

# Failure Mode and Effects Analysis Using Data Envelopment Analysis

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## Abstract

In this paper, we propose a bounded DEA based model to measure the overall risk of failure modes. In the proposed model risk is measured within the range of an interval, whose performance is definitely superior to any one. The risks, obtained from bounded DEA models, turn out to be all intervals and are referred to as interval risk, which combine the best and the worst relative risk in a reasonable manner to give an overall assessment of performances for all failure modes. Assessor's preference information on input and output weights is also incorporated into bounded DEA models reasonably and conveniently. A practical example is provided to compare the proposed model with those in the literature.

*Keywords* : Failure mode and effect analysis (FMEA); Data envelopment analysis (DEA); Risk priority number.

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## 1 Introduction

Failure mode and effects analysis (FMEA) has proven to be a useful and powerful tool in assessing potential failures and preventing them from occurring. According to definition of Chrysler [7] failure mode and effect analysis can be described as a set of purposeful activities to identify and evaluate potential failures in productions, processes and their effects. Failure means inability to fulfill to desired process or necessity function that results in a low quality a bind of problem or service as perceived as a reason of dissatisfaction by the customer. FMEA is a prevention methodology that have the capacity to with engineering and permanent method. This method is very significant in showing potential failures in production, process and provides effective management for

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risk factors [12]. FMEA was first proposed by NASA in 1963 for their obvious reliability requirements [15]. Since then, it has been extensively used as a powerful technique for system safety and reliability analysis of products and processes in wide range of industries—particularly aerospace, nuclear, automotive and medical [8, 9]. This technique is yet another powerful tool used by system safety and reliability engineers/analysts to identify critical parts, functions and components whose failure will lead to undesirable outcomes such as production loss, injury or even an accident. The main objective of FMEA is to discover and prioritize the potential failure modes by computing risk priority numbers (RPNs), which is a product of the risk factors occurrence (O), severity (S) and detection (D) [18]. Occurrence and severity are the frequency and seriousness (effects) of the failure, and detection is the ability to detect the failure before it reaches the customer. The three risk factors are evaluated using the ratings (also called ranks or scores) from 1 to 10. Generally, the higher RPN of a failure mode, the more important degree it should be assigned. With respect to the scores of RPNs, the failure modes can be ranked and then proper actions will be preferentially taken on the high-risk failure modes. In fact, different combinations of O, S and D may produce exactly the same value of RPN, but their hidden risk implications may be totally different. For example, two different events with the values of 2, 3, 2 and 4, 1, 3 for O, S and D, respectively, have the same RPN value of 12. However, the hidden risk implications of the two events may not necessarily be the same. This may cause a waste of resources and time, and in some cases a high risk event may go unnoticed. The relative importance among O, S and D is not taken into consideration. The three risk factors are assumed to be equally important. This may not be the case when considering a practical application of FMEA. To overcome the drawbacks listed above, a number of approaches have been suggested in the literature. For Example, Bevilacqua et al. [2] defined RPN as the weighted sum of six parameters (safety, machine importance for the process, maintenance costs, failure frequency, downtime length, and operating conditions) multiplied by a seventh factor (machine access difficulty), where the relative importance of the six attributes was estimated using pairwise comparisons. Braglia et al. [3] presented the method of technique for order preference by similarity to ideal solution (TOPSIS) for prioritizing failure modes. In the method, fuzzy logic theory was used to evaluate O, S, D and their relative importance weights. In [4], the authors utilized the grey theory for FMEA, but the grey relational degrees were computed using the traditional scores 1–10 for the three risk factors rather than fuzzy linguistic terms. Bowles and Pelez [13] described a fuzzy logic based approach for prioritizing failures in a system FMEA, which uses linguistic terms to describe O, S, D, and the risks of failures. The relationships between the risks and O, S, D were characterized by fuzzy if–then rules extracted from expert knowledge and expertise. Crisp rankings for O, S, D were fuzzified to match the premise of each possible if–then rule. All the rules that have any truth in their premises were fired to contribute to a fuzzy conclusion. The fuzzy conclusion was then defuzzified by the weighted mean of maximum method (WMoM) as the ranking value of the risk priority. Yang et al. [22] presented an efficient fuzzy rule–based Bayesian reasoning (FuRBaR) approach for prioritizing failures in FMEA. The technique was specifically developed to deal with some of the drawbacks concerning the use of conventional fuzzy logic (i.e. rule–based) methods in FMEA. In their approach, subjective belief degrees were assigned to the consequent part of the rules to model the incompleteness encountered in establishing the knowledge base. A Bayesian reasoning mechanism was then used to aggregate all relevant rules for assessing and prioritizing potential failure modes. The applicability of

