



Steady Mixed Convection Flow of Water at 4°C Along A Non-Isothermal Vertical Moving Plate with Transverse Magnetic Field

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Abstract

This paper is focused on the investigation of heat transfer characteristics of mixed convection flow of water at 4°C along a continuously moving vertical non-isothermal, non-conducting plate in the presence of a transverse magnetic field. The governing equations of continuity, momentum and energy for this boundary-layer flow are transformed into self-similar ordinary differential equations using the similarity transformation technique. The resulting coupled and non-linear ordinary differential equations are solved using the fourth-order Runge-Kutta method along with the shooting technique. The fluid flow and heat transfer characteristics are discussed and presented graphically. The values of the skin-friction coefficient and the Nusselt number at the plate surface are obtained for various values of the physical parameters and presented in tabular form and the physical aspects of these results are discussed.

Keywords : Steady flow; free stream; MHD; mixed convection; moving plate.

1 List of Symbols

A : a constant

C_f : skin-friction coefficient

C_p : specific heat at constant pressure

f : dimensionless stream function

g : acceleration due to gravity of the Earth

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Gr : Grashof Number $\left\{ \frac{2g\gamma(T_w - T_\infty)^2 x^3}{\nu^2} \right\}$

G : buoyancy parameter $\left\{ \frac{Gr}{Re^2} \right\}$

Ha : Hartmann number $\left\{ \sqrt{\frac{\sigma B_0^2 x^2}{\rho\nu}} \right\}$

M : Magnetic parameter $\left\{ \frac{Ha^2}{Re} \right\}$

m : parameter (= 0, 1, 2)

Nu : Nusselt number

Pr : Prandtl number $\left\{ \frac{\mu C_p}{\kappa} \right\}$

Re : Reynolds number $\left\{ \frac{U_r x}{\nu} \right\}$

T : temperature of the fluid

T_∞ : temperature of free stream

T_w : temperature of the plate

u, v : velocity components along x - and y -directions

U_r : reference velocity

U_w : velocity of plate

U_∞ : free stream velocity

x, y : Cartesian coordinates

Greek Letters

γ : coefficient of thermal expansion for water at 4°C

η : similarity variable

ρ : electrical conductivity of fluid

λ : velocity parameter $\left\{ \frac{U_\infty}{U_w + U_\infty} \right\}$

θ : dimensionless temperature $\left\{ \frac{T - T_\infty}{T_w - T_\infty} \right\}$

μ : coefficient of viscosity

κ : coefficient of thermal conductivity

ν, ρ : kinematic viscosity $\left\{ \frac{\mu}{\rho} \right\}$, fluid density, respectively

ψ : stream function

Superscript

' : differentiation with respect to η

2 Introduction

Mixed convection flow along a flat surface through free stream have many applications such as polymer sheets continuously drawn from a die, cooling of metallic plates in a bath, hot rolling, continuous casting, extrusion of plastic sheets, etc. For cooling purposes, cold water can be utilized. At ordinary temperature and atmospheric pressure, the variation in density is given by $\Delta \rho = \rho\beta(T_w - T_\infty)$ where $\beta = 2.07 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$. But when water is at 4°C, it has maximum density and the density variation is represented by $\Delta \rho = \rho\gamma(T_w - T_\infty)$ where $\gamma = 8.0 \times 10^{-6} \text{ }^\circ\text{C}^{-2}$ (Goren [10]). Sakiadis [18] introduced the concept of a continuously-moving surface through a fluid and studied the boundary layer behaviour. Gorla and Reddy [11] discussed flow and heat transfer characteristics for a fluid moving parallel to a continuous surface. Takhar and Ram [20] studied MHD free convection flow of water at 4°C through a porous medium on a vertical surface. Chen [5,6] considered forced convection over a continuously-moving horizontal isothermal/non-isothermal plate in free stream and discussed the effects of suction and injection on the

heat transfer characteristics. Raptis and Perdikis [17] studied the unsteady free convection flow of water at 4°C past a moving porous plate. Bhargava et al. [3] considered the mixed convection flow with viscous dissipation on fluid from a continuous surface in a parallel moving free stream. Ganesan and Palani [9] discussed free convection of water at its maximum density past an inclined plate.

The present paper aims to investigate steady mixed convection flow on a continuously moving non-isothermal vertical plate in the same direction as the free stream but with different velocities, taking into the account the density variation suggested by Goren [10], in the presence of transverse magnetic field.

3 Formulation of the Problem

Consider a continuous vertical plate at a temperature $T_w = T_\infty + Ax^m$ issuing from a slot at a constant velocity U_w and is being cooled by a free stream of water at 4°C having a constant velocity U_∞ . The origin of the coordinate system is placed at the point where the plate issues into the fluid medium. The x -axis is taken along the plate and the y -axis is taken perpendicular to the plate. A magnetic field of intensity B_0 is applied in the y -direction. It is assumed that the magnetic Reynolds number is small so that the induced magnetic field is neglected. Also, the electric field owing to polarization of charges and the Hall Effect are also neglected. The governing boundary-layer equations of continuity, momentum and energy (Jeffery[14], Bansal[2]), incorporating the usual Boussinesq approximation (Takhar and Ram [20]), for flow of water at 4°C and neglecting the viscous dissipation term are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (3.1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\gamma (T - T_\infty)^2 + \frac{\sigma B_0^2}{\rho} (U_\infty - u), \quad (3.2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T}{\partial y^2}. \quad (3.3)$$

where all the parameters are defined in the List of Symbols section.

The appropriate boundary conditions for this problem are

$$\begin{aligned} y = 0 : u = U_w, v = 0, T = T_w = T_\infty + Ax^m, \\ y \rightarrow \infty, u = U_\infty, T = T_\infty. \end{aligned} \quad (3.4)$$

where $m = 0$ means that the plate is isothermal, $m = 1$ or 2 means that the plate temperature variation is linear or quadratic with respect to x , respectively.

