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A Hybrid Decision-making System Using Data Envelopment Analysis and Fuzzy Models for Supplier Selection in the Presence of Multiple Decision Makers

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Abstract

Nowadays, improving the competitive condition of organizations greatly depends upon the process of outsourcing. Raw materials, products, services, or some parts of the organization activities can be outsourced. Thus, the process of outsourcing is regarded as a strategic decision. At the same time, the first step after making decision on outsourcing is selecting the appropriate supplier in the given area. Due to the importance of this issue, so far many extensive studies have been conducted on offering appropriate solutions to the problem of supplier selection. In this paper, a hybrid system consisting of Data Envelopment Analysis (DEA) and group decision making based on fuzzy models is proposed for solving the problem of supplier selection. In this hybrid system, first the weights of the criteria are obtained from every decision maker as fuzzy numbers and group decision making, and after being integrated, they are incorporated into the DEA model using the concept of intersection in fuzzy numbers. Then, DEA model is solved through Assurance Region (AR) method in order to select the best supplier.

Keywords : Supplier Selection, Group Decision Making, Fuzzy Models, DEA, Assurance Region.

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1 Introduction

Nowadays, Supply Chain Management (SCM) has gained particular importance due to globalization and increased competition among agencies [26]. This competitive atmosphere exerts a doubled pressure on the companies for decreasing expenses, improving qualities, and reducing lead-times [13]. Thus, this complex situation makes managers to focus on all activities of supply chain process from suppliers to the end users, and use numerous strategies and operational instruments to improve this chain. One of the strategies considered by most organizations is the strategy of keeping the set of core competencies within the organizations and outsourcing and delegating other competencies to other suppliers [12]. Hence, outsourcing is a very important process and in order to get a better competitive position, organizations must effectively manage this process. In this regard, purposeful selection of an appropriate supplier for outsourcing is one of the most important decisions at the organizational level, regardless of meeting operational needs of the organization, so suppliers are considered as parts of the executors of the strategic goals of the organization [13]. Therefore, the issue of supplier selection is important in the sense that selecting a weak supplier has direct and significant influence upon the quality of product delivered to the customer [3]. At the same time, selection of criteria for judging suppliers is one of the main aspects of supplier selection process. Dickson [10] proposed and prioritized 23 different criteria for the evaluation and selection of the appropriate suppliers. Weber et al. [40] reviewed 74 papers published since 1966 on the issues of supplier selection. They indicated that from among the selection criteria proposed in these papers and the study conducted by Dickson in 1966, 7 criteria have more importance. These criteria include quality, cost, on time delivery, production facility, production capacity, technical capability, and geographical location. They found out that the problem of vendor selection is essentially a multiple objective problem in which the specific criteria such as cost, quality, delivery time, etc. must be considered simultaneously and the best vendor is selected according to them. For this reason, so far various methods have been proposed for solving the problem of supplier selection.

In 1998, Analytic Hierarchy process (AHP) technique was utilized for ranking the companies [1]. In 2003, Kahraman et al. [27] used fuzzy AHP for selecting the best contractor based on meeting the specific criteria. Hou and Su [23] developed AHP method for the problem of selecting suppliers in mass customization environments. However, AHP technique is not without its faults. First, when more than one person uses this method in decision-making area, different opinions of decision-makers on the weight of each criterion makes the model complex. Second, this technique greatly depends upon the information and experience level of the decision maker regarding the decision issue [42]. The last criticism of this technique is not considering the interrelationships of the criteria in the model [34]. Braglia and Petroni [5] proposed the theory of multiple attribute utility on the basis of DEA. They used this method for the formulation of viable sourcing strategies in changing environments. Later, Bross and Zhao [6] indicated that multi attribute utility theory (MAUT) is an appropriate and useful method. It enables purchase managers to formulate their viable sourcing strategies. Technique for Order-Preference by Similarity to Ideal Solution (TOPSIS) is one of the well-known classical techniques for Multiple Attribute Decision Making (MADM) problems. This technique was invented by Hwang and Yoon in 1981 [24]. Chen et al. [8] used fuzzy TOPSIS for solving the evaluation problem in supplier selection process. Then, fuzzy hierarchical TOPSIS was utilized for solving supplier selection problem by Wang et al. [38]. Kumar et al. [28] employed fuzzy goal programming for solving the problem of supplier selection with multiple objectives. Weber [39] indicated how DEA technique can be applied for evaluating suppliers with multiple criteria and weights assigned for them. Forker and Mendez [18] proposed an analytical method for using DEA technique that can help companies identify the most efficient suppliers. Garfamy [19] utilized DEA technique for measuring the total performance of the suppliers based on the concept of total cost of ownership (TCO). Farzipoor Saen and Zohrehbandian [16] proposed a super efficiency model for ranking suppliers according to volume discount condition. Again, Farzipoor Saen [14] introduced a model in which the best suppliers are selected according to quantitative (cardinal) and qualitative (ordinal) data in environments in which the issue of volume discount is addressed. In addition, fuzzy logic and its application are among the techniques used for designing decision making models. Chen et al. [8] presented a hierarchical model on the basis of fuzzy sets theory for supplier selection problem. Florez-Lopez [17] employed fuzzy-linguistic models in order to select the best supplier. In recent years, researchers have used hybrid approaches (combination of various methods) for the evaluation and selection of suppliers. Ghodsypour and O'brien [20] offered a hybrid model of AHP and linear programming in which quantitative and qualitative criteria are used simultaneously. In order to reduce the number of suppliers from among the suppliers present for the purpose of better management, Mendoza et al. [30] offered a hybrid model of AHP and Goal Programming (GP). Sevkli et al. [36] proposed a model in which a combination of AHP and DEA is utilized for supplier selection. Farzipoor Saen [11] employed a hybrid model of AHP-DEA for evaluating and selecting from among slightly non- homogeneous suppliers. Ramanathan [33] offered a hybrid model consisting of AHP, DEA, and TCO in which quantitative and qualitative information are used concurrently.

As it can be inferred from this brief review, so far, various models have been designed and proposed for the issue of supplier selection. However, to the best of knowledge of the authors, there is no model which uses the combination of intersection concept in fuzzy numbers and DEA technique for solving the problem of supplier selection.