the proposed approach was demonstrated by studying a maritime collision risk due to technical failures. Sharma et al. [17] used a fuzzy rule-based inference method and the grey theory for prioritizing failure modes. Fuzzy linguistic terms are used to represent the risk degree for O, S, D and RPNs in the fuzzy rule base. Chin et al. [14] proposed an FMEA using the group-based evidential reasoning (ER) approach to capture FMEA team members' diversity opinions and prioritize failure modes under different types of uncertainties such as incomplete assessment, ignorance and intervals. The risk priority model was developed using the group-based ER approach, which includes assessing risk factors using belief structures, synthesizing individual belief structures into group belief structures, aggregating the group belief structures into overall belief structures, converting the overall belief structures into expected risk scores, and ranking the expected risk scores using the minimax regret approach (MRA). Wang et al. [21] proposed a definition for the fuzzy RPNs using fuzzy weighted geometric means (FWGM). The fuzzy RPNs can be calculated by using  $\alpha$ -level sets and a linear programming model and defuzzified by the centroid defuzzification method for the final ranking of the failure modes. In the method, different combinations of O, S and D can produce different fuzzy RPNs only when assigning different importance weights to O, S and D. In spite of the fact that much effort has been paid to the improvement of RPN, the improved methods either need to specify or determine the weights of risk factors in advance or take no account of them at all. It is argued that the specification or determination of risk factor weights is not easy because different decision makers (DMs) may have distinct judgments or preferences. Different failure modes have different consequences. The specification or determination of a fixed set of risk factor weights for all the failure modes might be inappropriate, particularly in the case with a large number of failure modes. In other words, it might be a better choice to use different sets of risk factor weights for different failure modes when there are a large number of failure modes to be prioritized. In this aspect, Garcia et al. [11] proposed a fuzzy data envelopment analysis (DEA) approach for FMEA, which does not require specifying or determining risk factor weights subjectively. Their approach, however, was computationally very complicated and also could not produce a full ranking for the failure modes to be prioritized. In this study we present an integrated model based on a new DEA model and Chin's approach [6] to prioritize the risk factors. It is shown that the proposed model has better discriminating power than the traditional DEA efficiency and Chin's model [6].

The rest of the paper is organized as follows. In the following section, we review DEA models for FMEA. In section 3, we illustrate our proposed method. A numerical examples is provided in section 4 to demonstrate the potential applications of the proposed FMEA and its advantages. Conclusions appear in section 5.

## 2 DEA models for FMEA

In this section we give a brief description of DEA and the geometric average methodology. Suppose we have a set of  $n$  peer DMUs,  $\{\text{DMU}_j : j = 1, 2, \dots, n\}$ , which produce multiple outputs  $y_{rj}$  ( $r = 1, 2, \dots, s$ ), by utilizing multiple inputs  $x_{ij}$  ( $i = 1, 2, \dots, m$ ). Let the inputs and outputs for  $\text{DMU}_j$  be  $X_j = (x_{1j}, x_{2j}, \dots, x_{mj})^t$  and  $Y_j = (y_{1j}, y_{2j}, \dots, y_{sj})^t$ ,

respectively. The efficiency of the  $DMU_o (o = 1, 2, \dots, n)$  is measured as follow:

$$\theta = \frac{\sum_{r=1}^s \mu_r y_{ro}}{\sum_{i=1}^m v_i x_{io}}, \tag{2.1}$$

where  $\mu_r$  is the output weight and  $v_i$  is the input weight. The optimistic efficiency and pessimistic efficiency of  $DMU_o$  is measured by the following DEA models, respectively:

$$\begin{aligned} \max \quad & \theta_o = \frac{\sum_{r=1}^s \mu_r y_{ro}}{\sum_{i=1}^m v_i x_{io}} \\ \text{s.t.} \quad & \theta_j = \frac{\sum_{r=1}^s \mu_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \leq 1 \quad j = 1, \dots, n \\ & \mu_r, v_i \geq \varepsilon \quad r = 1, \dots, s, \quad i = 1, \dots, m \end{aligned} \tag{2.2}$$

$$\begin{aligned} \min \quad & \varphi_o = \frac{\sum_{r=1}^s \mu_r y_{ro}}{\sum_{i=1}^m v_i x_{io}} \\ \text{s.t.} \quad & \varphi_j = \frac{\sum_{r=1}^s \mu_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \geq 1 \quad j = 1, \dots, n \\ & \mu_r, v_i \geq \varepsilon \quad r = 1, \dots, s, \quad i = 1, \dots, m \end{aligned} \tag{2.3}$$

It is a common knowledge that optimistic efficiency and pessimistic efficiency should form an interval when measured under the same constraints such as  $\alpha \leq \sum_{r=1}^n u_r y_{rj} \leq 1, (j = 1, \dots, n)$  with  $0 < \alpha < \min \{ \theta_j^* / \varphi_j^* \}, j = 1, \dots, n$ . The efficiency interval of  $DMU_j$  could accordingly be expressed as  $[\alpha \varphi_j^*, \theta_j^*]$  if the value of  $\alpha$  is small enough. To avoid the difficulty in determining the value of  $\alpha$ , Wang et al. [21] suggested a geometric average efficiency, determined by

$$\phi_j^* = \sqrt{\alpha \varphi_j^* \theta_j^*} \quad j = 1, \dots, n, \tag{2.4}$$

where  $\theta_j^*$  and  $\varphi_j^*$  are respectively the optimistic and pessimistic efficiencies of  $DMU_j (j = 1, \dots, n)$ . The geometric average efficiency considers not only the optimistic efficiency of a

DMU, but also its pessimistic efficiency. It measures the overall efficiency of a DMU and considers both sides of a coin. The integration of two extreme efficiencies, optimistic and pessimistic, into a geometric average efficiency is undoubtedly more meaningful and more comprehensive than the use of either of the two efficiencies.

When efficiency intervals  $[\alpha\varphi_j^*, \theta_j^*]$ ,  $j = 1, \dots, n$ , are compared through their geometric midpoints  $\sqrt{\varphi_j^*\theta_j^*}$ , the rankings among the  $n$  DMUs depend only upon their geometric average efficiencies  $\phi_j^* = \sqrt{\varphi_j^*\theta_j^*}$ ,  $j = 1, \dots, n$ , and have nothing to do with the value of  $\alpha$ . This good property enables the decision maker not to worry about how to determine the value of  $\alpha$ . He/She can therefore leave it alone and compare directly the geometric average efficiencies of the  $n$  DMUs to determine their overall performances and rankings [6].

Suppose there are  $n$  failure modes denoted by  $FM_i (i = 1, \dots, n)$  to be prioritized, each being evaluated against  $m$  risk factors denoted by  $RF_j (j = 1, \dots, m)$ . Let  $r_{ij}$ , ( $i = 1, \dots, n; j = 1, \dots, m$ ) be the ratings of  $FM_i$  on  $RF_j$  and  $w_j$  be the weight of risk factor  $RF_j$ , ( $j = 1, \dots, m$ ). Since the RPN defined as the product of three risk factors O, S and D has been largely criticized for its mathematical formula and equal treatment of the risk factors, we define in this paper the risks of failures with a different mathematical form, which can be either of the following:

$$R_i = \sum_{j=1}^m w_j r_{ij}, \quad i = 1, \dots, n, \tag{2.5}$$

$$R_i = \prod_{j=1}^m r_{ij}^{w_j}, \quad i = 1, \dots, n, \tag{2.6}$$

Eq.(2.5) defines the risk of each failure mode as the weighted sum of  $m$  risk factors, whereas Eq.(2.6) as the weighted product of  $m$  risk factors. For convenience to distinguish between the two risks, we refer to the risk determined by Eq.(2.5) as additive risk and the risk by Eq.(2.6) as multiplicative risk, respectively. It is worthwhile to point out that the definition for additive risks was first proposed by Braglia et al. [1], who defined the RPN as the weighted sum of O, S and D, whereas the definition for multiplicative risks was first proposed by Wang et al [21], who defined the RPN as the fuzzy weighted geometric mean of the three risk factors O, S, and D, which they referred to as fuzzy risk priority number (FRPN).

