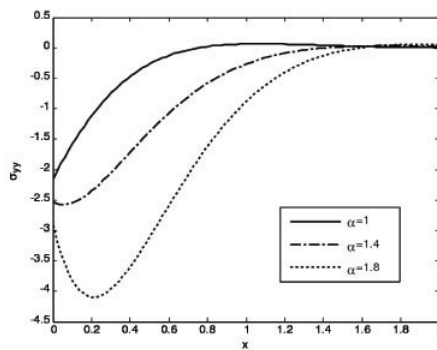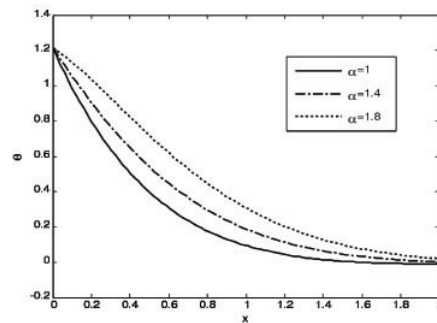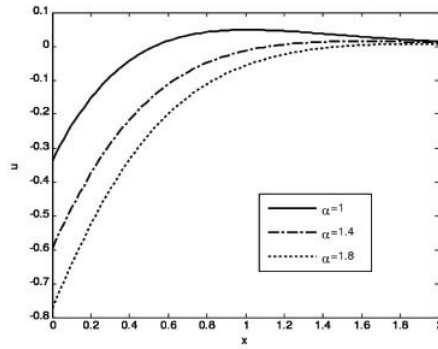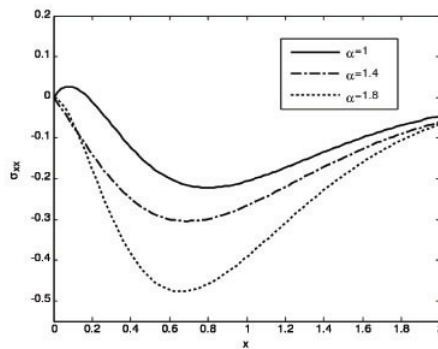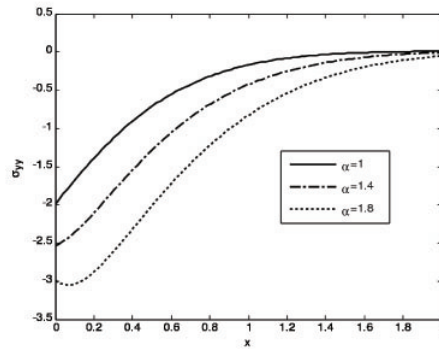

Fig. 1. (d) Stress distribution σ_{yy} for $y = 0$, $\varepsilon_2 = 0.4$ and $\varepsilon_3 = 0$

Figs. 4(a, b, c, d) give the variations of the temperature, the horizontal component of displacement and the stress components with x for different values of frequencyω_0. Here, we have $\alpha = 1.8, \varepsilon_1 = 0.0168, \varepsilon_2 = 0.4, \varepsilon_3 = 0$(G-N II theory) and $\varepsilon_1 = 0.0168$, $\varepsilon_2 = 0.4, \varepsilon_3 = 0.6$ (G-N III theory). We observed from Fig. 4(a) that the frequency has increasing effect on the temperature for type II and type III when $0 < x < 1.4$ and in type II is greater than that in type III for $0 < x < 1$. However, it converges to zero with increasing the distance x. Fig. 4(b) shows that the horizontal component of displacement decreases with increasing ω_0 for $0 < x < 1.6$, and converges to zero with increasing the distance x for $x > 1.6$. Figs. 4(c), 4(d) show that the stress components σ_{xx}, σ_{yy} decrease as ω_0 increases and type III is greater than type II for $0 < x < 1.1$, while type II is greater than type III for $x > 1.1$, then converges to zero with increasing the distance x.