The model proposed in this paper has the following contributions:

- For the first time, the proposed model utilizes the intersection concept in fuzzy numbers for integrating the views of decision-makers.
- For the first time, quasi-Gaussian fuzzy number is used in the definition of fuzzy linguistic variables for determining the importance of supplier selection criteria.
- Real data obtained from field study is used for defining fuzzy linguistic variables.
- The proposed model is a hybrid one in which the weight of each criterion, after being calculated through the concept of intersection in fuzzy numbers, is added to the classical DEA model and the resulting Assurance Region (AR) model is solved for the evaluation of the suppliers.

This paper proceeds as follows: In section 2 theoretical fundamentals and primary definitions of the tools and techniques used in the study are explained. In section 3, the proposed hybrid system and administrative stages are presented and finally, the proposed model is solved with an example in section 4. At the end, some outlooks of model development are suggested as the conclusion in section 5.

2 Theoretical fundamentals and primary definitions

2.1 Fuzzy set theory

The theory of fuzzy set was introduced by Zadeh [43] for expressing uncertain variables and concepts. The fuzzy set theory involves fuzzy logic, fuzzy arithmetic, fuzzy mathematical programming, fuzzy topology, fuzzy graph theory, and fuzzy data analysis [27]. In this subsection, some basic definitions of fuzzy set, i.e. fuzzy numbers and linguistic variables are illustrated.

- Gaussian Fuzzy Number (GFN)

As it is pointed out by [4] and [21], GFN is often used in practical and operational assumptions because its parameters are empirically determined though experience. Gaussian density function of probability is defined as below:

$$f(x) = \exp\left(-\frac{1}{2} \times \frac{(\overline{m}-x)^2}{\sigma^2}\right) \tag{2.1}$$

- Quasi-Gaussian Fuzzy Numbers (QGFN)

GFN is not bounded. This is considered as a disadvantage for its numerical treatment. The following procedure is used for bounding GFN [22]:

$$f(x) = \begin{cases} \exp\left(-\frac{1}{2} \times \frac{(\overline{m}-x)^2}{\sigma^2}\right) & \text{If } |\overline{m}-x| \le r\sigma \\ \\ 0 & \text{If } |\overline{m}-x| > r\sigma \end{cases} \qquad \text{where } \; r \in \mathbb{R}^+ \tag{2.2}$$

- Operations of fuzzy set

Fuzzy union: In general, the union of the two fuzzy sets of $\widetilde{A}$ and $\widetilde{B}$ is defined as below [29]:

$$\mu_{\widetilde{A}\cup\widetilde{B}}(x) = \max\left[\mu_{\widetilde{A}}(x), \mu_{\widetilde{B}}(x)\right] \tag{2.3}$$

Fuzzy intersection: Intersection of the two fuzzy sets of $\widetilde{A}$ and $\widetilde{B}$ is defined as below [29]:

$$\mu_{\widetilde{A}\cap\widetilde{B}}(x) = \min\left[\mu_{\widetilde{A}}(x), \mu_{\widetilde{B}}(x)\right] \tag{2.4}$$

The union and intersection of two fuzzy sets with the quasi-Gaussian membership function are depicted in (1.a) and (1.b) respectively:

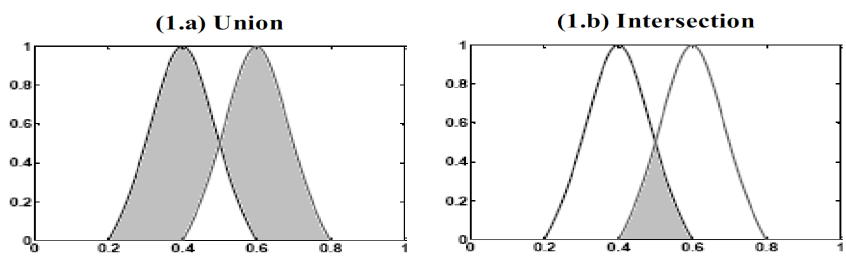


Fig. 1. Union and Intersection

- Fuzzy linguistic variables

In general, when a variable is considered, it is assigned a number as its value. Now, if linguistic terms are assigned to these variables, they are called linguistic variables [29]. Linguistic variables are defined via membership functions. Gaussian and quasi-Gaussian membership functions are two types of them. The characteristics of these membership functions in comparison to other common membership functions are as below:

1. Gaussian and quasi-Gaussian membership functions are closer to human behavior and thought.

2. Triangular or trapezoidal membership functions consider only 3 and 4 points from the given interval, respectively, and other points of the specific interval are not considered [32].

3. Adapting Gaussian and quasi-Gaussian membership functions with reality is easily achieved through changing the mean and variance of membership function [32].

4. Quasi-Gaussian membership function is the same as Gaussian membership function; the only difference is that the problem of being unbounded has been solved in it for numerical treatment.

Nevertheless, one of the most important decisions in the definition of linguistic variables is selecting the number of linguistic terms for describing each criterion. Miller [31] claimed that the number of words or sentences that an individual is able to distinguish is 7 ± 2 . In Fig. 2, the linguistic variable of temperature is expressed as the quasi-Gaussian fuzzy number.

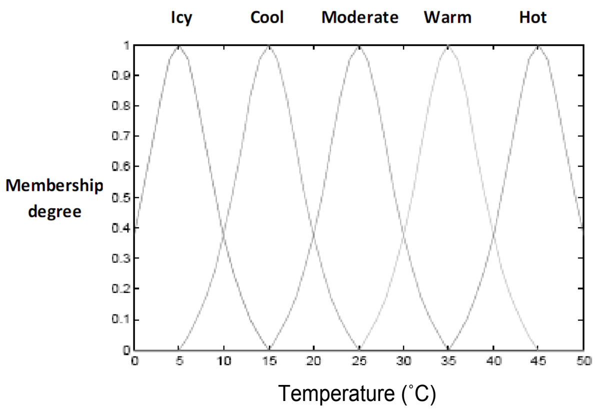


Fig. 2. The fuzzy linguistic variable of temperature

2.2 Data Envelopment Analysis

DEA is a decisional technique that has been widely used for performance analysis in public and private sectors. DEA developed by Charnes et al. [7], is a non-parametric estimation method, in the sense that no choice of a parametric functional form is needed in the estimation of the frontier. Later, in 1984, another model was proposed by Banker et al., called BCC [2].