4 Method of Solution

Introducing the stream function $\psi(x, y)$ such that

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x}, \quad (4.5)$$

where $\psi = \nu \sqrt{Re} f(\eta)$ and $\eta = \frac{y}{x} \sqrt{Re}$, is the similarity variable. It is observed that equation (3.1) is identically satisfied by $\psi(x, y)$. Substituting equation (4.5) into equations

(3.2) and (3.3), the resulting non-linear coupled ordinary differential equations are given by

$$f''' + \frac{1}{2}ff'' + G\theta^2 + M\left(\frac{U_\infty}{U_r} - f'\right) = 0, \quad (4.6)$$

and

$$\theta'' + Pr\left(\frac{1}{2}\theta'f - m\theta f'\right) = 0. \quad (4.7)$$

In equations (4.6) and (4.7) $\theta = \frac{T-T_\infty}{T_w-T_\infty}$. It is noted that $Pr = 11.4$ for water at 4°C . The corresponding boundary conditions become

$$f(0) = 0, f'(0) = \left(\frac{U_w}{U_r}\right), \theta(0) = 1, f'(\infty) = \left(\frac{U_\infty}{U_r}\right), \theta(\infty) = 0, \quad (4.8)$$

where U_r is a reference velocity and in the available literature it is taken as:

$$\begin{array}{lll} U_r = U_w & \text{if} & U_w > U_\infty \\ & \text{or} & \\ U_r = U_\infty & \text{if} & U_w < U_\infty \end{array} \quad (4.9)$$

which leads to formation of two sets of boundary conditions as given below

$$f(0) = 0, f'(0) = 1, \theta(0) = 1, f'(\infty) = \left(\frac{U_\infty}{U_w}\right), \theta(\infty) = 0, \quad \text{if} \quad U_w > U_\infty \quad (4.10)$$

and

$$f(0) = 0, f'(0) = \left(\frac{U_w}{U_\infty}\right), \theta(0) = 1, f'(\infty) = 1, \theta(\infty) = 0, \quad \text{if} \quad U_w < U_\infty \quad (4.11)$$

Following Afzal et al. [1], in the present work, U_r is taken as given below

$$U_r = U_w + U_\infty \quad (4.12)$$

which forms one set of boundary conditions as follows:

$$f(0) = 0, f'(0) = 1 - \lambda, \theta(0) = 1, f'(\infty) = \lambda, \theta(\infty) = 1. \quad (4.13)$$

where $\lambda = \frac{U_\infty}{U_\infty + U_w}$ is the velocity parameter. It should be noted that when the plate velocity and the free stream velocity are in the same direction, this corresponds to $0 < \lambda < 1$. In particular, when $0 < \lambda < 0.5$ the plate velocity is higher than the free stream velocity, at $\lambda = 0.5$, the plate velocity equals to the free stream velocity and when $0.5 < \lambda < 1$ the plate velocity is less than the free stream velocity.

5 Special Cases

1. For $M = 0.0$, $G = 0.0$ and $\lambda = 0.0$ or 1.0 the equation (4.6) reduces to equation obtained by Sakiadis[18] or Blasius[4], respectively for flow on a continuous horizontal plate.
2. For $M = 0.0$, $G = 0.0$ and $\lambda = 0.5$, there would be no formation of boundary layer as the fluid velocity equals the plate velocity.

Of special importance for this flow and heat transfer problem are the skin-friction coefficient and the Nusselt number. These are defined next.

6 Skin-Friction Coefficient

The skin-friction coefficient at the plate is given by

$$C_f = \frac{\tau}{\frac{1}{2}\rho U_r^2} = 2Re^{-\frac{1}{2}} f''(0), \quad (6.14)$$

where $\tau = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}$ is the shear stress.

7 Nusselt Number

The rate of heat transfer in terms of the Nusselt number at the plate is given by

$$Nu = \frac{qx}{\kappa(T_w - T_\infty)} = -Re^{\frac{1}{2}} \theta'(0), \quad (7.15)$$

where $q = -\kappa \left(\frac{\partial T}{\partial y} \right)_{y=0}$.

8 Numerical Method

The governing boundary-layer equations (4.6) and (4.7) with the boundary conditions (4.13) are solved numerically using the fourth-order Runge-Kutta technique (Jain et al. [13], Krishnamurthy and Sen [16] and Jain [12]) along with shooting technique (Conte and Boor [7]). The equations (4.6) and (4.7) are reduced to a set of simultaneous differential equations by defining new variables, as given below

$$\frac{df}{d\eta} = \phi_1(\eta, f, u, v) = u, f(0) = 0 \quad (8.16)$$

$$\frac{du}{d\eta} = \phi_2(\eta, f, u, v) = v, u(0) = f'(0) = 1 - \lambda \quad (8.17)$$

$$\frac{dv}{d\eta} = \phi_3(\eta, f, u, v) = -\frac{1}{2}fv - G[\theta(\eta)]^2 - M(\lambda - u), v(0) = f''(0) = A^{(0)} \quad (8.18)$$

$$\frac{d\theta}{d\eta} = \phi_4(\eta, \theta, w) = w, \theta(0) = 1 \quad (8.19)$$

$$\frac{dw}{d\eta} = \phi_5(\eta, \theta, w) = -Pr\left(\frac{1}{2}wf - m\theta u\right), w(0) = \theta'(0) = B^{(0)} \quad (8.20)$$

The values of $v(0)$ and $w(0)$ are unknown. Therefore, their initial guess values are (say), $A^{(0)}$ and $B^{(0)}$, respectively. Now, applying the Runge-Kutta method for a system of equations, then the system of differential equations (8.16) to (8.20) is solved and the values of $f''(\infty)$ and $\theta'(\infty)$ are looked into. The shooting method (Conte and Boor [7]) is then applied in such a way that the initial guess of $f''(0)$ and $\theta'(0)$ are improved to get the correct values of $f''(\infty)$ and $\theta'(\infty)$. The process is iterative and carried up to a stage, when reasonable accuracy is obtained such that $|A^{(k+1)} - A^{(k)}| < 10^{-5}$ and $|B^{(k+1)} - B^{(k)}| < 10^{-5}$.

