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Research Article

A new algorithm for solving Van der Pol equation based on piecewise spectral Adomian decomposition method

S. Gh. Hosseini *†, E. Babolian ‡, S. Abbasbandy §

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Abstract

In this article, a new method is introduced to give approximate solution to Van der Pol equation. The proposed method is based on the combination of two different methods, the spectral Adomian decomposition method (SADM) and piecewise method, called the piecewise Adomian decomposition method (PSADM). The numerical results obtained from the proposed method show that this method is an effective, accurate and powerful tool for solving Van der Pol equation and, the comparison show that the proposed technique is in good agreement with the numerical results obtained using Runge-Kutta method. The advantage of piecewise spectral Adomian decomposition method over piecewise Adomian decomposition method is that it does not need to calculate complex integrals. Another merit of this method is that, unlike the spectral method, it does not require the solution of any linear or nonlinear system of equations. Furthermore, the proposed method is easy to implement and computationally very attractive.

Keywords : Van der Pol equation; Spectral Adomian decomposition method; Piecewise method; Runge-Kutta method.

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1 Introduction

Many problems in chemistry, biology, physics and engineering are related to nonlinear self-excited oscillators [9, 16]. The Van der Pol oscillator is a mode developed to describe the behavior of nonlinear vacuum tube circuits in the relatively early days of the development of electronics technology. Balthazar Van der Pol introduced his equation in order to describe triode oscillations in electrical circuits[17, 6]. Van der Pol discovered stable oscillations, now known as limit cycles, in electrical circuits employing vacuum tubes. When these circuits are driven near the limit cycle, they become entrained, i.e. the driving signal pulls the current along with it. The mathematical model for the system is a well-known second order ordinary differential equation with cubic non linearity of Van der Pol equation. Since then thousands of papers have been published attempting to achieve better approximations to the solutions occurring in such non linear systems. The Van der Pol oscillator is a classical example of self-oscillatory system and is now considered as a very useful mathematical model that can be used in much more complicated and modified systems. However, this equation is so important for mathematicians, physicists and engineers to be extensively studied. The Van der

*Corresponding author. ghasem602@yahoo.com
†Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran.
‡Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran.
§Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran.

Pol equation has a long history of being used in both the physical and biological sciences. For instance, Fitzhugh [8] and Nagumo[15] used the equation in a planner field as a model for action potential of neurons. Additionally, the equation has also been extended to the Burridge–Knopoff model which characterizes earthquake faults with viscous friction[2].

During the first half of the twentieth century, Balthazar van der Pol pioneered the field of radio telecommunication[4, 3, 18]. The Van der Pol equation with large value of nonlinearity parameter has been studied by Cartwright and Littlewood [5]; they showed that the singular solution exists. Furthermore, analytically, Lavinson [14], analyzed the Van der Pol equation by substituting the cubic non linearity for piecewise linear version and showed that the equation has singular solution, as well. In addition, the Van der Pol equation for Nonlinear Plasma Oscillations has been studied by Hafeez and Chifu[11]; they showed that the Van der Pol equation depends on the damping coefficient μ which has a varying behaviour.

In the recent work, the Van der Pol equation will be described in section 2. In section 3, a new method, called the piecewise spectral Adomian decomposition method(PSADM), will be presented. Solution of Van der Pol equation by PSADM will be interpreted in section 4, and finally in section 5, the detailed conclusion is provided.

2 The van der Pol Equation

In this section a description of the Van der Pol equation can be expressed[12]. The Van der Pol oscillator is a self-maintained electrical circuit consisted of an inductor (L), a capacitor initially charged with a capacitance (C) and a non-linear resistance (R); all of which are connected in series as indicated in Figure 1. This oscillator was invented by Van der Pol while he was trying to discover a new way to model the oscillations of a self maintained electrical circuit. The characteristic intensity-tension U_R of the nonlinear resistance (R) is given as:

$$U_R = -R_0 i_0 [\frac{i}{i_0} - \frac{1}{3}(\frac{i}{i_0})^3] \tag{2.1}$$

where i_0 and R_0 are the current and the resistance of the normalization, respectively. This