- CCR Model

Suppose there is a set of n decision making units, $\{DMU_j : j = 1, 2, \ldots, n\}$, which produce multiple outputs $y_{rj} (r = 1, 2, \ldots, s)$, by utilizing multiple inputs $x_{ij} (i = 1, 2, \ldots, m)$.

When a DMU_p is under evaluation by the CCR model, there is:

$$\begin{aligned} \max \quad & W = \textstyle\sum_{r=1}^{s} u_r y_{rp} \\ s.t. \quad & \textstyle\sum_{i=1}^{m} v_i x_{ip} = 1 \\ & \textstyle\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad \forall j, \\ & u_r, v_i \geq 0 \qquad \forall r, i \end{aligned} \tag{2.5}$$

where u_r is the weight of rth output and v_i is the weight of the ith input in model (2.5), DMU_p is said to be efficient ($W = 1$) if no other DMU or combination of DMUs can produce more than DMU_p on at least one output without producing less in some other output or requiring more of at least one input.

- Assurance Region (AR) technique in DEA

One serious drawback of DEA applications in supplier selection has been the absence of decision maker judgment, allowing total freedom when allocating weights to input and output data of supplier under analysis. This allows suppliers to achieve artificially high efficiency scores by indulging in inappropriate input and output weights [15]. The most widespread method for considering judgments in DEA models is, perhaps, the weight restrictions inclusion. Weight restrictions allow for the integration of managerial preferences in terms of relative importance levels of various inputs and outputs. The idea of conditioning the DEA calculations to allow for the presence of additional information arose first in the context of bounds on factor weights in DEAs multiplier side problem. This led to the development of the cone-ratio and assurance region models [15].

In general, there are three methods for entering the restrictions of weights into multiplier models of DEA [15]:

1. Absolute weight restrictions:

$$\delta_i \leq v_i \leq \tau_i \qquad \rho_r \leq u_r \leq \eta_r \tag{2.6}$$

2. Assurance region of Type I (relative weight restrictions):

$$\alpha_i \leq \frac{v_i}{v_{i+1}} \leq \psi_i \qquad \theta_r \leq \frac{u_r}{u_{r+1}} \leq \xi_r$$

3. Assurance region of Type II (input-output weight restrictions):

$$\varphi_i v_i \geq u_r$$

where, Greek characters $(\delta_i, \tau_i, \rho_r, \eta_r, \alpha_i, \psi_i, \theta_r, \zeta_r, \varphi_i)$ are upper and lower limit of the weights assigned by the decision maker who desires that the model determines the weights of input and output factors in this limit.

For instance, by bounding the weights in model (2.5) and using the first method for applying weight restrictions, the CCR model is written as below:

$$\begin{aligned} \max \quad & W = \textstyle\sum_{r=1}^{s} u_r y_{rp} \\ s.t. \quad & \textstyle\sum_{i=1}^{m} v_i x_{ip} = 1 \\ & \textstyle\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \quad \forall j, \\ & \rho_r \leq u_r \leq \eta_r \qquad \forall r \\ & \delta_i \leq v_i \leq \tau_i \qquad \forall i \end{aligned} \tag{2.7}$$

where, (τ_i, δ_i) and (η_r, ρ_r) are the upper and lower limits of inputs and outputs, respectively. The important point is that assigning limits is not totally free and it must be noticed if the problem is feasible.

- l_1-norm method for ranking efficient units

In some cases, there are more than one efficient DMUs with relative efficiency of 1. In these situations, various ranking methods can be used for determining the efficient unit from among them. Jahanshahloo et al. [25] proposed l_1-norm method for ranking efficient units. They indicated that this method does not have the problem of infeasible solution which is found in other methods. When DEA model with constant returns to scale is assumed for ranking efficient DMUs, the following model will be utilized:

$$
\begin{array}{lll}
\min & \Gamma_c^o(X,Y) = \sum_{i=1}^{m} x_i - \sum_{r=1}^{s} y_r + \alpha & \\
s.t. & \sum_{j=1, j\neq o} \lambda_j x_{ij} \leq x_i & i = 1, \ldots, m, \\
& \sum_{j=1, j\neq o} \lambda_j y_{rj} \geq y_r & r = 1, \ldots, s, \\
& x_i \geq x_{io} & i = 1, \ldots, m, \\
& 0 \leq y_r \leq y_{ro} & r = 1, \ldots, s, \\
& \lambda_j \geq 0 & j = 1, \ldots, n, \quad j \neq 0
\end{array}
$$

where, $\alpha = \sum_{r=1}^{s} y_{ro} - \sum_{i=1}^{m} x_{io}$, and $\lambda = (\lambda_1, \ldots, \lambda_{o-1}, \lambda_{o+1}, \ldots, \lambda_n)$ is a non-negative vector of variables (envelopment from), α is the constant, and $\Gamma_c^o(X,Y)$ is the distance (X_o, Y_o) from (X,Y) by using l_1-norm.

3 The proposed model

Based on what stated in previous sections, the process of supplier selection is proposed as a hybrid system in 5 stages as below:

1. Identifying important criteria for the selection of suppliers
2. Eliciting the weight of every selected criteria
3. Evaluating suppliers and determining their relative efficiency
4. Ranking suppliers having tie in their relative efficiency (if necessary)
5. Reviewing the weights of criteria and re-evaluating the suppliers (if necessary)

The first step is identifying necessary and important criteria for evaluating the suppliers. It is worth noting that identifying important and applicable criteria is vital for a rational and unbiased selection. In the second step, every decision maker assigns an appropriate weight to each selected criterion, and then these opinions are integrated. This is done by using fuzzy linguistic variables and the concept of intersection in fuzzy numbers. In the third step, DEA is employed for calculating the relative efficiency of suppliers and selecting the best of them on the basis of the highest relative efficiency obtained. In this technique, AR method is used for incorporating the weights of criteria obtained in step 2. At the same time, if more than one supplier has tie in the relative efficiency, the fourth step is executed. In this case, l1-norm method is utilized for ranking the efficient units in order to determine the superior supplier. Finally, if entering criteria weights into AR model through DEA does not provide the problem with a feasible solution or an intersection is not achieved in integrating the opinions of the decision makers in step 2, the fifth step is activated and the weights assigned by decision makers are reviewed by analyzing the information and their causes. The model of these procedures is indicated in Fig 3.

