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Research Article

Positive-additive functional equations in non-Archimedean C^*-algebras

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Abstract

Hensel [K. Hensel, Deutsch. Math. Verein, 6 (1897), 83-88.] discovered the p-adic number as a number theoretical analogue of power series in complex analysis. Fix a prime number p. for any nonzero rational number x, there exists a unique integer $n_x \in \mathbb{Z}$ such that $x = \frac{a}{b}p^{n_x}$, where a and b are integers not divisible by p. Then $|x|_p := p^{-n_x}$ defines a non-Archimedean norm on $\mathbb{Q}$. The completion of $\mathbb{Q}$ with respect to metric $d(x, y) = |x - y|_p$, which is denoted by $\mathbb{Q}_p$, is called *p-adic number field*. In fact, $\mathbb{Q}_p$ is the set of all formal series $x = \sum_{k\geq n_x}^{\infty} a_k p^k$, where $|a_k| \leq p-1$ are integers. The addition and multiplication between any two elements of $\mathbb{Q}_p$ are defined naturally. The norm $\left|\sum_{k\geq n_x}^{\infty} a_k p^k\right|_p = p^{-n_x}$ is a non-Archimedean norm on $\mathbb{Q}_p$ and it makes $\mathbb{Q}_p$ a locally compact field. In this paper, we consider non-Archimedean C^*-algebras and, using the fixed point method, we provide an approximation of the positive-additive functional equations in non-Archimedean C^*-algebras.

Keywords : Functional equation; Fixed point; Positive-additive functional equation; Linear mapping; Non-Archimedean C^*-algebra.

1 Introduction

Park et al. [29] introduced the following functional equation:

$$f\left(\left(\sqrt{x}+\sqrt{y}\right)^2\right) = \left(\sqrt{f(x)}+\sqrt{f(y)}\right)^2$$

in the set of non-negative real numbers.

In this paper, we consider the following functional equation

$$T\left(\left(x^{\frac{1}{m}}+y^{\frac{1}{m}}\right)^m\right) = \left(T(x)^{\frac{1}{m}}+T(y)^{\frac{1}{m}}\right)^m \quad (1.1)$$

for all $x, y \in A^+$ and a fixed integer m greater than 1, which is called a *positive-additive functional equation* (see [16]). Each solution of the positive-additive functional equation is called a *positive-additive mapping*.

Note that the function $f(x) = cx$ for any $c \geq 0$ in the set of non-negative real numbers is a solution of the functional equation (1.1).

Let X be a set. A function $d : X \times X \to [0, \infty]$ is called a *generalized metric* on X if d satisfies the following conditions: for all $x, y, z \in X$,

(1) $d(x, y) = 0$ if and only if $x = y$;

(2) $d(x, y) = d(y, x)$;

(3) $d(x, z) \leq d(x, y) + d(y, z)$.

The set X with a generalized metric d is called a *generalized metric space*.

We recall a fundamental result in fixed point

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theory.

Theorem 1.1 *[[2, 10]] Let (X,d) be a complete generalized metric space and let $J : X \to X$ be a strictly contractive mapping with Lipschitz constant $L < 1$. Then, for each $x \in X$, either*

$$d(J^n x, J^{n+1}x) = \infty$$

for all $n \geq 0$ or there exists a positive integer n_0 such that

(1) *$d(J^n x, J^{n+1}x) < \infty$ for all $n \geq n_0$;*

(2) *the sequence $\{J^n x\}$ converges to a fixed point y^* of J;*

(3) *y^* is the unique fixed point of J in the set $Y = \{y \in X \mid d(J^{n_0}x, y) < \infty\}$;*

(4) *$d(y, y^*) \leq \frac{1}{1-L} d(y, Jy)$ for all $y \in Y$.*

In 1996, Isac and Th.M. Rassias [19] were the first to provide applications of stability theory of functional equations for the proof of new fixed point theorems with applications. By using the fixed point method, the stability problems of several functional equations have been extensively investigated by a number of authors (see [3], [4], [27]–[34], [38] and [1, 5, 6, 7, 8, 9, 13, 14, 15, 17, 18, 20, 21, 22, 24, 25, 26, 28, 31, 33, 36, 35, 37, 39, 40, 43, 44]).

2 non–Archimedean C^*- algebras

In this section, we consider non-Archimedean C^*-algebras. By a *non-Archimedean field* we mean a field K equipped with a function (valuation) $|\cdot|$ from K into $[0,\infty)$ such that

(a) $|r| = 0$ if and only if $r = 0$;

(b) $|rs| = |r|\,|s|$;

(c) $|r+s| \leq \max\{|r|, |s|\}$ for all $r, s \in K$.