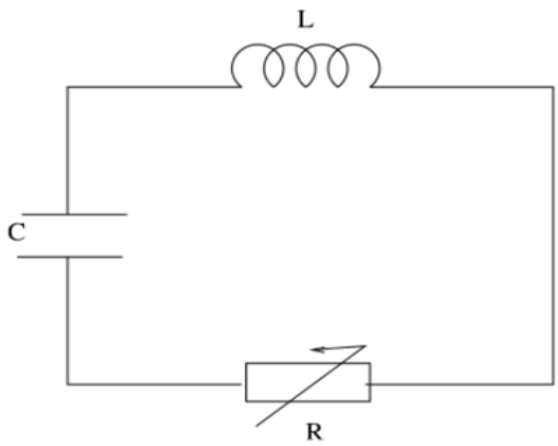

Figure 1: Electric circuit modelizing the Van der Pol oscillator in an autonomous regime.

non-linear resistance can be obtained by using the operational amplifier (op-amp). By applying the links law to Figure 1 we have:

$$U_L + U_R + U_C = 0 \tag{2.2}$$

where U_L and U_C are the tension to the limits of the inductor and capacitor, respectively, and are defined as:

$$U_L = L\frac{di}{d\tau}, \tag{2.3}$$

$$U_C = \frac{1}{C}\int i d\tau. \tag{2.4}$$

Substituting (2.1), (2.3) and (2.4) in (2.2), we have:

$$L\frac{di}{d\tau} - R_0 i_0 [\frac{i}{i_0} - \frac{1}{3}(\frac{i}{i_0})^3] + \frac{1}{C}\int i d\tau = 0. \tag{2.5}$$

Differentiating (2.5) with respect to τ, we have

$$L\frac{d^2 i}{d\tau^2} - R_0[1 - \frac{i^2}{i_0^2}]\frac{di}{d\tau} + \frac{i}{C} = 0. \tag{2.6}$$

Setting

$$y = \frac{i}{i_0} \tag{2.7}$$

and

$$t = \omega_e \tau \tag{2.8}$$

where $\omega_e = \frac{1}{\sqrt{LC}}$ is an electric pulsation, we get:

$$\frac{d}{d\tau} = \omega_e \frac{d}{dt} \tag{2.9}$$

$$\frac{d^2}{d\tau^2} = \omega_e^2 \frac{d^2}{dt^2}. \tag{2.10}$$

Substituting (2.9) and (2.10) in (2.6), yields

$$\frac{d^2y}{dt^2}R_0\sqrt{\frac{C}{L}}(1-y^2)\frac{dy}{dt}+y=0. \qquad (2.11)$$

By setting $\mu = R_0\sqrt{\frac{C}{L}}$, Eq.(2.11) takes dimensional form as follows

$$y'' - \mu(1-y^2)y' + y = 0. \qquad (2.12)$$

where μ is the scalar parameter indicating the strength of the nonlinear damping, and (2.12) is called the Van der Pol equation in the autonomous regime. This equation is expressed with the initial conditions as:

$$y(0) = \alpha, \qquad y'(0) = \beta. \qquad (2.13)$$

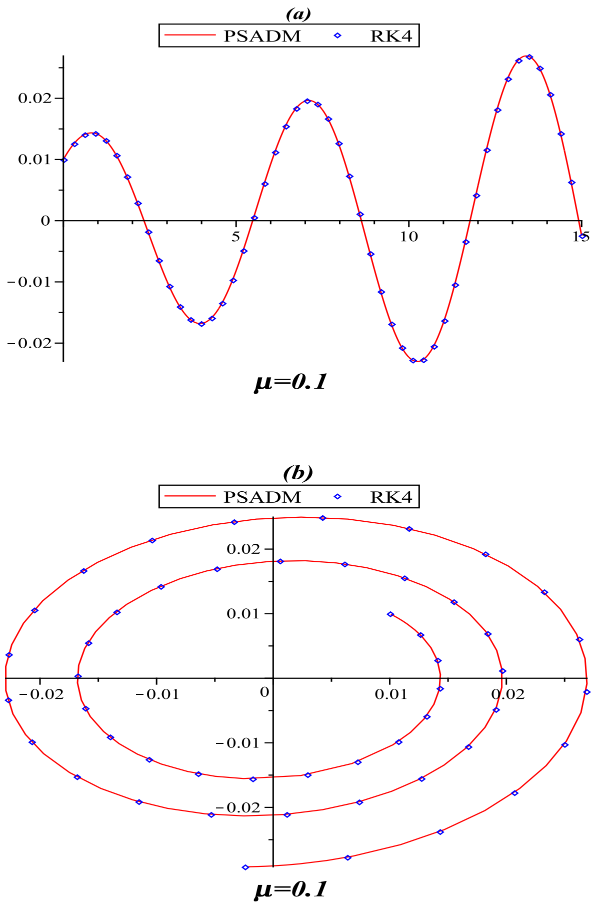

Figure 2: ***(a)***: plot of displacement y versus time t; ***(b)***: phase plane. Solid line: PSADM; Solid circle: RK4.

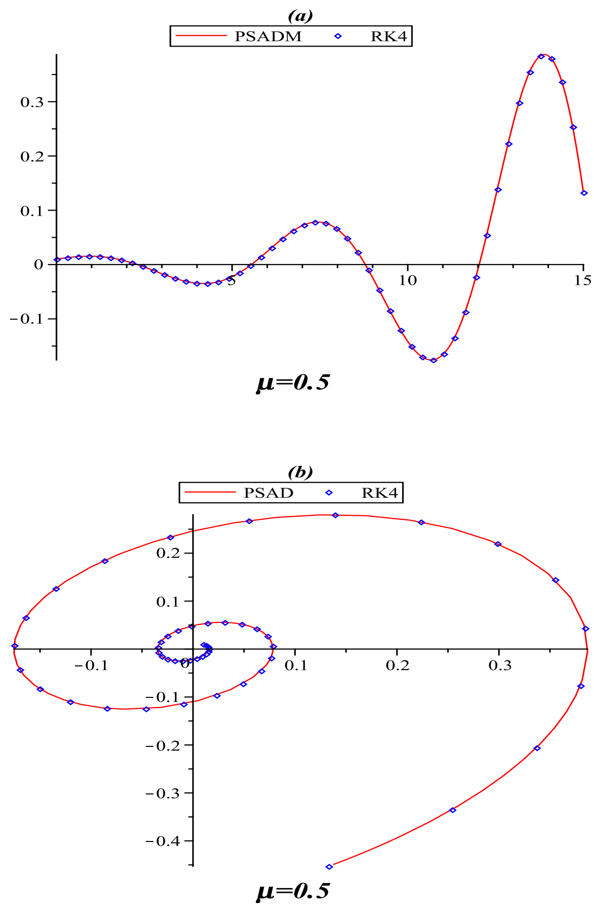

Figure 3: ***(a)***: plot of displacement y versus time t; ***(b)***: phase plane. Solid line: PSADM; Solid circle: RK4.

3 The methodology

3.1 Adomian decomposition method for Van der Pol Equation

The Adomian decomposition method (ADM) is a semi-analytical method for ordinary and partial nonlinear differential equations. The details of this method are presented by G. Adomian[1]. The ADM presented the equation in an operator form by considering the highest-order of derivative in the problem. Hence, in this problem we choose the differential operator $\mathcal{L}$ in terms of y'', then (2.12) can be rewritten in the following form:

$$\mathcal{L}y = \mu y' - \mu y^2 y' - y, \qquad (3.14)$$

where the differential operator $\mathcal{L}$ is

$$\mathcal{L} = \frac{d^2}{dt^2}. \qquad (3.15)$$

The inverse operator $\mathcal{L}^{-1}$ is

$$\mathcal{L}^{-1}(.) = \int_0^t \int_0^t (.)dtdt. \qquad (3.16)$$

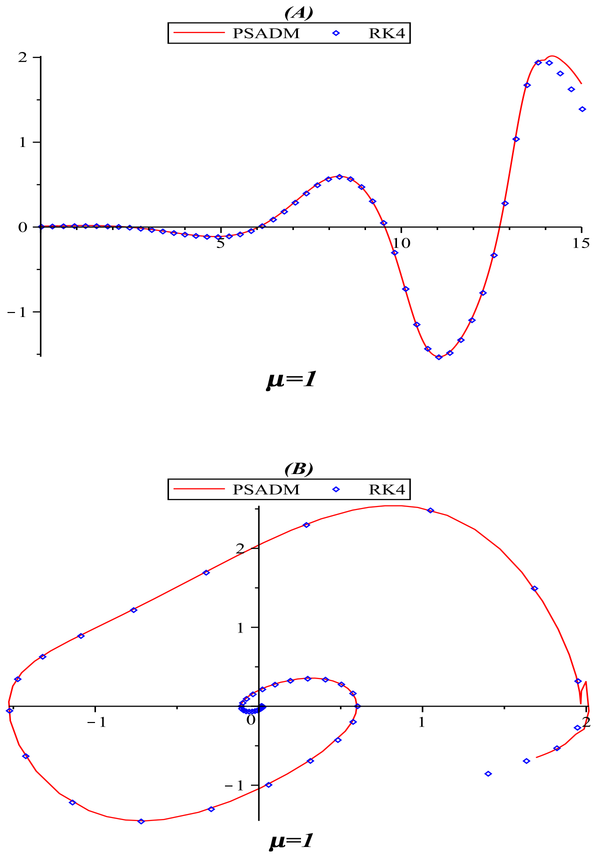

Figure 4: ***(A)***: plot of displacement y versus time t; ***(B)***: phase plane. Solid line: PSADM; Solid circle: RK4.

Operating with $\mathcal{L}^{-1}$ on (3.14) , it follows

$$y(t) = y(0) - ty'(0) + \mu\mathcal{L}^{-1}y'(t)$$
$$-\mu\mathcal{L}^{-1}y^2(t)y'(t) - \mathcal{L}^{-1}y(t). \quad (3.17)$$

According to the ADM, the solution $y(t)$ is represented by the decomposition series

$$y(t) = \sum_{n=0}^{\infty} y_n(t), \quad (3.18)$$

and the nonlinear part of Eq. (3.17) is represented by the decomposition series

$$N(y(t)) = y^2(t)y'(t) = \sum_{n=0}^{\infty} A_n(t), \quad (3.19)$$

where $A_n(t)$, the Adomian polynomials, are obtained as follows:

$$A_n = \frac{1}{n_!}\frac{d^n}{d\lambda^n}[N(\sum_{i=0}^{n}\lambda^i y_i)]_{\lambda=0}$$

$$n = 0, 1, 2, ... \quad (3.20)$$

By setting (3.18) and (3.19) in (3.17), we obtain

$$\sum_{n=0}^{\infty} y_n(t) = y(0) - ty'(0) + \mu\mathcal{L}^{-1}\frac{d}{dt}(\sum_{n=0}^{\infty} y_n(t))$$
$$-\mu\mathcal{L}^{-1}(\sum_{n=0}^{\infty} A_n(t) - \mathcal{L}^{-1}(\sum_{n=0}^{\infty} y_n(t)). \quad (3.21)$$

To specify the components $y_n(x)$, ADM wich indicates the use of recursive relation will be applied,

$$y_0(t) = y(0) - ty'(0), \quad (3.22)$$

and

$$y_{n+1}(t) = \mu\mathcal{L}^{-1}\frac{d}{dt}y_n(t) - \mu\mathcal{L}^{-1}A_n(t) -$$
$$\mathcal{L}^{-1}y_n(t), \quad n \geq 0. \quad (3.23)$$