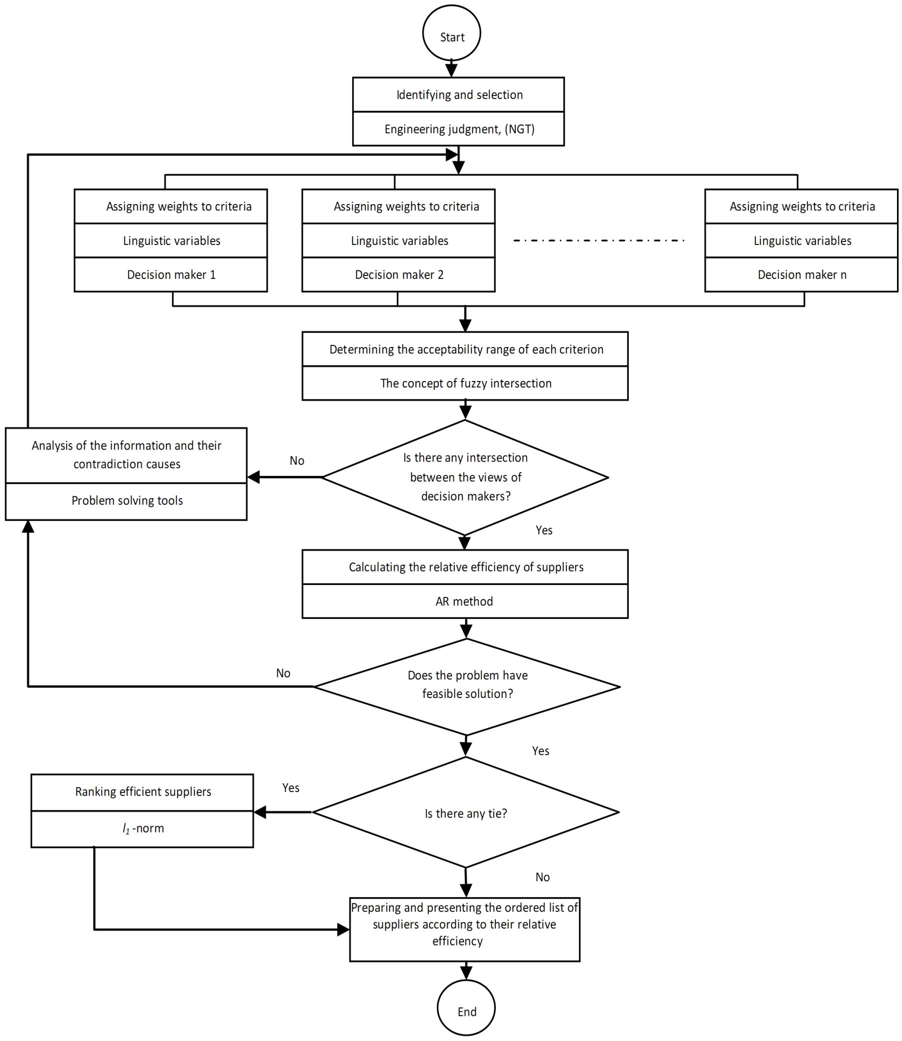


Fig. 3. Depiction of the proposed hybrid model

3.1 Identifying important criteria

Making a rational and correct decision is very difficult in the process of evaluating and selecting suppliers. In this respect, many criteria must be considered with great care for problem solving. Research conducted by Dickson [10] and Weber et al. [40] can be considered as a guide for selecting appropriate criteria in supplier selection problem. In this case, appropriate criteria can be identified and used through engineering judgment or using the expert opinions of the organization or through any other techniques such as nominal group technique (NGT) [9].

3.2 Eliciting the weight of every selected criteria

The second step in the process of supplier selection is specifying the weights of the selected criteria. For this purpose, first an appropriate ranking system must be designed for assigning weights to criteria by the decision makers, and then these weights must be integrated and the final weight of each criterion should be determined.

3.2.1 Defining the linguistic variables

Linguistic variables are useful for stating complex situations or situations which cannot be converted into quantitative terms, because the evaluation of these variables is done on the basis of subjective judgment of the decision makers. in this study, as discussed in section 2, the linguistic variable of importance degree with 5 fuzzy linguistics terms having quasi-Gaussian membership function will be used for specifying weights of the criteria. The linguistic variables used for stating the importance of supplier selection criteria in this study include:

Very low	low	middle	high	Very high
VL	L	M	H	VH

To determine the shape and range of each linguistic term, a questionnaire was developed and the opinion of each expert regarding the importance of the selected criteria in the numerical example of the paper was obtained. Since supplier selection in each organization is conducted by experts and senior directors, judgmental sampling method was utilized to survey opinions on the importance of the selected criteria and the views of this group of experts and directors were obtained [35]. To this end, the experts and directors of various organizations were provided with a guiding diagram of definition of linguistic variables that has been shown in Fig. 4 to express their views on the shape and range of each linguistic variable.

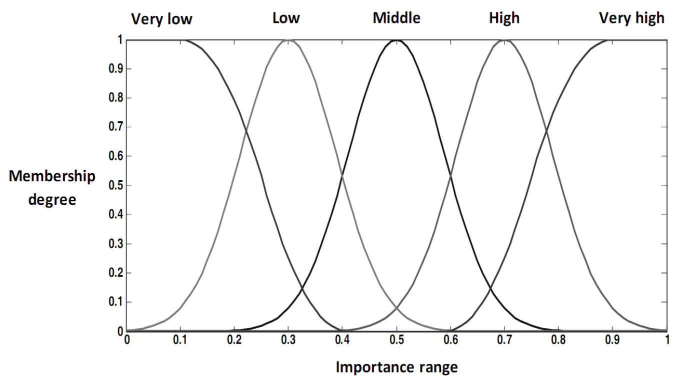
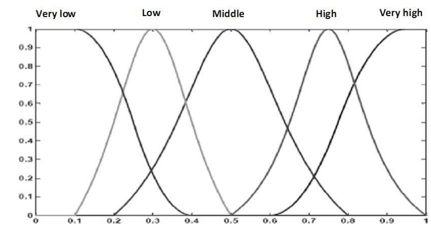
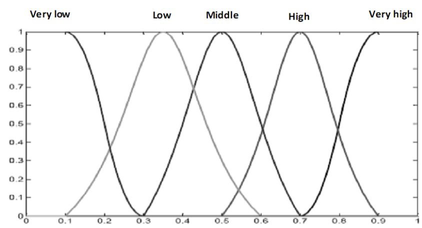