Clearly, $|1| = |-1| = 1$ and $|n| \leq 1$ for all $n \in \mathbb{N}$. By the trivial valuation we mean the mapping $|\cdot|$ taking everything but 0 into 1 and $|0| = 0$.

Let X be a vector space over a field K with a non-Archimedean non-trivial valuation $|\cdot|$. A function $\|\cdot\|: X \to [0,\infty)$ is called a *non-Archimedean norm* if it satisfies the following conditions:

(a) $\|x\| = 0$ if and only if $x = 0$;

(b) for any $r \in K, x \in X$, $\|rx\| = |r|\|x\|$;

(c) the strong triangle inequality (ultrametric) holds; namely,

$$\|x+y\| \leq \max\{\|x\|, \|y\|\}$$

for all $x, y \in X$.

Then $(X, \|\cdot\|)$ is called a *non-Archimedean normed space*. From the fact that

$$\begin{aligned} &\|x_n - x_m\| \\ \leq\ &\max\{\|x_{j+1} - x_j\|: m \leq j \leq n-1\} \end{aligned}$$

for all $n, m \in \mathbb{N}$ with $n > m$ holds, a sequence $\{x_n\}$ is a Cauchy sequence if and only if $\{x_{n+1} - x_n\}$ converges to zero in a non-Archimedean normed space. By a *complete non-Archimedean normed space* we mean one in which every Cauchy sequence is convergent.

For any nonzero rational number x, there exists a unique integer $n_x \in \mathbb{Z}$ such that $x = \frac{a}{b}p^{n_x}$, where a and b are integers not divisible by p. Then $|x|_p := p^{-n_x}$ defines a non-Archimedean norm on $\mathbb{Q}$. The completion of $\mathbb{Q}$ with respect to the metric $d(x,y) = |x-y|_p$ is denoted by $\mathbb{Q}_p$, which is called the *p-adic number field*.

A *non-Archimedean Banach algebra* is a complete non-Archimedean algebra $\mathcal{A}$ which satisfies

$$\|ab\| \leq \|a\| \cdot \|b\|$$

for all $a, b \in \mathcal{A}$. For more detailed definitions of non-Archimedean Banach algebras, refer to [7, 12, 41].

If $\mathcal{U}$ is a non-Archimedean Banach algebra, then an *involution* on $\mathcal{U}$ is a mapping $t \to t^*$ from $\mathcal{U}$ into $\mathcal{U}$ which satisfies the following conditions:

(a) $t^{**} = t$ for all $t \in \mathcal{U}$;

(b) $(\alpha s + \beta t)^* = \overline{\alpha} s^* + \overline{\beta} t^*$;

(c) $(st)^* = t^* s^*$ for $s, t \in \mathcal{U}$.

If, in addition, $\|t^*t\| = \|t\|^2$ for all $t \in \mathcal{U}$, then $\mathcal{U}$ is a *non-Archimedean C^*-algebra.*

Definition 2.1 **([11])** *Let A be a non-Archimedean C^*-algebra and $x \in A$ be a self-adjoint element, i.e., $x^* = x$. Then x is said to be positive if it is of the form yy^* for some $y \in A$.*

The set of positive elements of A is denoted by A^+. Note that A^+ is a closed convex cone (see [11]). It is well-known that, for a positive element x and a positive integer n, there exists a unique positive element $y \in A^+$ such that $x = y^n$. We denote y by $x^{\frac{1}{n}}$ (see [16]).

Throughout this paper, let A^+ and B^+ be the sets of positive elements in non-Archimedean C^*-algebras A and B, respectively. Assume that m is a fixed integer greater than 1.

3 Approximation of the positive-additive functional equation 1.1: fixed point approach

In this section, we investigate the positive-additive functional equation 1.1 in non-Archimedean C^*-algebras.

Lemma 3.1 **([29])** *Let $T : A^+ \to B^+$ be a positive-additive mapping satisfying 1.1. Then T satisfies*

$$T(2^{mn}x) = 2^{mn}T(x)$$

for all $x \in A^+$ and $n \in \mathbb{Z}$.

Using the fixed point method, we provide an approximation of the positive-additive functional equation 1.1 in non-Archimedean C^*-algebras. Note that the fundamental ideas in the proofs of the main results in this section are contained in [2, 3, 4].