In practice, not all terms of the series in Eq. (3.23) need to be determined and hence, the solution will be approximated by the truncated series

$$\psi_k(t) = \sum_{n=0}^{k-1} y_n(t)$$

with

$$\lim_{k\to\infty} \psi_k(t) = y(t). \quad (3.24)$$

3.2 Chebyshev polynomials

Chebyshev polynomials of the first kind are orthogonal with respect to the weight function $\omega(x) = \frac{1}{\sqrt{1-x^2}}$ on the interval$[-1, 1]$, and satisfy the following recursive formula:

$$T_0(x) = 1, \;\; T_1(x) = x,$$
$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x),$$
$$n = 1, 2, 3, ... \quad (3.25)$$

This system is orthogonal basis with weight function $\omega(t) = (1 - x^2)^{-1/2}$ and orthogonality property:

$$\int_{-1}^{1} T_n(x)T_m(x)(1 - x^2)^{-1/2}dx = \frac{\pi}{2}c_n\delta_{nm}, \quad (3.26)$$

where $c_0 = 2$, $c_n = 1$ for $n \geq 1$ and δ_{nm} is the Kronecker delta function.

A function $u(x) \in L^2_\omega(-1,1)$ can be expanded by Chebyshev polynomials as follows:

$$u(x) = {\sum_{j=0}^{\infty}}' u_j T_j(x), \tag{3.27}$$

where the coefficients u_j are

$$u_j = \frac{2}{\pi} < u(x), T_j(x) >_\omega$$
$$j = 0, 1, 2, ...$$

(3.28)

Here, $< .,. >_\omega$ is the inner product of $L^2_\omega(-1,1)$. The grid (interpolation) points are chosen to be the exterma

$$x_i = -\cos(\frac{i\pi}{m}), \quad i = 0, 1, ..., m, \tag{3.29}$$

of the $T_m(x)$. The following approximation of the function $u(x)$ can be introduced:

$$u(x) \simeq u^{[m]}(x) = \sum_{j=0}^{m} \tilde{u}_j T_j(x), \tag{3.30}$$

where $\tilde{u}_j$ are the Chebyshev coefficients. These coefficients are determined as follows:

$$\tilde{u}_j = \frac{2(-1)^j}{m\tilde{c}_j} \sum_{i=0}^{m} \frac{1}{\tilde{c}_i} u(x_i) \cos(\frac{\pi i j}{m}),$$

j=0,1,...,m ,
(3.31)

where

$$\tilde{c}_i = \begin{cases} 2, & i = 0, m \\ 1, & 1 \leq i \leq m-1. \end{cases} \tag{3.32}$$

3.3 Spectral Adomian decomposition method(SADM)

At first, based on initial conditions (2.13), the initial approximation $y_0(t) = \alpha + \beta t$ is selected. By applying iteration formula (3.23), the following will be obtained

$$y_1(t) = -\mathcal{L}^{-1}(A_0(t)). \tag{3.33}$$

From (3.30), the function $y_1(x)$ on $[0, \xi]$ can be approximated as follows:

$$y_1(t) \simeq y_1^{[m]}(t) = \sum_{j=0}^{m} \tilde{y}_{1j} T_j(\frac{2}{\xi}t - 1), \tag{3.34}$$

where $\tilde{y}_{1j}$ are the Chebyshev coefficients derived from (3.31) as follows:

$$\tilde{y}_{1j} = \frac{2(-1)^j}{m\tilde{c}_j} \sum_{i=0}^{m} \frac{1}{\tilde{c}_i} y_1(\tilde{t}_i). \cos(\frac{\pi i j}{m}),$$
$$j = 0, 1, ..., m,$$
$$\tilde{t}_i = \frac{\xi}{2}(t_i + 1), \quad i = 0, ..., m. \tag{3.35}$$