Fig. 4. Linguistic Variables Defining Importance

In Table 1, the pattern of defining criteria importance by two experts is presented as a sample:

Table 1
Collecting the views of experts on linguistic terms

Experts' view	**Criteria**	**Importance**	**The proposed pattern for defining the importance of criteria**
Expert 1	Price	High	
	Percent of rejected materials	Very high	
	Percent of on time delivery	Middle	
	Supplier capacity	Low	
Expert 2	Price	Middle	
	Percent of rejected materials	High	
	Percent of on time delivery	High	
	Supplier capacity	Very low	

After obtaining views of 100 experts and directors, the data of the presented figures was derived and the frequency table of each defined linguistic term was prepared. Table 2 presents this information.

Table 2
Frequency of views obtained from the experts

Importance range	**VH**	**H**	**M**	**L**	**VL**
0	0	0	0	1	5
0.1	0	0	0	3	5
0.2	0	0	1	19	4
0.3	0	0	5	27	2
0.4	0	3	35	25	0
0.5	2	23	47	10	0
0.6	7	53	46	3	0
0.7	26	74	22	0	0
0.8	35	57	5	0	0
0.9	40	3	0	0	0
1	40	1	0	0	0

Then, the frequencies obtained were normalized through linear normalization method via equation (*3.2.*1). This method is useful in that all results become equally linear and thus the condition of criteria and their results remain the same.

$$n_{ij} = \frac{c_{ij}}{c_j^*} \quad with \quad c_j^* = \max_j c_{ij}$$

where, c_{ij} is the frequency of the ith importance range relative to jth term.

By normalizing the frequencies obtained, the membership degree of each element of importance range is obtained. Table 3 summarizes these results.

Table 3
Membership degree of the elements of importance range

Importance range	**VH**	**H**	**M**	**L**	**VL**
0	0.00	0.00	0.00	0.04	1.00
0.1	0.00	0.00	0.00	0.11	1.00
0.2	0.00	0.00	0.02	0.70	0.80
0.3	0.00	0.00	0.11	1.00	0.40
0.4	0.00	0.04	0.74	0.93	0.00
0.5	0.05	0.31	1.00	0.37	0.00
0.6	0.18	0.72	0.98	0.11	0.00
0.7	0.65	1.00	0.47	0.00	0.00
0.8	0.88	0.77	0.11	0.00	0.00
0.9	1.00	0.04	0.00	0.00	0.00
1	1.00	0.01	0.00	0.00	0.00

In the next stage, the Gaussian membership function is fitted to this data in order to determine the shape of membership function of each linguistic variable. This is easily done by MATLAB 7.5 software. Fig. 5 shows the membership function of each linguistic variable.

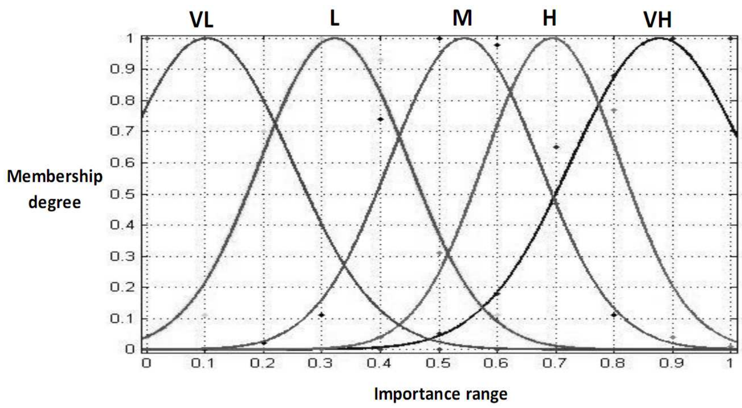


Fig. 5. Fitted functions to each linguistic variable

The statistical information of the fitted functions to each derived data is presented in Table 4.

Table 4
Statistical information of the fitted functions

Linguistic variable	Function	SSE	R-square	RMSE	Adj.R-sq.	Mean	Sigma
Very high	Gaussian	0.015	0.992	0.040	0.991	0.88	0.15
High	Gaussian	0.019	0.986	0.046	0.985	0.69	0.13
Middle	Gaussian	0.018	0.989	0.045	0.988	0.54	0.13
Low	Gaussian	0.011	0.993	0.035	0.992	0.32	0.13
Very low	Gaussian	0.014	0.992	0.039	0.991	0.1	0.15

In Table 4, SSE is the sum of squares due to error, R-square is coefficient of determination, RMSE is the root mean squared errors (standard error), Adj. R-square is adjusted coefficient of determination, and Sigma is the standard deviation of the fitted function to the data. Considering these results and Adj.R-square, which is above 0.9 in all functions, it can be concluded that the fitted functions to data are appropriate and can be used as the basis of defining linguistic variables in this study.

Since in the fuzzy sets with Gaussian membership function, the interval $\pm\sigma$ from the mean is considered for investigation of function behavior [37], it is possible to draw the figure of membership function of each linguistic term using the information presented in Table 4, so that the linguistic variables used for determining the importance of criteria are defined as Fig. 6. It is clear that because terms are placed in the upper and lower limit of importance range of the criteria, the ranges $Mean - 3\sigma$ and $Mean + 3\sigma$ are respectively used for defining terms very high and very low in the fuzzy definitions, and values lower or higher than mean will have the membership degree of 1.