The traditional DEA often assigns too many zeros to input and output weights, leading to optimistic efficiency being unreasonably high and pessimistic efficiency being extraordinarily low. To avoid this from happening in FMEA, we consider imposing a constraint on the ratio of maximum weight to minimum weight. According to Saaty's AHP [19] method, the maximum value, as a ratio of the comparative importance of a criterion over another, can assume to be 9. We therefore constrain the ratio of maximum weight to minimum weight within the range of one and nine. That is,

$$1 \leq \frac{\max(w_1, \dots, w_m)}{\min(w_1, \dots, w_m)} \leq 9 \tag{2.7}$$

The main reasons for us to set the maximum ratio as 9 are based on the following observations:

• The pairwise comparison matrices in the AHP are the most widely used approaches for estimating the relative importance weights of decision attributes or criteria, in which the maximum ratio scale between the importance of two attributes or criteria are usually not greater than 9.

• Risk factors O, S and D are all evaluated using the ratings between 1 and 10, where 1 represents no risk. Accordingly, their relative importance should also be evaluated using similar ratings. Due to the fact that no importance makes no sense, the ratings used for evaluating the relative importance of risk factors should therefore be defined as 1–9 rather than 1–10. As a result, the maximum ratio between the importance of two risk factors is less than or equal to 9.

The left-hand-side of Eq.(2.7) is trivial and holds always. Its right hand side is equivalent to the following:

$$\max \left\{ \frac{w_j}{w_k}, j, k = 1, \dots, m; j \neq k \right\} \quad (2.8)$$

which can be further rewritten as

$$w_j - 9w_k \leq 0, j, k = 1, \dots, m; j \neq k \quad (2.9)$$

According to the DEA models introduced, we know that optimistic efficiency and pessimistic efficiency of  $DMU_o$  is measured by (2.10) and (2.11):

$$\begin{aligned} \max \theta_o &= \frac{\sum_{r=1}^s u_r y_{r0}}{\sum_{i=1}^m v_i x_{i0}} \\ s.t. \theta_j &= \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \leq 1, j = 1, \dots, n, \\ u_r, v_i &\geq \varepsilon, r = 1, \dots, s; i = 1, \dots, m \end{aligned} \quad (2.10)$$

$$\begin{aligned} \min \varphi_o &= \frac{\sum_{r=1}^s u_r y_{r0}}{\sum_{i=1}^m v_i x_{i0}} \\ s.t. \varphi_j &= \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \geq 1, j = 1, \dots, n, \\ u_r, v_i &\geq \varepsilon, r = 1, \dots, s; i = 1, \dots, m \end{aligned} \quad (2.11)$$

According to above models , FMEA models built for measuring the maximum and minimum risks of each failure mode, as shown below:

$$R_o^{\max} = \max R_o$$

$$s.t. \begin{cases} R_i \leq 1, & i = 1, \dots, n, \\ w_j - 9w_k \leq 0, & j, k = 1, \dots, m; k \neq j \end{cases} \quad (2.12)$$

$$R_o^{\min} = \min R_o$$

$$s.t. \begin{cases} R_i \geq 1, & i = 1, \dots, n, \\ w_j - 9w_k \leq 0, & j, k = 1, \dots, m; k \neq j \end{cases} \quad (2.13)$$

where  $R_o$  is the risk of the failure mode under evaluation. The overall risk of each failure mode is defined by Eq.(2.4) as the geometric average of the maximum and minimum risks of the failure mode. That is,

$$\bar{R}_i = \sqrt{R_i^{\max}.R_i^{\min}}, \quad i = 1, \dots, n \quad (2.14)$$

Therefor,  $n$  failure modes  $FM_i$  ( $i = 1, \dots, n$ ) can be easily prioritized by their geometric average risks  $R_i$  ( $i = 1, \dots, n$ ). The above models (2.12) and (2.13) are developed for additive risks. For multiplicative risks defined by Eq.(2.6), the maximum and minimum risk models can be built in the same way, but the ratings and risks need to be transformed into logarithmic scales for linearity. The two models are constructed as follows:

$$\ln R_o^{\max} = \max \ln R_o$$

$$s.t. \begin{cases} \ln R_i \leq 1, & i = 1, \dots, n, \\ w_j - 9w_k \leq 0, & j, k = 1, \dots, m; k \neq j \end{cases} \quad (2.15)$$

$$\ln R_o^{\min} = \min \ln R_o$$

$$s.t. \begin{cases} \ln R_i \geq 1, & i = 1, \dots, n, \\ w_j - 9w_k \leq 0, & j, k = 1, \dots, m; k \neq j \end{cases} \quad (2.16)$$

Accordingly, the geometric average risk is defined as

$$\bar{R}_i = \sqrt{\exp(\ln R_i^{\max}). \exp(\ln R_i^{\min})}, \quad i = 1, \dots, n. \quad (2.17)$$

where  $EXP(.)$  is the exponential function.

### 3 Proposed model

In order to apply DEA method in determining  $\alpha$  value for FMEA risk methods, first we should define ideal failure item and anti-ideal one. It is known that in FMEA, failure item has the first priority an high risk. In other word it has the highest degree for risk factors.

**Definition 1.** Anti-ideal failure item is a virtual item that has the lowest degree among risk factors.

**Definition 2.** Ideal failure item is a virtual item that has the highest degree among risk

factors.

Based on the above definitions, we denote the input and output values of ideal DMU (IDMU) by  $x_i^{\min}(i = 1, \dots, m)$  &  $y_r^{\max}(i = 1, \dots, s)$  and denote the input and output values of anti-ideal dMU (ADMU) by  $x_i^{\max}(i = 1, \dots, m)$ ,  $y_r^{\min}(i = 1, \dots, s)$ . These values are determined as follows:

$$x_i^{\min} = \min_j \{x_{ij}\} \text{ and } x_i^{\max} = \max_j \{x_{ij}\}, \quad i = 1, \dots, m \quad (3.18)$$

$$y_r^{\min} = \min_r \{y_{rj}\} \text{ and } y_r^{\max} = \max_r \{y_{rj}\}, \quad r = 1, \dots, s \quad (3.19)$$

According to the concept of efficiency, the efficiency of ADMU is defined as follows:

$$\theta_{ADMU} = \frac{\sum_{r=1}^s u_r y_r^{\min}}{\sum_{i=1}^m v_i x_i^{\min}} \quad (3.20)$$

Let  $\theta_{ADMU}^*$  be the optimistic efficiency of ADMU; then it can be obtained from the following fractional programming model:

$$\begin{aligned} \max \theta_{ADMU} &= \frac{\sum_{r=1}^s u_r y_r^{\min}}{\sum_{i=1}^m v_i x_i^{\max}} \\ \text{s.t. } \theta_j &= \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \leq 1, \quad j = 1, \dots, n, \\ u_r, v_i &\geq \varepsilon, \quad r = 1, \dots, s; \quad i = 1, \dots, m, \end{aligned} \quad (3.21)$$

The fractional programming model (3.21) is converted to the following LP model, which can be solved readily.

$$\begin{aligned} \max \theta_{ADMU} &= \sum_{r=1}^s u_r y_r^{\min} \\ \text{s.t. } \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} &\leq 0, \quad j = 1, \dots, n, \\ \sum_{i=1}^m v_i x_i^{\max} &= 1 \\ u_r, v_i &\geq \varepsilon, \quad r = 1, \dots, s; \quad i = 1, \dots, m, \end{aligned} \quad (3.22)$$



Similarly, efficiency of IDMU is defined as

$$\varphi_{IDMU} = \frac{\sum_{r=1}^s u_r y_r^{\max}}{\sum_{i=1}^m v_i x_i^{\min}} \tag{3.23}$$