For finding the unknown coefficients $y_1(\tilde{t}_i)$, $i = 0, 1, ..., m$, by substituting the grid points $\tilde{t}_i$, $i = 0, 1, ..., m$ in (3.31), the following will be concluded:

$$y_1(\tilde{t}_i) = -\mathcal{L}^{-1}(A_0(\tilde{t}_i)), \tag{3.36}$$

from (3.35) and (3.36)

$$\tilde{y}_{1j} = \frac{2(-1)^j}{m\tilde{c}_j} \sum_{i=0}^{m} \frac{-1}{\tilde{c}_i} \mathcal{L}^{-1}(A_0(\tilde{t}_i)). \cos(\frac{\pi i j}{m}),$$
$$j = 0, 1, ..., m, \tag{3.37}$$

can be gained. Therefore, from (3.34) and (3.37) the approximation of $y_1(t)$ can be obtained.
For finding the approximation of $y_2(t)$ of (3.23), the following will be gained:

$$y_2(t) = -\mathcal{L}^{-1}(A_1), \tag{3.38}$$

in a similar way, the function $y_2(t)$ on $[0, \xi]$ can be approximated as

$$y_2(t) \simeq y_2^{[m]}(t) = \sum_{j=0}^{m} \tilde{y}_{2j} T_j(\frac{2}{\xi}t - 1), \tag{3.39}$$

where

$$\tilde{y}_{2j} = \frac{2(-1)^j}{m\tilde{c}_j} \sum_{i=0}^{m} \frac{1}{\tilde{c}_i} y_2(\tilde{t}_i). \cos(\frac{\pi i j}{m}),$$
$$j = 0, 1, ..., m, \tag{3.40}$$

similarly, for finding the unknown coefficients $y_2(\tilde{t}_i)$, $i = 0, 1, ..., m$, by substituting the grid points $\tilde{t}_i, i = 0, 1, ..., m$ in (3.38)

$$y_2(\tilde{t}_i) = -\mathcal{L}^{-1}(A_1(\tilde{t}_i)), \tag{3.41}$$

can be concluded, therefore, from (3.40) and (3.41)

$$\tilde{y}_{2j} = \frac{2(-1)^j}{m\tilde{c}_j} \sum_{i=0}^{m} \frac{-1}{\tilde{c}_i} \mathcal{L}^{-1}(A_1(\tilde{t}_i)).\cos(\frac{\pi ij}{m}),$$
$$j = 0, 1, ..., m, \tag{3.42}$$

and from (3.39) and (3.42) the approximation of $y_2(t)$ can be obtained.
Generally, for $n \geq 2$, according to the above method, the approximation of $y_n(x)$ will be achieved as follows:

$$y_n(t) \simeq y_n^{[m]}(t) = \sum_{j=0}^{m} \tilde{y}_{nj} T_j(\frac{2}{\xi}t - 1), \tag{3.43}$$

where

$$\tilde{y}_{nj} = \frac{2(-1)^j}{m\tilde{c}_j} \sum_{i=0}^{m} \frac{-1}{\tilde{c}_i} \mathcal{L}^{-1}(A_{n-1}(\tilde{t}_i)).\cos(\frac{\pi ij}{m}),$$
$$j = 0, 1, ..., m. \tag{3.44}$$

At the end, $y_0^{[m]}(t)+y_1^{[m]}(t)+y_2^{[m]}(t)+...+y_n^{[m]}(t)$ is the (n, m)-term approximation of the series solution.

3.4 Piecewise spectral Adomian decomposition method

It is clear that the hybrid spectral Adomian decomposition method is ideally suited for solving differential equations whose solutions do not change rapidly or oscillate over small parts of the domain of the governing problem. For solving strongly-nonlinear oscillators on large domains, we introduce the main idea of the piecewise spectral Adomian decomposition method.

We first divide the interval $[0, \xi]$ into subintervals $\Omega_r = [\xi_{r-1}, \xi_r]$ where $r = 1, ..., M$ and $\Delta_r = \xi_r - \xi_{r-1}$. Moreover, we define the linear mappings $\psi_r : \Omega_r \to [-1, 1]$ by

$$\psi_r(t) = \frac{2(t - \xi_{r-1})}{\Delta_r} - 1,$$
$$r = 1, 2, ..., M, \tag{3.45}$$

and choose grid points $\tilde{t}_i^r$ as:

$$\tilde{t}_i^r = \psi_r^{-1}(t_i) = \frac{\Delta_r}{2}(t_i + 1) + \xi_{r-1},$$
$$r = 1, 2, ..., M, \ \ i = 0, 1, ..., m, \tag{3.46}$$