Theorem 3.1 *Let $\varphi : A^+ \times A^+ \to [0, \infty)$ be a function such that there exists $L < 1$ with*

$$|2|\varphi\left(\frac{x}{2}, \frac{y}{2}\right) \leq L\varphi(x, y) \tag{3.2}$$

for all $x, y \in A^+$. Let $f : A^+ \to B^+$ be a mapping satisfying

$$\begin{aligned} &\left\| f\left(\left(x^{\frac{1}{m}} + y^{\frac{1}{m}}\right)^m\right)\right. \\ - &\left.\left(f(x)^{\frac{1}{m}} + f(y)^{\frac{1}{m}}\right)^m \right\| \\ \leq &\ \varphi(x, y) \end{aligned} \tag{3.3}$$

for all $x, y \in A^+$. Then there exists a unique positive-additive mapping $T : A^+ \to A^+$ satisfying 1.1 and

$$\|f(x) - T(x)\| \leq \frac{L}{|2|^m - |2|^m L}\varphi(x, x) \tag{3.4}$$

for all $x \in A^+$.

Proof. It follows from (3.2) that

$$\lim_{m\to\infty} |2|^m \varphi\left(\frac{x}{2^m}, \frac{y}{2^m}\right) = 0 \tag{3.5}$$

for all $x, y \in A^+$. Letting $y = x$ in (3.3), we get

$$\|f(2^m x) - 2^m f(x)\| \leq \varphi(x, x) \tag{3.6}$$

for all $x \in A^+$. Consider the set

$$X := \{g : A^+ \to B^+\}$$

and introduce the generalized metric on X as follows:

$$\begin{aligned} &d(g, h) = \inf\{\mu \in \mathbb{R}_+ \\ &: \|g(x) - h(x)\| \leq \mu\varphi(x, x), \quad \forall x \in A^+\}, \end{aligned}$$

where, as usual, $\inf \phi = +\infty$.

It is easy to show that (X, d) is complete (see [23]).

Now, we consider the linear mapping $J : X \to X$ such that

$$Jg(x) := 2^m g\left(\frac{x}{2^m}\right)$$

for all $x \in A^+$. Let $g, h \in X$ be given such that $d(g, h) \leq \varepsilon$. Then we have

$$\|g(x) - h(x)\| \leq \varepsilon\varphi(x, x)$$

for all $x \in A^+$ and so

$$\left\|2^m g\left(\frac{x}{2^m}\right) - 2^m h\left(\frac{x}{2^m}\right)\right\| \leq \varepsilon|2|^m\varphi\left(\frac{x}{2^m}, \frac{x}{2^m}\right)$$

for all $x \in A^+$ and $m \in \mathbb{N}$. Hence it follows that

$$\|Jg(x) - Jh(x)\| \leq L\varepsilon\varphi(x, x)$$

for all $x \in A^+$ and So $d(g,h) \leq \varepsilon$ implies that $d(Jg, Jh) \leq L\varepsilon$. This means that

$$d(Jg, Jh) \leq Ld(g,h)$$

for all $g, h \in X$. It follows from (3.6) that

$$\left\| f(x) - 2^m f\left(\frac{x}{2^m}\right) \right\| \leq \frac{L}{|2|^m}\varphi(x,x)$$

for all $x \in A^+$ and so $d(f, Jf) \leq \frac{L}{|2|^m}$.

By Theorem 1.1, there exists a mapping $T : A^+ \to B^+$ satisfying the following:

(1) T is a fixed point of J, i.e.,

$$T\left(\frac{x}{2^m}\right) = \frac{1}{2^m}T(x) \tag{3.7}$$

for all $x \in A^+$. The mapping T is a unique fixed point of J in the set

$$M = \{g \in X : d(f,g) < \infty\}.$$

This implies that T is a unique mapping satisfying (3.7) such that there exists $\mu \in (0,\infty)$ satisfying

$$\|f(x) - T(x)\| \leq \mu\varphi(x,x)$$

for all $x \in A^+$;

(2) $d(J^n f, T) \to 0$ as $n \to \infty$. This implies the equality

$$\lim_{n\to\infty} 2^{mn} f\left(\frac{x}{2^{mn}}\right) = T(x)$$

for all $x \in A^+$;