9 Results and Discussion

Numerical values of $f''(0)$ and $-\theta'(0)$ obtained in the present paper are compared with various previously published results when $M = 0.0$, $G = 0.0$, $\lambda = 0.0 / 1.0$, $m = 0.0, 1.0, 2.0$ and presented through Tables 1 and 2. The results are in good agreement, which in turn validates the results produced by the numerical scheme used in the present paper. It is seen from Table 3 that with the increase in the value of λ , $f''(0)$ increases irrespective of the value of m . Physically, negative value of $f''(0)$ implies that plate exerts drag on the fluid while positive value of $f''(0)$ means that the fluid exerts drag on the plate. The increase in the value of λ means increase in the free stream velocity, therefore the drag increases. The value of $-\theta'(0)$ decreases with the increase in the value of λ , irrespective of value of m . It means that the heat loss from the plate to the fluid is higher for higher values of λ . Furthermore, it is observed that with the increase in the value of m at a given λ , $f''(0)$ decreases slightly, whereas $-\theta'(0)$ increases considerably. Thus, the heat loss from plate to the fluid decreases with the increase in m .

Table 4 shows that for $\lambda = 0.2$, $f''(0)$ and $-\theta'(0)$ decrease while for $\lambda = 0.8$, $f''(0)$ and $-\theta'(0)$ increase with the increase in the magnetic parameter M . Thus, the flow and heat transfer characteristics are different when the plate velocity is higher or less than the free stream velocity. A similar behaviour is obtained for different value of m . Table 5 reflects that with the increase in the buoyancy parameter G , the values of $f''(0)$ and $-\theta'(0)$ increase. This totally agrees with natural phenomena as buoyancy assists the free convective flow. Similar flow and heat transfer characteristics are seen for different values of λ and m .

Figure 1 shows the fluid velocity profiles at different values of λ . Figure 2 is a representation of the dimensionless fluid velocity ($\frac{u-U_\infty}{U_w-U_\infty}$) profiles within the range $1 \rightarrow 0$, which reveals that for different values of λ the boundary-layer thickness is almost the same. Figures 3 and 4 show that a change in the value of the parameter m has no significant effect on the fluid velocity for a low value of the buoyancy parameter G . Figures 5-7 reveal that the fluid temperature decreases with the decrease in the value of λ , which would mean that more cooling of plate would take place at large values of λ . Figure 8 shows that with the increase in the parameter m , the fluid temperature and the thermal boundary-layer thickness decrease. Hence, more cooling is observed when the value of m is low. It is observed from figure 9 that with the increase in the magnetic parameter M , the fluid velocity decreases for $\lambda = 0.2$ (i.e. when the plate velocity is higher than the free stream velocity) which agrees to the fact that the Lorentz force is a retarding force and it increases with the increase in the magnetic parameter within the boundary layer. A contrast is observed from figure 10 when $\lambda = 0.8$ (i.e. the plate velocity is less than the free stream velocity), the fluid velocity increases with increases in the magnetic parameter. This is attributed to the fact that the higher free stream velocity overcomes the effect of the Lorentz force. It is also seen from these figures (9 and 10) that the boundary-layer thickness decreases with the increase in the magnetic parameter. Figures 11-13 show that with the increase in the magnetic parameter, the fluid temperature increases. Thus, the heat flux from the plate surface to the fluid would get reduced, which in turn, would slow down the cooling of the plate surface. However, the effect of the magnetic parameter on the fluid temperature diminishes with the increase in the value of m , which can be observed with decreased separation in the fluid temperature profiles as the value of m increases. Figures 14-16 reveal the opposite trend to the behaviour as discussed above for temperature distribution.

It is seen from figures 17-19 that the fluid velocity increases within the boundary layer with the increase in the buoyancy parameter G , which is true because the buoyancy parameter assists free convective flow. A similar trend is viewed in figures 20-22 for $\lambda = 0.8$. Also, it can be suitably added that with the increase in the parameter m , the effect of the buoyancy parameter diminishes. Figures 23 and 24 when compared with figures 3 and 4 reveal an interesting fluid velocity characteristic that, at a high value of the buoyancy parameter G , the parameter m has significant effect and it is clearly seen that with the increase in the parameter m , the fluid velocity decreases. Figures 25-27 show that for $\lambda = 0.2$, the fluid temperature decreases with increases in the buoyancy parameter with a remark that the profoundness of the buoyancy parameter reduces with the increase in the parameter m . A similar trend occurs for the case when $\lambda = 0.8$, as is seen from figures 28-30.

10 Conclusions

The problem of steady mixed convection boundary-layer flow of water at 4°C along a non-isothermal continuously stretched vertical flat plate in the presence of magnetic field is investigated. The governing partial differential equations are converted into ordinary differential equations by using a suitable similarity transformation, and solved numerically by employing the fourth-order Runge-Kutta integration scheme along with the shooting technique. The following observations are concluded:

1. The heat loss from the plate to the fluid is higher for higher values of the velocity parameter λ .
2. The heat loss from plate to the fluid decreases with the increase in the parameter m .
3. When the velocity parameter $\lambda < 0.5$, the skin-friction coefficient and the rate of heat transfer decrease while for a velocity parameter $\lambda > 0.5$, the skin-friction coefficient and the rate of heat transfer increase with the increase in the magnetic parameter.
4. The fluid velocity decreases with the increase in the magnetic parameter when $\lambda < 0.5$, but for $\lambda > 0.5$ the fluid velocity increases with the increase in the magnetic parameter, which is in full agreement with physical phenomenon.
5. The fluid temperature increases with the increase in the magnetic parameter, thereby slowing cooling of the plate.
6. The variation in the parameter m has no significant effect on the fluid velocity for a low value of the buoyancy parameter. However, for a large buoyancy parameter, with the increase in the parameter m , the fluid velocity decreases.

It is hoped that the present investigation will serve as a vehicle for understanding more complex problems involving the physical effects.

Table 1. Comparison of numerical values of $f''(0)$ with previously published results.

$\lambda = 0.0, M = 0.0, G = 0.0.$			$\lambda = 1.0, M = 0.0, G = 0.0.$	
Fox et al. [8]	Chen [5]	Present results	Korovkin & Andrievskii [15]	Present results
-0.4437	-0.4438	-0.44374	0.33205	0.33205

Table 2. Comparison of numerical values of $-\theta'(0)$ with previously published results when

$$\lambda = 0.0, M = 0.0, G = 0.0.$$

Pr	Soundalgekar and Murthy [21]			Chen [7]			Jacobi [22]	Present results		
	$m = 0.0$	$m = 1.0$	$m = 2.0$	$m = 0.0$	$m = 1.0$	$m = 2.0$	$m = 0.0$	$m = 0.0$	$m = 1.0$	$m = 2.0$
0.7	0.3508	0.8028	1.2111	0.3509	0.8029	1.2111	0.3492	0.3492	0.8026	1.1210
10	1.6808	3.4515	4.6431	1.6802	3.4517	4.6411	1.679	1.6802	3.4518	4.6411

Table 3. Numerical values of $f''(0)$ and $-\theta'(0)$ when $M = 0.5$ and $G = 0.5$.