where $\psi_r^{-1}(t_i)$ is inverse map of $\psi_r(t)$. On $\Omega_1 = [\xi_0, \xi_1]$, let $y_{1,0}(t) = y(\xi_0)+y'(\xi_0)(t-\xi_0) = \alpha+\beta t$ and for $k \geq 1$

$$y_{1,k}(t) \simeq y_{1,k}^{[m]}(t) = \sum_{j=0}^{m} \tilde{y}_{kj}^{(1)} T_j(\psi_1(t)), \tag{3.47}$$

where $\tilde{y}_{kj}^{(1)}$ are the Chebyshev coefficients. These coefficients are determined by

$$\tilde{y}_{kj}^{(1)} = \frac{2(-1)^j}{m\tilde{c}_j} \sum_{i=0}^{m} \frac{1}{\tilde{c}_i} y_{1,k}(\tilde{t}_i^1).\cos(\frac{\pi ij}{m}),$$
$$j = 0, 1, ..., m. \tag{3.48}$$

For finding the unknown coefficients $y_{1,k}(\tilde{t}_i^1)$, $i = 0, 1, ..., m$, by substituting the grid points $\tilde{t}_i^1$, $i = 0, 1, ..., m$ in (3.31), the following will be concluded:

$$y_{1,k}(\tilde{t}_i^1) = -\mathcal{L}^{-1}(A_0(\tilde{t}_i^1)), \tag{3.49}$$

from (3.48) and (3.49)

$$\tilde{y}_{kj}^{(1)} = \frac{2(-1)^j}{m\tilde{c}_j} \sum_{i=0}^{m} \frac{-1}{\tilde{c}_i} \mathcal{L}^{-1}(A_{k-1}(\tilde{t}_i^1)).\cos(\frac{\pi ij}{m}),$$
$$j = 0, 1, ..., m. \tag{3.50}$$

Now, the (n, m)-term approximation on $\Omega_1 = [0, \xi_1]$ is introduced as follows:

$$\Phi_{1,n}^{[m]}(t) = y_{1,0}^{[m]}(t) + y_{1,1}^{[m]}(t) + y_{1,2}^{[m]}(t) + ... + y_{1,n}^{[m]}(t). \tag{3.51}$$

Similarly, on $\Omega_2 = [\xi_1, \xi_2]$, we have $y_{2,0}(t) = \Phi_{1,n}^{[m]}(\xi_1) + \Phi_{1,n}^{'[m]}(\xi_1)(t - \xi_1)$ and for $k \geq 1$,

$$y_{2,k}(t) \simeq y_{2,k}^{[m]}(t) = \sum_{j=0}^{m} \tilde{y}_{kj}^{(2)} T_j(\psi_2(t)), \tag{3.52}$$

where $\tilde{y}_{kj}^{(2)}$ are the Chebyshev coefficients. These coefficients are determined as follows:

$$\tilde{y}_{kj}^{(2)} = \frac{2(-1)^j}{m\tilde{c}_j} \sum_{i=0}^{m} \frac{1}{\tilde{c}_i} y_{2,k}(\tilde{t}_i^1).\cos(\frac{\pi ij}{m}),$$
$$j = 0, 1, ..., m. \tag{3.53}$$

For finding the unknown coefficients $y_{2,k}(\tilde{t}_i^2)$, $i = 0, 1, ..., m$, by substituting the grid points $\tilde{t}_i^2$, $i = 0, 1, ..., m$ in (3.31), the following will be concluded:

$$y_{2,k}(\tilde{t}_i^2) = -\mathcal{L}^{-1}(A_0(\tilde{t}_i^2)), \tag{3.54}$$

from (3.53) and (3.54)

$$\tilde{y}_{kj}^{(2)} = \frac{2(-1)^j}{m\tilde{c}_j} \sum_{i=0}^{m} \frac{-1}{\tilde{c}_i} \mathcal{L}^{-1}(A_{k-1}(\tilde{t}_i^2)).\cos(\frac{\pi i j}{m}),$$

$$j = 0, 1, ..., m. \tag{3.55}$$

Now, the (n, m)-term approximation on $\Omega_2 = [\xi_1, \xi_2]$ is introduced as:

$$\Phi_{2,n}^{[m]}(t) = y_{2,0}^{[m]}(t) + y_{2,1}^{[m]}(t) + y_{2,2}^{[m]}(t) + ... + y_{2,n}^{[m]}(t). \tag{3.56}$$