Fig. 2. (a) Temperature distribution θ for $y = 0$, $\varepsilon_2 = 0.4$ and $\varepsilon_3 = 0.6$

Fig. 2. (b) Displacement distribution u for $y = 0$ $\varepsilon_2 = 0.4$ and $\varepsilon_3 = 0.6$

Fig. 2. (c) Stress distribution σ_{xx} for $y = 0$, $\varepsilon_2 = 0.4$ and $\varepsilon_3 = 0.6$

Fig. 2. (d) Stress distribution σ_{yy} for $y = 0$, $\varepsilon_2 = 0.4$ and $\varepsilon_3 = 0.6$
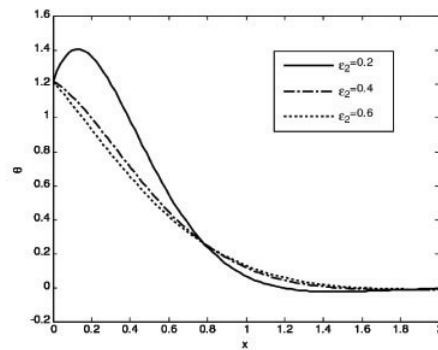
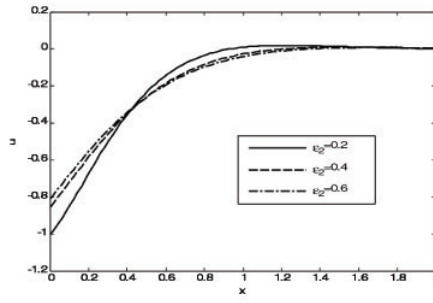Fig. 3. (a) Temperature distribution θ for $y = 0$, $\alpha = 1.4$ and $\varepsilon_3 = 0$

Fig. 3. (b) Displacement distribution u for $y = 0$, $\alpha = 1.4$ and $\varepsilon_3 = 0$
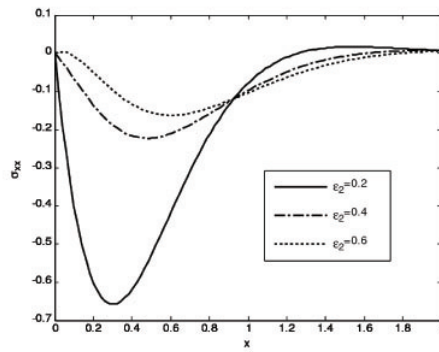
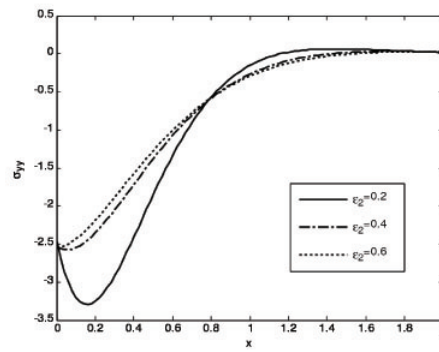Fig. 3. (c) Stress distribution σ_{xx} for $y = 0$, $\alpha = 1.4$ and $\varepsilon_3 = 0$

Fig. 3. (d) Stress distribution σ_{yy} for $y = 0$, $\alpha = 1.4$ and $\varepsilon_3 = 0$
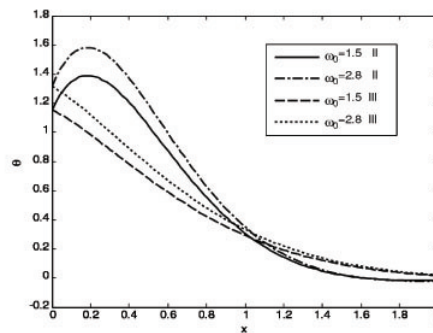

Fig. 4. (a) Temperature distribution θ for $y = 0$, $\alpha = 1.8$, $\varepsilon_2 = 0.4$ and $\varepsilon_3 = 0$
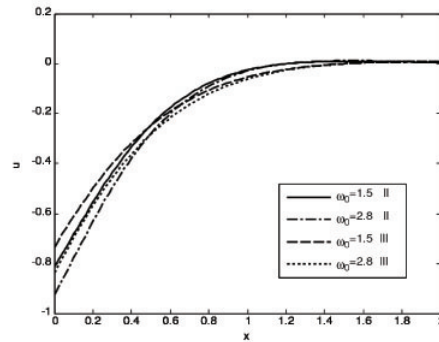
Fig. 4. (b) Displacement distribution u for $y = 0$, $\alpha = 1.8$, $\varepsilon_2 = 0.4$, $\varepsilon_3 = 0$ and $\varepsilon_2 = 0.4$, $\varepsilon_3 = 0.6$
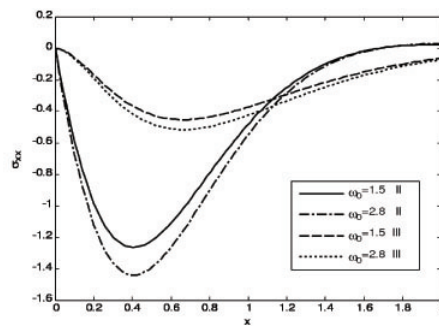