	$m = 0.0$		$m = 1.0$		$m = 2.0$	
	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
$\lambda = 0.2$	-0.38623	1.58764	-0.42343	3.26682	-0.43712	4.39849
$\lambda = 0.4$	-0.05009	1.43992	-0.08966	2.90221	-0.10429	3.88191
$\lambda = 0.6$	0.27975	1.28865	0.23783	2.50760	0.22231	3.31288
$\lambda = 0.8$	0.60323	1.13503	0.55935	2.07670	0.54320	2.67612

Table 4. Numerical values of $f''(0)$ and $-\theta'(0)$ when $G = 0.5$.

	$m = 0.0$		$m = 1.0$		$m = 2.0$	
$\lambda = 0.2$	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
$M = 0.0$	-0.15329	1.64337	-0.19205	3.33460	-0.20599	4.46825
$M = 1.0$	-0.54572	1.55092	-0.58148	3.22111	-0.59489	4.35120
$M = 2.0$	-0.78448	1.49899	-0.81790	3.15434	-0.83080	4.28147
$\lambda = 0.8$	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
$M = 0.0$	0.37924	1.05688	0.32339	1.94860	0.30308	2.52526
$M = 1.0$	0.75352	1.17531	0.71465	2.14419	0.70020	2.75665
$M = 2.0$	0.97937	1.22438	0.94601	2.22871	0.93343	2.85880

Table 5. Numerical values of $f''(0)$ and $-\theta'(0)$ when $M = 0.5$.

	$m = 0.0$		$m = 1.0$		$m = 2.0$	
$\lambda = 0.2$	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
$G = 0.0$	-0.48778	1.57483	-0.48778	3.25699	-0.48778	4.39056
$G = 1.0$	-0.28619	1.59994	-0.35955	3.27646	-0.38671	4.40633
$G = 2.0$	-0.09024	1.62320	-0.23314	3.29523	-0.28657	4.42175
$G = 5.0$	0.47072	1.68463	0.13643	3.34797	0.00865	4.46605
$\lambda = 0.8$	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
$G = 0.0$	0.46915	1.11030	0.46915	2.05166	0.46915	2.65260
$G = 1.0$	0.73272	1.15777	0.64760	2.10047	0.61601	2.69872
$G = 2.0$	0.98054	1.19860	0.81901	2.14480	0.75829	2.74150
$G = 5.0$	1.66155	1.29695	1.30151	2.25873	1.16332	2.85466

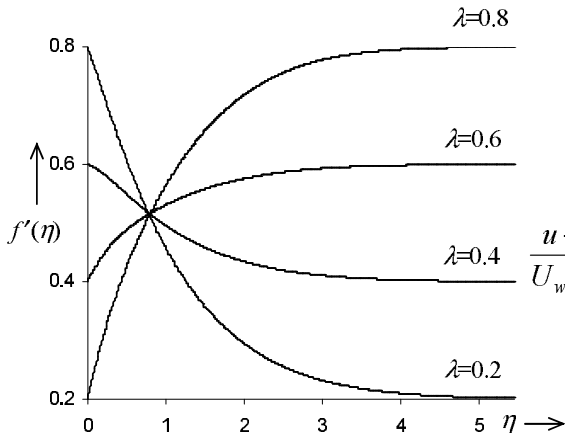


Fig.1 Velocity distribution versus η when $m=0.0$, $M=0.5$ and $G=0.5$.

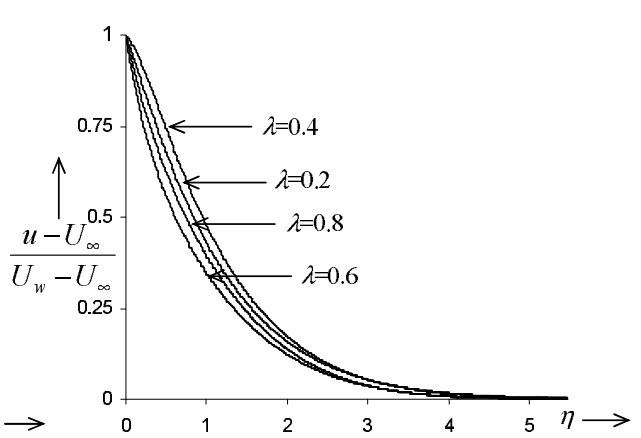


Fig.2 Dimensionless fluid velocity versus η when $m=0.0$, $M=0.5$ and $G=0.5$.

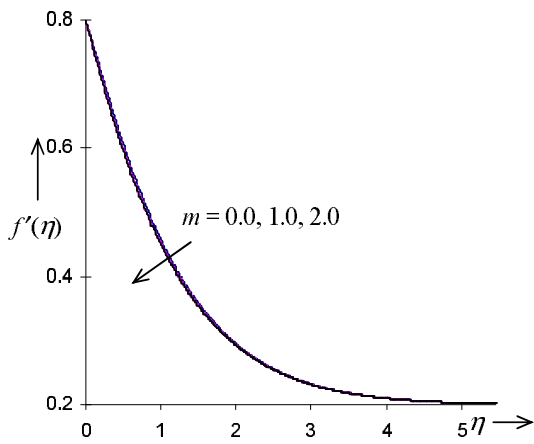


Fig.3 Velocity distribution versus η when $\lambda=0.2$, $M=0.5$ and $G=0.5$.