(3) $d(f,T) \leq \frac{1}{1-L}d(f, Jf)$, which implies the inequality

$$d(f,T) \leq \frac{L}{|2|^m - |2|^m L}.$$

This implies that the inequality (3.4) holds. By (3.3) and (3.5), we have

$$\begin{aligned} &|2|^{mn} \left\| f\left(\frac{\left(x^{\frac{1}{m}} + y^{\frac{1}{m}}\right)^m}{2^{mn}}\right) - \right. \\ &\left(\left(2^{mn} f\left(\frac{x}{2^{mn}}\right)\right)^{\frac{1}{m}} \right. \\ &\left.\left. + \left(2^{mn} f\left(\frac{y}{2^{mn}}\right)\right)^{\frac{1}{m}}\right)^m \right\| \\ \leq\ & |2|^{mn}\varphi\left(\frac{x}{2^{mn}}, \frac{y}{2^{mn}}\right) \end{aligned}$$

for all $x, y \in A^+$ and $n \in \mathbb{N}$ and so

$$\begin{aligned} &\left\| T\left(\left(x^{\frac{1}{m}} + y^{\frac{1}{m}}\right)^m\right) \right. \\ - &\left. \left(T(x)^{\frac{1}{m}} + T(y)^{\frac{1}{m}}\right)^m \right\| \\ = &\ 0 \end{aligned}$$

for all $x, y \in A^+$. Thus the mapping $T : A^+ \to B^+$ is positive-additive. This completes the proof.

Corollary 3.1 *Let $p > 1$ and θ_1, θ_2 be non-negative real numbers, and let $f : A^+ \to B^+$ be a mapping such that*

$$\begin{aligned} &\left\| f\left(\left(x^{\frac{1}{m}} + y^{\frac{1}{m}}\right)^m\right) \right. \\ - &\left. \left(f(x)^{\frac{1}{m}} + f(y)^{\frac{1}{m}}\right)^m \right\| \\ \leq &\ \theta_1(\|x\|^p + \|y\|^p) + \theta_2 \cdot \|x\|^{\frac{p}{2}} \cdot \|y\|^{\frac{p}{2}} \end{aligned} \tag{3.8}$$

for all $x, y \in A^+$. Then there exists a unique positive-additive mapping $T : A^+ \to B^+$ satisfying 1.1 and

$$\|f(x) - T(x)\| \leq \frac{|2|\theta_1 + \theta_2}{|2|^{mp} - |2|^m}||x||^p$$

for all $x \in A^+$.

Proof. The proof follows from Theorem 3.1 by taking

$$\varphi(x,y) = \theta_1(\|x\|^p + \|y\|^p) + \theta_2 \cdot \|x\|^{\frac{p}{2}} \cdot \|y\|^{\frac{p}{2}}$$

for all $x, y \in A^+$. Then we can choose $L = |2|^{m-mp}$ and we get the desired result.

Theorem 3.2 *Let $\varphi : A^+ \times A^+ \to [0,\infty)$ be a function such that there exists $L < 1$ with*

$$\varphi(x,y) \leq |2|L\varphi\left(\frac{x}{2}, \frac{y}{2}\right)$$

for all $x, y \in A^+$. Let $f : A^+ \to B^+$ be a mapping satisfying (3.3). Then there exists a unique positive-additive mapping $T : A^+ \to A^+$ satisfying 1.1 and

$$\|f(x) - T(x)\| \leq \frac{1}{|2|^m - |2|^m L}\varphi(x,x)$$

for all $x \in A^+$.

Proof. Let (X, d) be the generalized metric space defined in the proof of Theorem 3.1. Consider the linear mapping $J : X \to X$ such that

$$Jg(x) := \frac{1}{2^m} g\left(2^m x\right)$$

for all $x \in A^+$. It follows from (3.6) that

$$\left\| f(x) - \frac{1}{2^m} f(2^m x) \right\| \leq \frac{1}{2^m} \varphi(x, x)$$

for all $x \in A^+$ and so $d(f, Jf) \leq \frac{1}{2^m}$.

The rest of the proof is similar to the proof of Theorem 3.1.

Corollary 3.2 *Let $0 < p < 1$ and θ_1, θ_2 be non-negative real numbers and let $f : A^+ \to B^+$ be a mapping satisfying (3.8). Then there exists a unique positive-additive mapping $T : A^+ \to B^+$ satisfying 1.1 and*

$$\| f(x) - T(x) \| \leq \frac{|2|\theta_1 + \theta_2}{|2|^m - |2|^{mp}} \|x\|^p$$

for all $x \in A^+$.

Proof. The proof follows from Theorem 3.2 by taking

$$\varphi(x, y) = \theta_1(\|x\|^p + \|y\|^p) + \theta_2 \cdot \|x\|^{\frac{p}{2}} \cdot \|y\|^{\frac{p}{2}}$$

for all $x, y \in A^+$. Then we can choose $L = |2|^{mp-m}$ and we get the desired result.

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