Fig. 4. (c) Stress distribution σ_{xx} for $y = 0$, $\alpha = 1.8$, $\varepsilon_2 = 0.4$, $\varepsilon_3 = 0$ and $\varepsilon_2 = 0.4$, $\varepsilon_3 = 0.6$
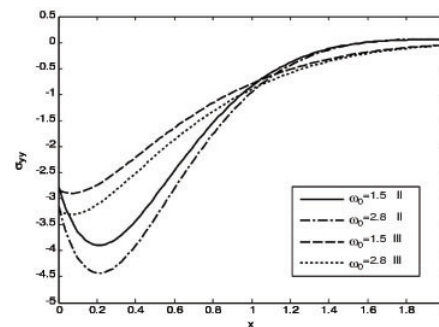

Fig. 4. (d) Stress distribution σ_{yy} for $y = 0$, $\alpha = 1.8$, $\varepsilon_2 = 0.4$, $\varepsilon_3 = 0$ and $\varepsilon_2 = 0.4$, $\varepsilon_3 = 0.6$

5 Concluding remarks

In this paper normal mode method used to study the problem of the effect of magnetic field on a two-dimensional problem of a generalized thermoelastic half-space based on Green-Naghdi theory (of both type II and III). The following conclusions have been obtained according to the analysis above:

1. The electro-magneto-thermoelasticity that coupled two-dimensional problem of a perfect
    conductivity half-space solid can be described by a fourth-order characteristic equation.

2. The magnetic field plays a dual role in the distribution of the field quantities.

3. The magnetic field has great influence on field quantities and this influence produces the same result with respect to G-N theory (of both type II and III).

4. ε_2 (of type II) plays an important role in the distribution of the field quantities.

5. The effect of frequency is prodigious on the distribution of the field quantities based on
   G-N theory (of both type II and III).

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Appendix I

$$a_1 = 2\,a^2 + \alpha_1\omega^2, a_2 = a^4 + \alpha_1 a^2\omega^2, a_3 = \varepsilon_2 + \omega\,\varepsilon_3, a_4 = a_1\,a_3 + \omega^2 + \varepsilon_1\beta_1^2\omega^2,$$

$$a_5 = a_2\,a_3 + \omega^2(a^2 + \alpha_1\omega^2) + a^2\varepsilon_1\beta_1^2\omega^2, A = a_4/a_3, B = a_5/a_3.$$

Appendix II

$$R_1 = n^*\left[M_2(a^2 + m^2) + 2\,i\,a\,m\,L_2\right]\Big/\left[(a^2 + m^2)(N_1M_2 - N_2M_1) - 2\,i\,a\,m\,(N_2L_1 - N_1L_2)\right],$$

$$R_2 = -n^*\left[M_1(a^2 + m^2) + 2\,i\,a\,m\,L_1\right]\Big/\left[(a^2 + m^2)(N_1M_2 - N_2M_1) - 2\,i\,a\,m\,(N_2L_1 - N_1L_2)\right],$$

$$C = n^*\left[M_1L_2 - M_2L_1\right]\Big/\left[(a^2 + m^2)(N_1M_2 - N_2M_1) - 2\,i\,a\,m\,(N_2L_1 - N_1L_2)\right],$$

$$M_j = \beta^2(k_j^2 - a^2) + 2a^2 - [\beta^2(k_j^2 - a^2 - \alpha_1\omega^2)/\beta_1^2], \quad j = 1, 2,$$

$$L_j = 2\,i\,a\,k_j, N_j = (k_j^2 - a^2 - \alpha_1\omega^2)/\beta_1^2, \quad j = 1, 2.$$