In a similar way, we can obtain the (n,m)-order approximation $\Phi_{s,n}^{[m]}(t) = \sum_{k=0}^{n} y_s k(t)$ on $\Omega_s = [\xi_{s-1}, \xi_s]$, s=3,4,...,M. Finally, the approximation solution $y(t)$ in entire interval $[0, \xi]$ is given by

$$y(t) \simeq \Phi_n^{[m]}(t) = \begin{cases} \Phi_{1,n}^{[m]}(t), & t \in \Omega_1, \\ \Phi_{2,n}^{[m]}(t), & t \in \Omega_2, \\ \vdots & \\ \Phi_{M,n}^{[m]}(t), & t \in \Omega_M, \end{cases} \tag{3.57}$$

where $\Omega_1 = [0, \xi_1]$, $\Omega_M = [\xi_{M-1}, \xi]$ and $\Omega_s = [\xi_{s-1}, \xi_s]$ for $s = 2, 3, ..., M-1$. It is clear that $[0, \xi] = \bigcup_{s=1}^{M} \Omega_s$. According to [13] SADM is convergent on $[0, \xi]$, consequently, it is concluded that PSADM is convergent.

4 Numerical results

According to (2.12) and (2.13), the Van der Pol equation in standard form is as follows:

$$y'' - \mu(1 - y^2)y' + y = 0, \quad y(0) = \alpha, \quad y'(0) = \beta, \tag{4.58}$$

where μ is a scalar parameter indicating the degree of nonlinearity and the strength of the damping. If $\mu = 0$, the equation reduces to the equation of simple harmonic motion $y'' + y = 0$. For $\mu > 0$, when $y > 1$, $-\mu(1-y^2)$ is positive and the system behaves as a damped (energy dissipating) system, and when $y < 1$, $-\mu(1 - y^2)$ is negative and the system behaves as a self excited (energy absorbing) system.

To demonstrate the validity and applicability of the PSADM, we compare the approximate results given by the presented method in three cases $\mu = 0.1$, $\mu = 0.5$ and $\mu = 1(\Delta_r = \Delta = 0.5, m = 15, n = 10, M = 30)$ with numerical solution obtained by $4th$ order Runge-Kutta method $(\Delta t = 0.001)$ on interval $[0, 15]$. In all cases, we take $\alpha = \beta = 0.01$, and the results are shown in Figures 2, 3 and 4, respectively. From these figures we find that PSADM results are close to the numerical solutions obtained using $RK4$.

5 Conclusion

In this paper, the piecewise Adomian decomposition method was introduced and this method was applied for solving Van der Pol equation, an strongly non-linear and swinging equation. The numerical results presented show that the proposed method is a powerful method for solving Van der Pol equation. The advantage of piecewise spectral Adomin decomposition method over piecewise Adomian decomposition method is that it does not need to calculate complex integrals. Another advantage of the proposed method is that, unlike the spectral method, there is no need to solve algebraic equations (linear or nonlinear). Furthermore, the result obtained from PSADM for Van der Pol equation shows that this method is an accurate and efficient method for solving such equations.

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Seyed Ghasem Hosseini is Ph.D of applied mathematics . He is member of Department of mathematics, Ashkezar Branch, Islamic Azad University, Yazd, Iran. His research interests include numerical solution of functional equations.

Esmail Babolian is Professor of applied mathematics and Faculty of Mathematical sciences and computer, Kharazmy University, Tehran, Iran. Interested in numerical solution of functional Equations, Numerical linear algebra and mathematical education.

Saeid Abbasbandy is Professor of applied mathematics and Faculty of Mathematical sciences, Imam Khomeini international University, Ghazvain, Iran. Interested in numerical solution of functional Equations and Fuzzy numerical Analysis.