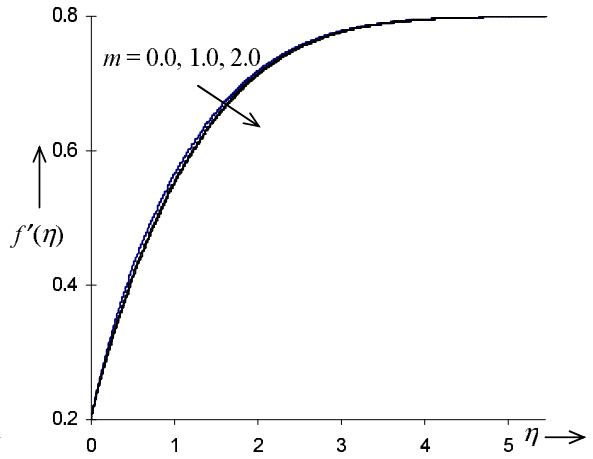


Fig.4 Velocity distribution versus η when $\lambda=0.8$, $M=0.5$ and $G=0.5$.

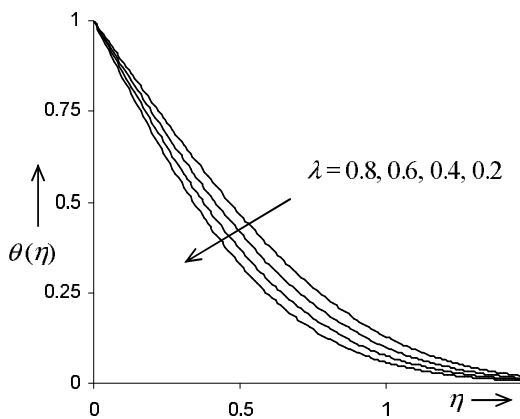


Fig.5 Temperature distribution versus η when $m=0.0$, $M=0.5$ and $G=0.5$.

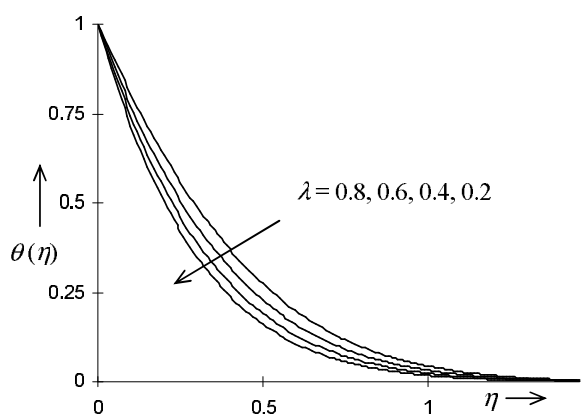


Fig.6 Temperature distribution versus η when $m=1.0$, $M=0.5$ and $G=0.5$.

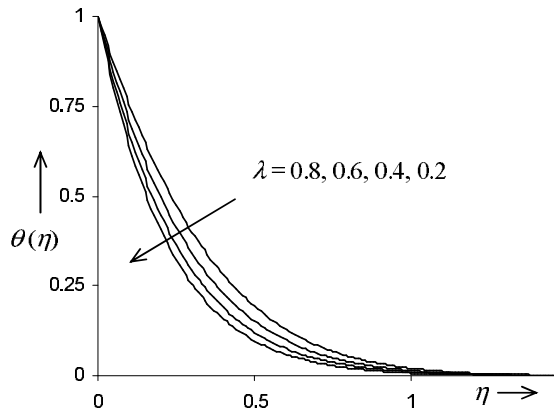


Fig.7 Temperature distribution versus η when $m=2.0, M=0.5$ and $G=0.5$.

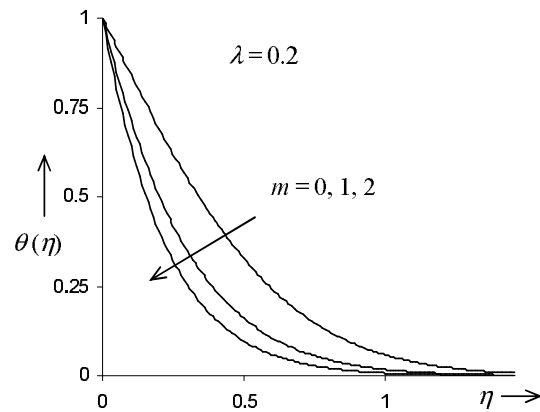


Fig.8 Temperature distribution versus η when $\lambda=0.2, M=0.5$ and $G=0.5$.

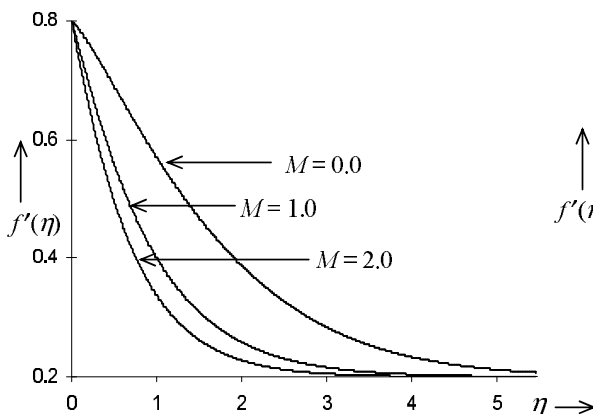


Fig.9 Velocity distribution versus η when $\lambda = 0.2, m = 0.0$ and $G = 0.5$.

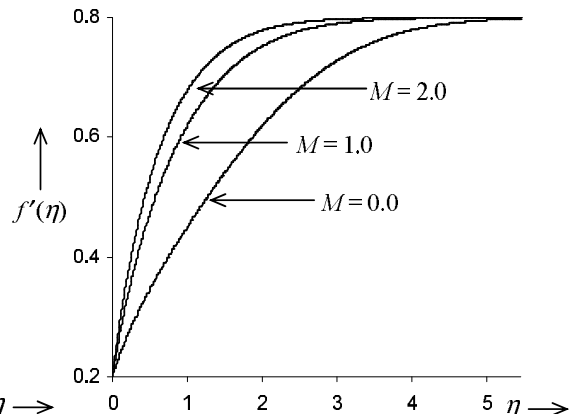


Fig.10 Velocity distribution versus η when $\lambda = 0.8, m = 0.0$ and $G = 0.5$.