Assuming that  $\varphi_{IDMU}^*$  is the pessimistic efficiency of IDMU, it can be obtained from the following fractional programming model

$$\begin{aligned} \min \varphi_{IDMU} &= \frac{\sum_{r=1}^s u_r y_r^{\max}}{\sum_{i=1}^m v_i x_i^{\min}} \\ \text{s.t. } \varphi_j &= \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \geq 1, j = 1, \dots, n, \\ u_r, v_i &\geq \varepsilon, r = 1, \dots, s; i = 1, \dots, m, \end{aligned} \tag{3.24}$$

which can be solved using the following LP model:

$$\begin{aligned} \min \varphi_{IDMU} &= \sum_{r=1}^s u_r y_r^{\max} \\ \text{s.t. } \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} &\geq 0, j = 1, \dots, n, \\ \sum_{i=1}^m v_i x_i^{\min} &= 1 \\ u_r, v_i &\geq \varepsilon, r = 1, \dots, s; i = 1, \dots, m, \end{aligned} \tag{3.25}$$

Based on the above discussion, we have  $\theta_{ADMU}^* \leq \min_{j=1, \dots, n} \{\theta_j^*\}$  and  $\theta_{IDMU}^* \geq \max_{j=1, \dots, n} \{\theta_j^*\}$ . Now we determine the parameter for all intervals  $[\alpha\varphi_o^*, \theta_o^*]$  ( $j = 1, \dots, n$ ).

$$\min_{j=1, \dots, n} \left\{ \theta_j^* / \varphi_j^* \right\} \geq \frac{\min_{j=1, \dots, n} \{\theta_j^*\}}{\max_{j=1, \dots, n} \{\varphi_j^*\}} \geq \frac{\theta_{ADMU}^*}{\varphi_{IDMU}^*} \tag{3.26}$$

If we set  $\alpha = \theta_{ADMU}^* / \varphi_{IDMU}^*$ , then we will have no problem in determining  $\alpha$ . After determining  $\alpha$ , we will see that the efficiencies of DMUs cannot be smaller than it. The above method based of interval DEA is defined for determination of  $\alpha$  value. So our proposed method is identified for application of DEA in FMEA.

In additive risk mode,  $R_o^{max}$  is calculated for ideal and anti-ideal failure item by using model (2.12). So the following eqnarray is used to calculate  $\alpha$  value:

$$\alpha = R_{AFM}^{max} / R_{IFM}^{max} \quad (3.27)$$

After determining  $\alpha$  value, model (2.13) is rewritten by adding a new constraint.

$$R_o^{max} = \max R_o$$

$$s.t. \begin{cases} R_i \leq 1, & i = 1, \dots, n, \\ R_i \geq \alpha, & i = 1, \dots, n, \\ w_j - 9w_k \leq 0, & j, k = 1, \dots, m; k \neq j \end{cases} \quad (3.28)$$

So the above model is used for determining values of  $R_o^{max}$  failure modes. As it is seen in model (2.13),  $\alpha$  value has no roles in this model. So model (3.28) is used for calculating maximum risk and model (2.13) is used for calculating minimum risk by considering  $\alpha$ . According to the results, the following eqnarray is used to calculate interval risk

$$[R_j^L, R_j^U] = [\alpha R_j^{min}, R_j^{max}] \quad (3.29)$$

But the question is how we rank interval numbers? Various methods have been presented for ranking interval numbers. Yue method [23] is used for ranking interval numbers in the present study. This method is on the basis of degree magnitude possibility of an interval number rather than another.

The advantage of Yue method against Wang method [21] in ranking interval numbers is that interval magnitude, lower bound and upper bound of comparative numbers have significant effect on the ranking of interval numbers. While in Wang method, two interval number with the same lower bound have the same ranking only if upper bound of these two numbers would not be the maximum upper bound of ranking numbers.

The above model is developed for additive risks. For multiplicative risks defined by Eq.(2.6), we should calculate values  $R_o^{max}$  for Anti-ideal Item and Ideal Item of failure modes in the same way. Then  $\alpha$  value is determined by using Eq.(3.27). Regarding to the estimated  $\alpha$ , model (2.15) is rewritten to determine  $R_o^{max}$  for each item as follows:

$$\ln R_o^{max} = \max R_o$$

$$s.t. \begin{cases} \ln R_i \leq 1, & i = 1, \dots, n, \\ \ln R_i \geq \ln(\alpha), & i = 1, \dots, n, \\ w_j - 9w_k \leq 0, & j, k = 1, \dots, m; k \neq j \end{cases} \quad (3.30)$$

After determination of maximum and minimum values by model (2.16), interval risk is calculated according to eqnarray (3.29) and the suggested method is used to prioritize failure items.