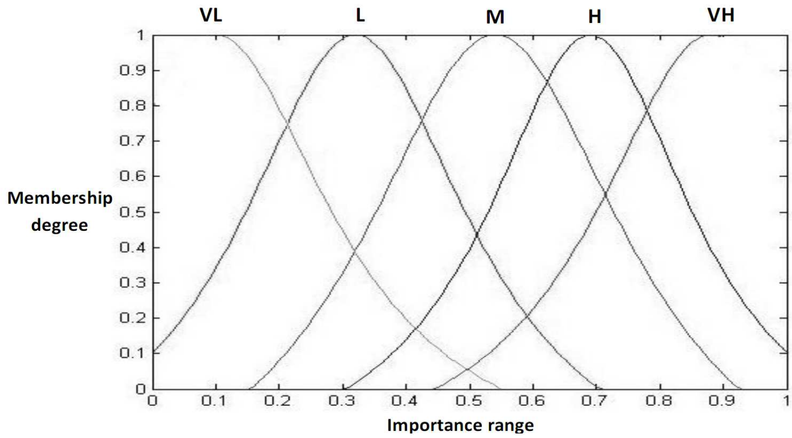


Fig. 6. Linguistic variables for determining the importance of criteria

Note 1: with regard to the method suggested in this paper for integrating the views of decision makers, what is important is the interval defined or the upper and lower limit of each linguistic term to obtain the intersection among the views. Thus the slope of these curves are not of much importance in this study.

3.2.2 Determining the weight range of each criterion

After assigning weights to criteria selected by each decision-maker, the obtained views must be integrated and a single view agreed upon by all decision makers must be announced. Since in AR method, the weight assigned by the decision maker to each criterion is added as a numerical interval to classical DEA model, and at the same time, fuzzy linguistic variables are used for specifying criteria weights in this model, the concept of intersection in fuzzy numbers can be used for integrating the views of decision makers and deriving their acceptable range. In fact, intersection among the views of decision makers which is usually used as a range in fuzzy numbers can be regarded as the common and agreed upon view of all decision makers and used as the output of group decision making for specifying the weight of each criterion.

For this purpose, the upper and lower limit of each defined linguistic variable presented in Table 5 can be used to extract the intersection of views.

Table 5
The Upper and Lower Limits of Linguistic Variables

Linguistic variable	Very low	Low	Middle	High	Very high
Upper limit	0.55	0.71	0.93	1.00	1.00
Lower limit	0	0	0.15	0.30	0.43

Based on what was mentioned above and definition of the given linguistic variables, the intersection of views can be easily calculated. Part of the intersection between decision-makers' views are presented in Table 6 as a sample.

Table 6
Sample of the range of weights based on the intersection of decision makers' views

Decision maker's views					Intersection of views
Very low	Low	Middle	High	Very high	
✓					[0.00 , 0.55]
✓	✓				[0.00 , 0.55]
✓	✓	✓			[0.15 , 0.55]
	✓				[0.00 , 0.71]
	✓	✓			[0.15 , 0.71]
	✓	✓	✓		[0.30 , 0.71]
		✓			[0.15 , 0.93]
		✓	✓		[0.30 , 0.93]
		✓	✓	✓	[0.43 , 0.93]
			✓		[0.30 , 1.00]
			✓	✓	[0.43 , 1.00]
				✓	[0.43 , 1.00]

For instance, if each decision maker assigns a specific weight according to Table 7 to hypothetical criterion of C_1 , the final weight range of C_1 would be [0, 0.55].

Table 7
An example of determining intersection of decision makers' views

	Criterion	Very low	low	Middle	High	Very high	Intersection of views
View of decision maker 1	C_1	✓					[0 , 0.55]
View of decision maker 2			✓				
View of decision maker 3			✓				
View of decision maker 4		✓					

Finally, at the end of this stage, the range of weights related to each of the selected criteria is determined and considered as the input of AR model through DEA technique.

3.3 Evaluating suppliers and determining their relative efficiency

After specifying the weight range of each selected criteria, these ranges are incorporated into DEA model as a restriction. In this study, assuming constant returns to scale and due to the improvement of efficiency of inefficient suppliers by decreasing inputs (e.g. reducing prices and reducing percentage of rejected items supplied by the suppliers), model (2.7) is utilized. As mentioned before, this method of controlling weights in DEA technique is called AR model.

3.4 Ranking suppliers having tie in their relative efficiencies

If the relative efficiency of more than one supplier equals 1, the suppliers must be ranked in order to discriminate the best and most appropriate supplier. In this case, the best supplier will be selected on the basis of l1-norm method explained before.

3.5 Reviewing the weights of criteria and re-evaluating the suppliers

If the discrepancy of decision makers' views in the third step of the proposed approach regarding assigning weights to each of the selected criteria is so high that the intersection range obtained is very small, then incorporating weight control restrictions to CCR model will cause the problem of infeasible solution. In this case, the present step is activated.

At this stage, the factor contributing to the problem is systematically analyzed and removed. The mechanism of this process is analyzing the obtained information and re-defining the range of common views. This means that after identifying the contradictions, the issue is investigated through interaction with decision makers and after obviating the contradictions, the weight range of each criterion is re-specified and is incorporated into CCR model to provide the problem with optimal solution.

4 Numerical example

Data used in this section is taken from Weber et al. [41]. The factory under investigation is one of the sub-branches of Fortune 500 Pharmacy Company which uses JIT system in its production lines. Hence, each of the criteria of price, quality, deliver, and capacity are considered as important criteria in the evaluation of the suppliers of the organization. Table 8 summarizes the information on the 6 suppliers discussed in this example.

Table 8

Information of the selected criteria in evaluation of suppliers

Criteria	**Suppliers**					
	1	**2**	**3**	**4**	**5**	**6**
Unit price[1]	0.1958	0.1881	0.2204	0.2081	0.2118	0.2096
Percent of rejected materials	1.2	0.8	0	2.1	2.3	1.2
Percent of on time delivery	95	93	100	100	97	96
Supplier capacity	2,400,000	360,000	2,783,000	3,000,000	2,966,000	2,500,000

1. Price is considered as the unit price.

Step 1: Specifying important criteria for supplier selection

As it can be seen in Table 8, the problem involves 4 criteria for the evaluation and selection of the best supplier. The criterion of price is measured by the unit price of goods purchased by the company. The criterion of quality is measured by the percentage of rejected items. The criterion of capacity is also measured on the basis of annual production volume of each supplier and the criterion of on time delivery is measured via late delivery of purchased items. The formula is presented in the following equation:

$$\text{Percent of on time delivery} = 1 - (\text{Percent of late delivery})$$

Nevertheless, to incorporate these data into CCR model, they must be homogenous with the data obtained from the weights assigned by each decision maker. Thus, data related to each criterion presented in Table 8 is normalized via equation (*3.2.*1). The results of these calculations are presented in Table 9.