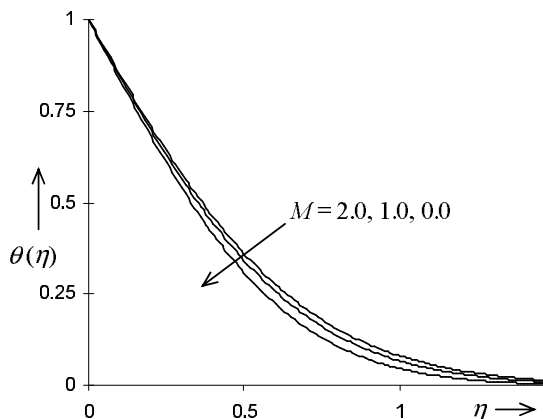


Fig.11 Temperature distribution versus η when $m = 0.0, \lambda = 0.2$ and $G = 0.5$.

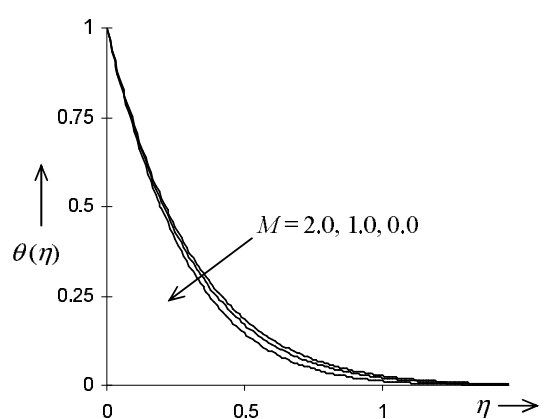


Fig.12 Temperature distribution versus η when $m = 1.0, \lambda = 0.2$ and $G = 0.5$.

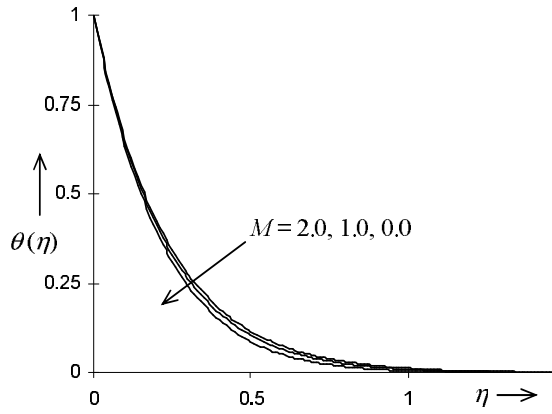


Fig.13 Temperature distribution versus η when $m=2.0$, $\lambda=0.2$ and $G=0.5$.

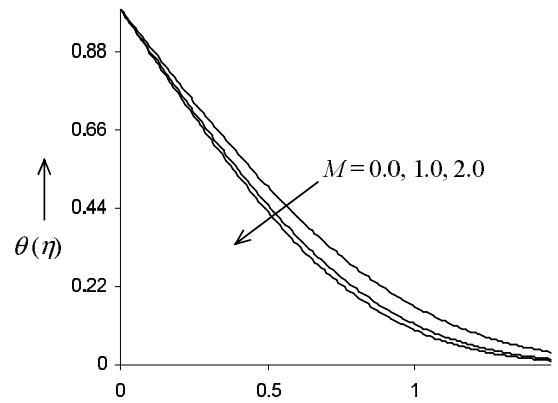


Fig.14 Temperature distribution versus η when $m=0.0$, $\lambda=0.8$ and $G=0.5$.

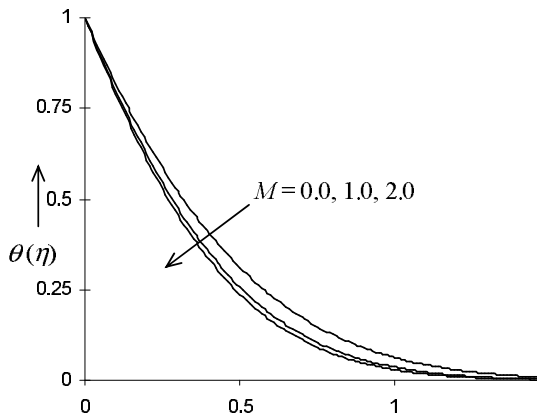


Fig.15 Temperature distribution versus η when $m=1.0$, $\lambda=0.8$ and $G=0.5$.

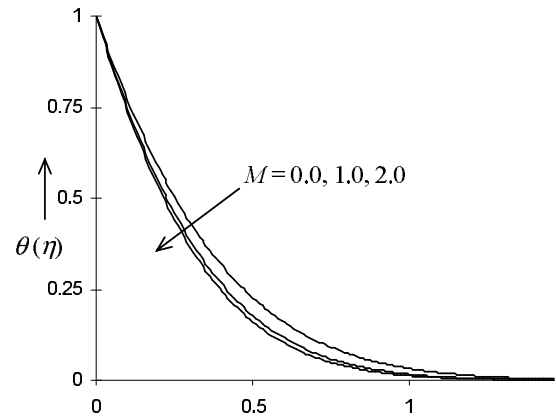


Fig.16 Temperature distribution versus η when $m=2.0$, $\lambda=0.8$ and $G=0.5$.

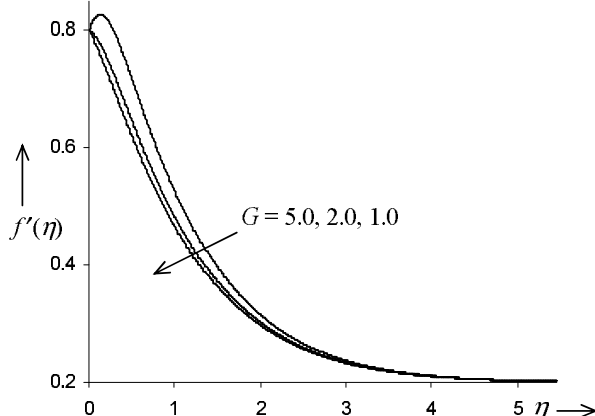


Fig.17 Velocity distribution versus η when $\lambda=0.2$, $m=0.0$ and $M=0.5$.