## 4 An illustrative example

In this section we provide a numerical example we illustrate the potential application of the proposal fuzzy FMEA. This example is taken from Pillay and Wang [16].

**Example.** The FMEA for the fishing vessel investigates four different systems which

are structure, propulsion, electrical, and auxiliary systems. Each system is considered for different failure modes that could lead to an accident with undesired consequences. The effects of each failure mode on the system and vessel are studied along with the provisions that are in place or available to mitigate or reduce risk. For each of the failure modes, the system is investigated for any alarms or condition monitoring arrangements, which are in place. Table 1 show the 21 identified failure modes and their ratings on the three risk factors O, S, and D.

Table 1.  
FMEA for the fishing vessel by RPN

Failure mode	O	S	D	RPN	Priority ranking
Seizure	1	8	3	24	14
Breakage	1	8	3	24	14
Structural	2	8	4	64	10
Loss of output	8	8	5	320	2
Auto	6	8	6	288	3
Shaft	2	8	1	16	19
Shaft seizure	2	9	2	36	12
Gearbox	1	4	3	12	20
Hydraulic	3	2	3	18	18
Prop. blade	1	2	4	8	21
No start air	4	2	3	24	14
Generator fail	9	3	7	189	4
Complete loss	8	3	6	144	7
Complete loss	3	7	4	84	9
Loss of output	3	3	4	36	12
Loss of output	1	8	3	24	14
Contamination	4	8	5	160	6
No fuel to	2	7	7	98	8
No cooling	7	2	4	56	11
System loss	9	8	9	648	1
Loos of	9	3	6	162	5

Table 2 shows the obtained results from the models (2.12) and (2.13) as well as (2.15) and (2.16) for additive risk and multiplicative risk, respectively, using geometric average method.

Table 2.  
FMEA for the fishing vessel by DEA [6]

Failure Mode	Additive risks				Multiplicative risks			
	Max risk	Min risk	Geo ave risk	Rank	Max risk	Min risk	Geo ave risk	Rank
1	0.84	1.12	0.97	12	2.36	2.95	2.64	13
2	0.84	1.12	0.97	12	2.36	2.95	2.64	13
3	0.87	1.60	1.18	9	2.46	5.46	3.66	7
4	0.94	2.16	1.43	3	2.64	7.27	4.38	3
5	0.93	2.32	1.47	2	2.62	7.73	4.51	2
6	0.83	1	0.91	16	2.32	2.72	2.51	17
7	0.94	1.23	1.08	11	2.5	4.06	3.18	11
8	0.44	1	0.67	18	1.8	2.72	2.21	19
9	0.33	1	0.57	21	1.64	2.72	2.11	21
10	0.40	1	0.63	20	1.73	2.72	2.17	20
11	0.42	1.02	0.65	19	1.81	2.76	2.24	18
12	0.93	1.85	1.31	5	2.58	5.43	3.75	6
13	0.83	1.74	1.20	8	2.46	5.11	3.54	10
14	0.78	1.60	1.12	10	2.38	5.43	3.59	9
15	0.43	1.27	0.74	17	1.84	3.75	2.63	16
16	0.84	1.12	0.97	12	2.36	2.95	2.64	13
17	0.90	1.96	1.33	4	2.56	6.69	4.14	4
18	0.80	2.05	1.28	6	2.41	6.82	4.05	5
19	0.70	1.22	0.93	15	2.26	3.22	2.7	12
20	1	3.16	1.78	1	2.72	10	5.21	1
21	0.92	1.78	1.28	7	2.57	5.14	3.63	8

### 4.1 Proposed model

In this case, first we should calculate values  $R_o^{\max}$  for Anti-ideal Item and Ideal Item. Anti-ideal Item and Ideal Item equals minimum and maximum failure modes. However risk factor raying of Anti-ideal Item and Ideal Item are as follow

Table 3  
Ratings of Anti-ideal Item and Ideal Item factor risks

Failure modes	O	S	D
Anti-ideal Item	1	2	1
ideal Item	9	9	9

By solving (3.22) and (3.25) for Anti-ideal Item and ideal Item, respectively, we get the following results for additive risk:

$\theta_{A-Ideal}^* = 0.2222$  and  $\varphi_{Ideal}^* = 1.1$ , hence  $\alpha = \frac{0.2222}{1.1} = 0.202$ . Regarding to the estimated  $\alpha$ , the following linear programming has been solved to determine  $R_o^{\max}$  for each 21 items:

$$R_o^{\max} = \max R_o$$

$$s.t. \begin{cases} R_i \leq 1, & i = 1, \dots, n, \\ R_i \geq 0.202, & i = 1, \dots, n, \\ w_j - 9w_k \leq 0, & j, k = 1, \dots, m; k \neq j \end{cases}$$

Since  $\alpha$  value has no role in evaluating  $R_o^{\min}$ , we use estimated values of  $R_o^{\min}$  in Table 2. Table 4 shows results of maximum and minimum evaluation, interval efficiency, and priority ranking of failure mode.

Table 4  
Interval efficiency evaluation for additive and multiplicative risk.

Failure Mode	Additive risks			Multiplicative risks		
	Max risk	Min risk	Interval risk	$p_i$	Rank	Rank
1	0.84	1.12	[0.2262, 0.8443]	10.98075	12	14
2	0.84	1.12	[0.2262, 0.8444]	10.98044	13	15
3	0.87	1.60	[0.3232, 0.8666]	12.37335	8	7
4	0.94	2.16	[0.4363, 0.9444]	14.37248	3	3
5	0.93	2.32	[0.4686, 0.9333]	14.61795	2	2
6	0.83	1	[0.202, 0.8333]	10.61201	15	17
7	0.94	1.23	[0.2484, 0.9444]	12.07564	9	11
8	0.44	1	[0.202, 0.4444]	5.062353	18	19
9	0.33	1	[0.202, 0.3294]	2.727613	21	21
10	0.40	1	[0.202, 0.3964]	4.074229	20	20
11	0.40	1.02	[0.2060, 0.399]	4.162893	19	18
12	0.89	1.85	[0.3737, 0.8942]	13.25079	5	6
13	0.79	1.74	[0.3514, 0.7901]	12.05094	10	10
14	0.78	1.60	[0.3232, 0.7777]	11.5493	11	9
15	0.43	1.27	[0.2565, 0.4280]	5.293842	17	13
16	0.84	1.12	[0.2262, 0.8446]	10.98014	14	16
17	0.90	1.96	[0.3959, 0.9]	13.60016	4	4
18	0.80	2.05	[0.4141, 0.8]	12.99997	6	5
19	0.65	1.22	[0.2464, 0.6471]	8.955935	16	12
20	1	3.16	[0.6383, 1]	16.98319	1	1
21	0.86	1.78	[0.3595, 0.8610]	12.79693	7	8

In order to estimate multiplicative risk, we solved model (2.15) for Anti-ideal Item and Ideal Item that following results were obtained

$\theta_{A-Ideal}^* = 1.3098$  and  $\varphi_{Ideal}^* = 2.8458$  then  $\alpha = \frac{1.3098}{2.8458} = 0.4630$ . Rankings prevented in the last column of Table 4 is calculated by determining  $\alpha$  value and solving model (3.30) for 21 failure modes. It is clear from Table 3 that the Chin's method could not provide a robust ranking. For example, FM<sub>1</sub>, FM<sub>2</sub> and FM<sub>16</sub> have the same additive risk rank. A similar situation holds for multiplicative risk ranking. While our method overcomes this shortcoming. Out of 21 failure modes 12 modes have the same additive risk rank in both methods. Whereas, when we consider multiplicative risk 17 modes have the same rank in both methods.

## 5 Conclusion

In this paper we proposed a DEA based methodology for ranking failure mode risk in FMEA. In the proposed model each FM is considered as a DMU and its best and worse relative efficiency computed. Furthermore, theoretically the best and worst relative efficiency showed as an interval. For this purpose, a parameter called as  $\alpha$  used to moderate the worst relative efficiency of DMUs. So it seems that the proposed model that uses  $\alpha$  measure to determine failure risk items is more effective and efficient than geometric average method.

## Acknowledgment

The authors are grateful for the constructive comments and suggestions made by the two anonymous referees.

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