Table 9
Normalized data of the criteria of supplier selection problem

Criteria	**Suppliers**					
	1	**2**	**3**	**4**	**5**	**6**
Unit price	0.8884	0.8534	1	0.9442	0.961	0.951
Percent of rejected materials	0.522	0.348	0	0.913	1	0.522
Percent of on time delivery	0.95	0.93	1	1	0.97	0.96
Supplier capacity	0.8	0.12	0.928	1	0.989	0.833

Step 2: Deriving the Criteria Weights

In order to measure and evaluate the importance of the criteria, the opinions of 5 decision makers ($DM_1, DM_2, \ldots, DM_5$) were surveyed. Each of these DMs assigns importance weights to each criterion according to linguistic weighing variables indicated in Figure 6. The weight importance of each criterion, assigned by each decision maker, is shown in Table 10.

Table 10
Weights assigned for criteria by the decision makers

		Decision makers				
		DM_1	DM_2	DM_3	DM_4	DM_5
Criteria	Unit price	VH	H	H	VH	H
	Percent of rejected materials	H	VH	VH	VH	H
	Percent of on time delivery	M	H	H	H	M
	Supplier capacity	M	H	M	M	L

Now, the intersection of views of DMs as the output of decision making group can be derived using Tables 5 and 6.

The intervals of the final weights of each criterion which is a numerical interval $[a, b]$, are presented in Table 11.

Table 11
Determination of the final weight of each criterion

Criteria	Category	VL	L	M	H	VH	Final interval
Unit price	Input 1				✓	✓	[0.43 , 1.00]
Percent of rejected materials	Input 2				✓	✓	[0.43 , 1.00]
Percent of on time dedelivery	Output 1			✓	✓		[0.30 , 0.93]
Supplier capacity	Output 2		✓	✓	✓		[0.30 , 0.71]

Step 3: Evaluation of the suppliers by their relative efficiencies

The criteria are classified into two categories, i.e. inputs and outputs. Each of the input and output factors is introduced in Table 11. According to this table, inputs and outputs and each weigh restriction of the criteria were incorporated into model (2.7). Model (2.7) was solved using LINDO 6.1. Software and the relative efficiency of each of the 6 suppliers were calculated. The results are presented in Table 12.

Table 12
Results of model (2.7)

	Relative efficiency	Output 1	Output 2	Input 1	Input 2
Supplier 1	0.8048	0.5945	0.3	0.8729	0.43
Supplier 2	0.7037	0.7180	0.3	0.9964	0.43
Supplier 3	1	0.7216	0.3	1	0.43
Supplier 4	0.6699	0.3	0.3699	0.6433	0.43
Supplier 5	0.6034	0.3	0.3158	0.5931	0.43
Supplier 6	0.7655	0.5371	0.3	0.8154	0.43

As it can be seen in Table 12, supplier 3 with the relative efficiency of 1 can be considered as the optimal choice. Also, all weights considered by the model are within the acceptable range of the decision makers.

If AR method is not used in this model, 4 out of 6 suppliers will have relative efficiency of 1 which can present challenges to decision making. Meanwhile, the weight of some criteria might be zero or more than 1 which is illogical and is not acceptable for decision makers. Table 13 presents these results.

Table 13
The results of CCR model

	Relative efficiency	Output 1	Output 2	Input 1	Input 2
Supplier 1	1	1.0066	0.0545	1.0573	0.1162
Supplier 2	1	1.0752	0	1.0752	0.2366
Supplier 3	1	0.7755	0.2418	1	0.0801
Supplier 4	1	0	1	0.928	0.1355
Supplier 5	0.9717	0	0.9825	1.040	0
Supplier 6	0.9528	0.7811	0.2436	1.0071	0.0807

5 Conclusions

In this paper, fuzzy group decision making techniques and DEA were utilized for solving the problem of supplier selection and a hybrid system was proposed accordingly. For this purpose, group decision making technique, using fuzzy linguistic data and the concept of intersection in fuzzy numbers was for integrating the opinions of decision makers, so that the weight of each criterion was determined within an interval.

Then, this interval was incorporated into DEA within the framework of absolute restrictions in order to calculate the relative efficiency of each of the suppliers through AR method and the best and most appropriate one was selected from among them.

According to the results of numerical example, the proposed hybrid system is an appropriate solution for selecting the best supplier.

The problem considered in this study is regarded as the first phase of research and complementary studies in the future can be conducted on the basis of the present results. Some of these future studies are as below:

- Similar studies can be conducted considering both cardinal and ordinal data in the model.

- In some cases, there is not sufficient intersection range to integrate the decision makers' views using the concept of intersection in fuzzy linguistic variables. Therefore, the proposed hybrid system will face infeasible solution. This problem can be the topic of future studies.

- The aim of the proposed model of this study is selecting suppliers. It seems that this model can be utilized in other areas such as technology selection, personnel selection, etc.

References

[1] Z. Babic, N. Plazibat, Ranking of enterprises based on multicriteria analysis, International Journal of Production Economics 56-57 (9) (1998) 29-35.

[2] R.D. Banker, A. Charnes, W.W. Cooper, Some methods for estimating technical and scale inefficiencies in data envelopment analysis, Management Science 30 (9) (1984) 1078-1092.

[3] W.C. Benton, L. Krajeski, Vendor performance and alternative manufacturing environment, Decision Science 21 (1) (1990) 403-415.

[4] I.F. Blake, An introduction to applied probability theory, John Wiley Son, New York-Chichester- Brisbane 1979.

[5] M. Braglia, A. Petroni, A Quality assurance oriented methodology for handling trade-offs in supplier selection, International Journal of Physical Distribution Logistics Management 30 (2) (2000) 96-111.

[6] M.E. Bross, G. Zhao, Supplier selection process in emerging markets- The case study of Volvo bus corporation in china [M.SC. Thesis]. School of Economics and Commercial Law Goteborg University 2004.

[7] A. Charnes, W.W. Cooper, E. Rhodes, Measuring the efficiency of decision making units, European Journal of Operational Research 2 (6) (1978) 429-444.

[8] C.T. Chen, C.T. Lin, S.F. Huang, A fuzzy approach for supplier evaluation and selection in supply chain management, International Journal of Production Economics 102 (2) (2006) 289-301.