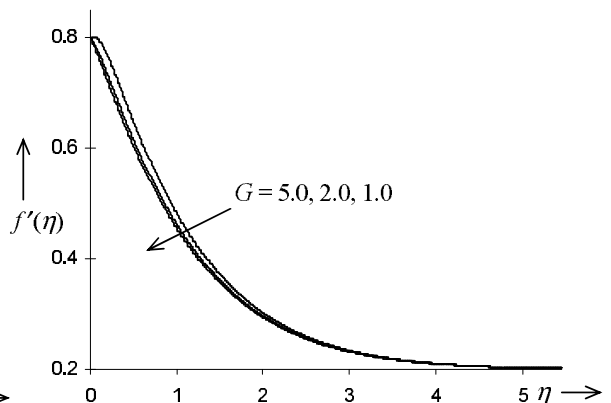


Fig.18 Velocity distribution versus η when $\lambda=0.2$, $m=1.0$ and $M=0.5$.

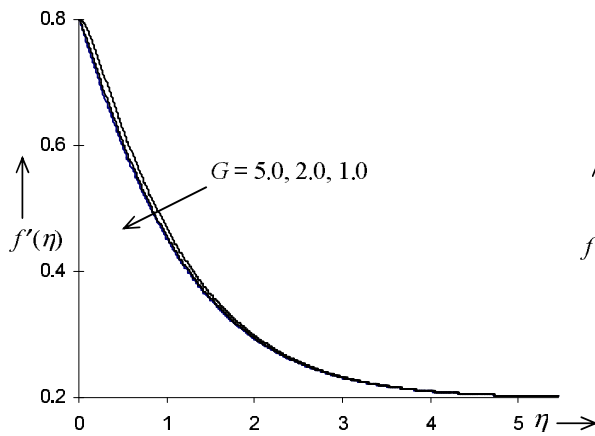


Fig.19 Velocity distribution versus η when $\lambda = 0.2$, $m = 2.0$ and $M = 0.5$.

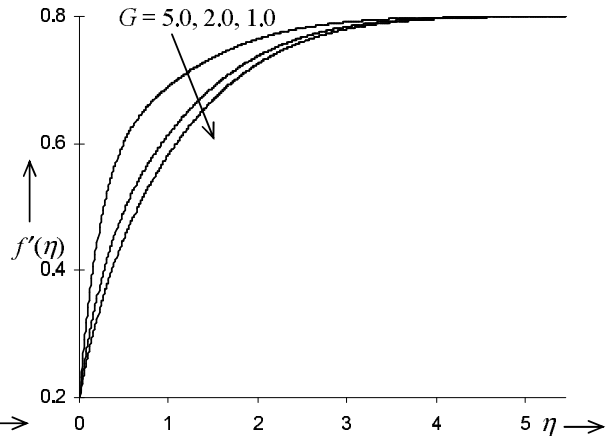


Fig.20 Velocity distribution versus η when $\lambda = 0.8$, $m = 0.0$ and $M = 0.5$.

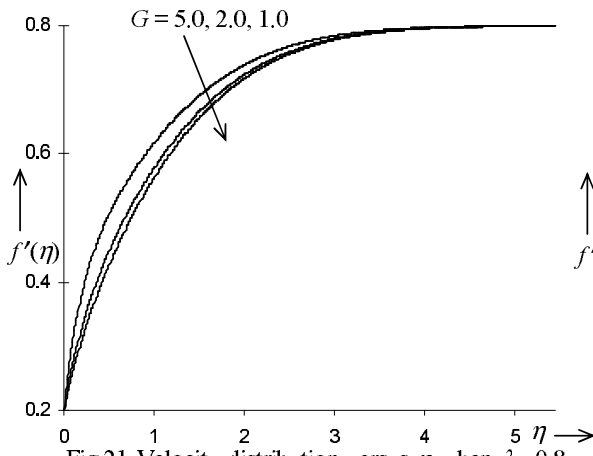


Fig.21 Velocity distribution versus η when $\lambda = 0.8$, $m = 1.0$ and $M = 0.5$.

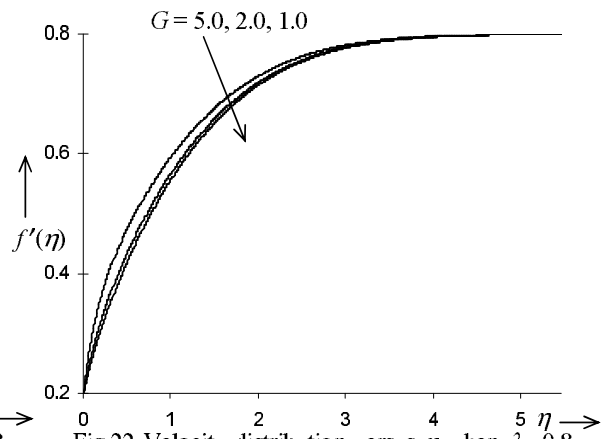


Fig.22 Velocity distribution versus η when $\lambda = 0.8$, $m = 2.0$ and $M = 0.5$.

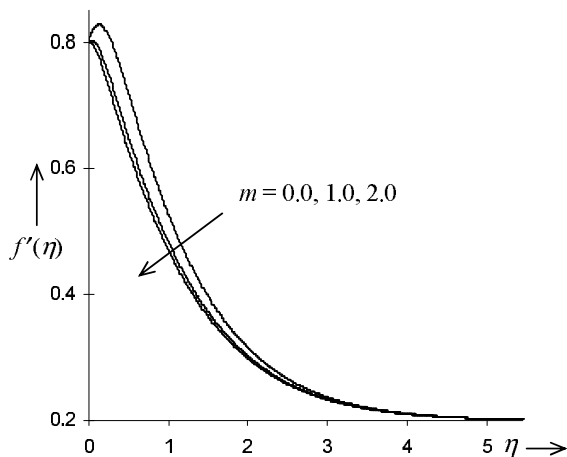


Fig.23 Velocity distribution versus η when $\lambda = 0.2$, $M = 0.5$ and $G = 5.0$.