[9] A.L. Delbecq, A.H. Van de Ven, D.H. Gustafson, Group techniques for program planning: A guide to nominal group and Delphi processes, Scott, Foresman in Glenview Ill 1975.

[10] G.W. Dickson, An analysis of vendor selection systems and decisions, Journal of Purchasing 2 (1) (1966) 5-17.

[11] R. Farzipoor Saen, A new mathematical approach for supplier selection: Accounting for non- homogeneity is important, Applied Mathematics and Computation 185 (1) (2007) 84-95.

[12] R. Farzipoor Saen, Using super-efficiency analysis for ranking suppliers in the presence of volume discount offers, International Journal of Physical Distribution Logistics Management 38 (8) (2008) 637-651.

[13] R. Farzipoor saen, A decision model for ranking suppliers in the presence of cardinal and ordinal data, weight restriction, and nondiscretionary factors, Annals of Operations Research 172 (1) (2009a) 177-192.

[14] R. Farzipoor Saen, Supplier selection in volume discount environments in the presence of both cardinal and ordinal data, International Journal of Information Systems and Supply Chain Management 2 (1) (2009b) 69-80.

[15] R. Farzipoor Saen, Restricting weights in supplier selection decisions in the presence of dual-role factors, Applied Mathematical Modeling 34 (10) (2010) 2820-2830.

[16] R. Farzipoor Saen, M. Zohrehbandian, A data envelopment analysis approach to supplier selection in volume discount environment, International Journal of Procurement Management 1 (4) (2008) 472-488.

[17] R. Florez-Lopez, Strategic supplier selection in the added-value perspective: A CI approach, Information Science 177 (5) (2007) 1169-1179.

[18] L.B. Forker, D. mendez, An analytical method for benchmarking best peer suppliers, International Journal of Operations and Production Management 21 (1/2) (2001) 195-209.

[19] R.M. Garfamy, A data envelopment analysis approach based on total cost of ownership for supplier selection, Journal of Enterprise Information Management 19 (6) (2006) 662-678.

[20] S.H. Ghodsypour, C. O'Brien, Decision support system for supplier selection using an integrated analytical hierarchy process and linear programming, International Journal of Production Economics 56-57 (9) (1998) 199-212.

[21] B.V. Gnedenko, Theory of probability, Taylor Francis, London, Great Britain 1998.

[22] M. Hanss, The transformation method for the Simulation and analysis of Systems with Uncertain Parameters, Fuzzy Sets and Systems 130 (3) (2002) 227-289.

[23] J. Hou, D. SU, Oriented supplier selection system for mass customization, Journal of Manufacturing Technology Management 18 (1) (2007) 54-71.

[24] C.L. Hwang, K. Yoon, Multiple Attribute Decision Making Methods and Applications, Berlin Heidelberg: Springer, New York 1981.

[25] G.R. Jahanshahloo, F. Hosseinzadeh Lotfi, N. Shoja, G. Tohidi, S. Razavyan, Ranking using l1-norm in data envelopment analysis, Applied Mathematics and Computation 153 (1) (2004) 215-224.

[26] X. Jiuping, L. Bin, W. Desheng, Rough data envelopment analysis and its application to supply chain performance evaluation, International Journal of Production Economics 122 (2) (2009) 628-638.

[27] C. Kahraman , U. Cebeci, Z. Ulukan, Multi criteria supplier selection using fuzzy AHP, Logistics Information Management 16 (6) (2003) 382-394.

[28] M. Kumar, P. Vrat, R. Shankar, A Fuzzy goal programming approach for vendor selection problem in supply chain, Computers and Industrial engineering 46 (1) (2004) 69-85.

[29] K.H. Lee, First Course on Fuzzy theory and applications, Springer-Verlag Berlin Heidelberg 2005.

[30] A. Mendoza, E. Santiago, A.R. Ravindran, A three- phase multi criteria method to supplier selection problem, International Journal of Industrial Engineering 15 (2) (2008) 195-210.

[31] G.A. Miller, The magical number seven or mine two: Some limits on our capacity of processing information, The Psychological Review 63 (2) (1956) 81-97.

[32] F. Qui, J.R. Jensen, Opening the black box of neural networks for remote sensing image classification, International Journal of Remote Sensing 25 (9) (2004) 1749-1768.

[33] R. Ramanathan, Supplier selection problem: Integrating DEA with the approaches of total cost of ownership and AHP, Supply Chain Management: An International Journal 12 (4) (2007) 258-261.

[34] T.L. Saaty, Decision making with dependence and Feedback: The Analytical Network Process, RWS Publications, Pittsburgh 1996.

[35] U. Sekaran, Research methods for business: A skill building approach (2nd edition), John Wiley Sons Incorporated, England 1992.

[36] M. Sevkli, S.C.L. Koh, S. Zamin, M. Demirbag, E. Totoglu, An application of data envelopment analytic hierarchy process for supplier selection a case study of BEKO in Turkey, International Journal of Production Research 45 (9) (2007) 1973-2003.

[37] J. Valente de Oliveira, W. Pedrycz, Advances in fuzzy clustering and its applications, John Wiley Sons, Ltd, England 2007.

[38] J. Wang, C. Cheng. , H. Kun-cheng, Fuzzy hierarchical TOPSIS for supplier selection. Applied Soft Computing 9 (1) (2009) 377-386.

[39] C.A. Weber, A data envelopment analysis approach to measuring vendor performance, Supply Chain Management 1 (1) (1996) 28-39.

[40] C.A.Weber, J.R. Current, W.C. Benton, Vendor selection criteria and methods, European Journal of Production Research 50 (1) (1991) 2-18.

[41] C.A. Weber, J.R. Current, A. Desai, Non-cooperative negotiation strategies for vendor selection, European Journal of Operational Research 108 (1) (1998) 208-223.

[42] R.D. Yussuff, K. Poh Yee, A Preliminary study on the potential use of the Analytical Hierarchical Process (AHP) to predict Advanced Manufacturing Technology (AMT) implementation, Robotic and Computer Integrated Manufacturing 17 (5) (2001) 421-427.

[43] L.A. Zadeh, Fuzzy Sets, Information and Control 8 (3) (1965) 338-353.