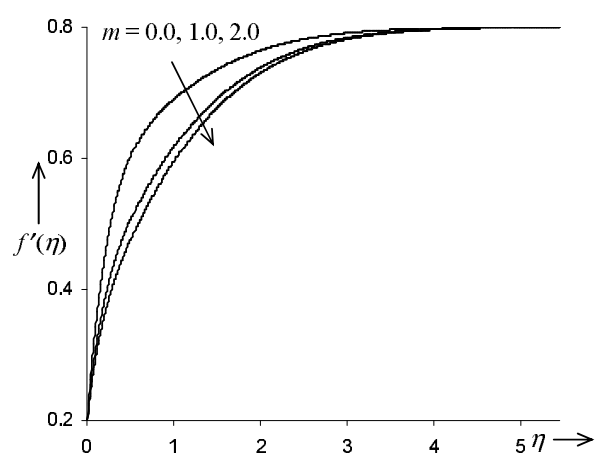


Fig.24 Velocity distribution versus η when $\lambda = 0.8$, $M = 0.5$ and $G = 5.0$.

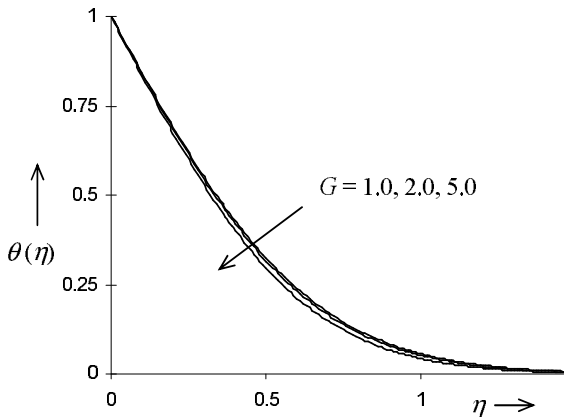


Fig.25 Temperature distribution versus η when $m=0.0$, $\lambda=0.2$ and $M=0.5$.

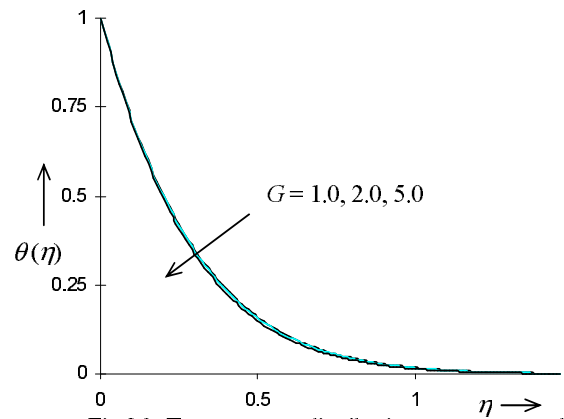


Fig.26 Temperature distribution versus η when $m=1.0$, $\lambda=0.2$ and $M=0.5$.

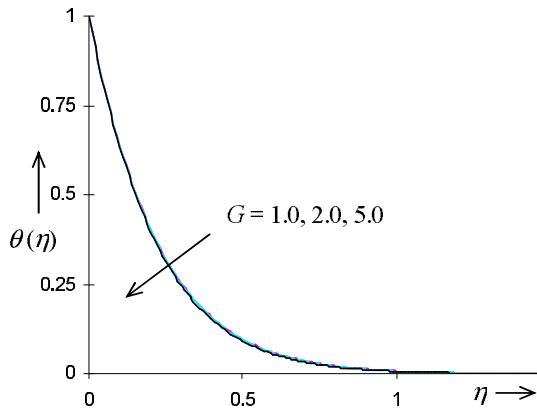


Fig.27 Temperature distribution versus η when $m=0.0$, $\lambda=0.2$ and $M=0.5$.

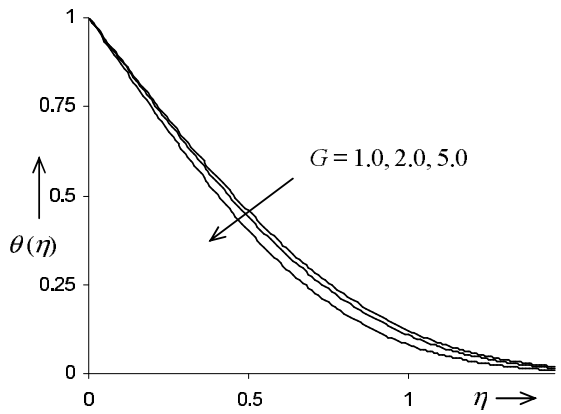


Fig.28 Temperature distribution versus η when $m=0.0$, $\lambda=0.8$ and $M=0.5$.

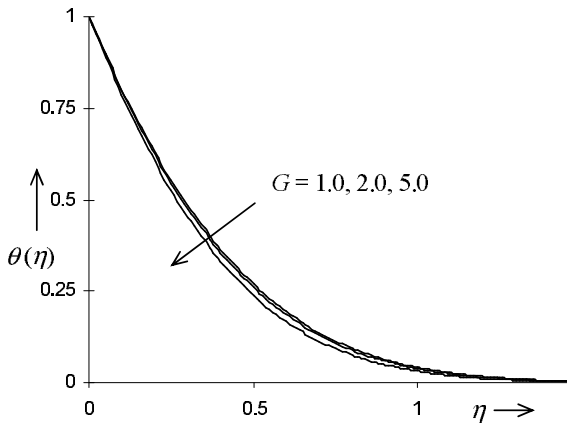


Fig.29 Temperature distribution versus η when $m=1.0$, $\lambda=0.8$ and $M=0.5$.

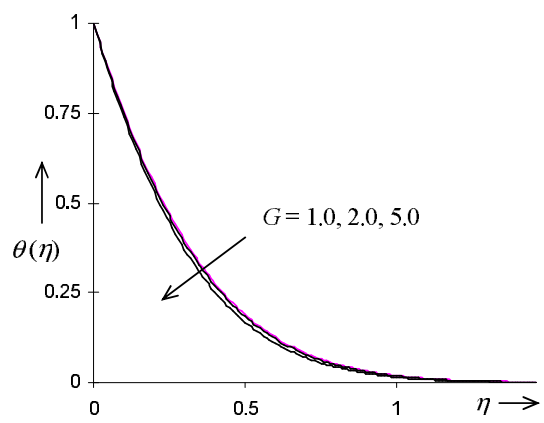


Fig.30 Temperature distribution versus η when $m=2.0$, $\lambda=0.8$ and $M=0.5$.

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