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Research Article

Modified Gauss Elimination Technique for Separable Nonlinear Programming Problem

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Abstract
Separable programming deals with such nonlinear programming problem in which the objective function as well as constraints are separable. For solving Separable Nonlinear Programming Problem (SNPP) is reduced first to Linear Programming Problem (LPP) by approximating each separable function by a piecewise linear function and than usual graphical, simplex method applied. A new form of Gauss elimination technique for inequalities has been proposed for solving a Separable Nonlinear Programming Problem. The technique is useful than the earlier existing methods because it takes least time and calculations involve in are also simple. The same has been illustrated by a numerical example of SNPP.

Keywords : Separable Nonlinear Programming; Elimination Technique; Inequalities; Breaking Points.

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1 Introduction

A nonlinear programming model seeks to optimize the objective function of non-negative variables subject to linear/ nonlinear constraints and in separable nonlinear programming problem, the objective function and constraints can be expressed as a linear combination of several different single variable functions, of which some or all are non-linear. Here, the aim is to reduce the computing time of the optimization process of the proposed problem in which objective function treated as constraint in nature. These constraints are encountered in econometric data fitting, logistics, design and management of water supply system, and electrical network analysis. Variable elimination techniques for linear constraints have particular significance in the context of constraint programming problem that have been

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developed in recent years. Variable elimination for linear equations (Gaussian elimination) is a fundamental technique in computational linear algebra and is therefore familiar to most of us. Variable elimination in linear inequalities is intimately related to polyhedral theory and aspect of linear programming that are not quite as familiar. In addition, the high complexity of elimination in inequalities has forced the consideration of intricate specializations of Gauss's original technique. The simplicity and transparency of variable elimination makes it an attractive computational paradigm for use in interpreters for new generation constraint programming problems. Variable elimination on more general linear constraints using Gaussian elimination and its extensions is an interesting topic in its own right. Chandru [12] studied variable elimination in linear constraints over the reals extends quite naturally to constraints in certain discrete domain. Firozja, Babakordi and Shahhosseini [5] proposed Gauss elimination algorithm for interval linear equation. Gunter [1] contributed on feasibility of interval Gaussian algorithm. Different elimination techniques for solving Linear Programming Problem are given earlier by Kanniappan and Thangvel [7], Karmarker [6], Kohler [2], Sharma and Bhargava [4], and Williams [3]. Jain and Mangal [9, 10, 11] studied various elimination techniques for Fractional Programming Problem. Jain [8] studied elimination technique for Multi-objective Linear Programming Problem. Many researcher's work on the eliminations techniques for constraints, Linear Programming Problem, Linear Fractional Programming Problem and Multi-objective Linear Programming Problem but eliminations techniques for non-linear programming problem is still untouched. The purpose of this paper is to give a brief taxonomy of an important group of Non-linear Programming Problem which occurs in certain branch of Industrial Mathematics. Here special emphasis is given on Separable Non-linear Programming Problem solved by modified Gauss elimination technique. In the next section, we describe in brief how modified Gauss elimination technique is applied on inequalities. In the proposed method, first we include objective function of SNPP into constraints set; using modified Gauss elimination technique to solve these systems of inequalities of SNPP.

2 Modified Gauss Elimination Technique for Inequalities

In Numerical Analysis, the systems of simultaneous linear equation is solved by Gauss elimination technique with the help of elimination of variables one by one and finally reduce to upper triangular system of equations, which can be solved by back substitution. Equation gives finite solution while inequality gives possibility of many solutions in bounded / region form, out of which we select maximum or minimum value according to the problem to optimize. This is the main theme to apply modified Gauss elimination technique for inequality in place of equation. It can easily verify that max. /mini. value of linear variables gives, max. /mini. value of objective function of $\Sigma c_i x_i$ from where all c_i are positive. If some c_i are not positive, then we take mini. /max. value of corresponding linear variables so it gives max. /mini. value of objective function. Here we apply modified Gauss elimination technique for a system of inequalities of the same sign i.e., either less than equal to ($\leqslant$) or greater than equal to ($\geqslant$) in nature. Here variables are eliminated by combining inequalities in such a way that the inequalities and variables reduced one by one in every iteration i.e., one variable and one inequality reduce in one iteration so at last there remains only one inequality with one variable remains. This last inequality gives value of last variable in bounded form and finally taking the value of last variable maximum or minimum according to objective function of SNPP. Finally, we get value of other variables by back substitution of value of the last variable.

2.1 Problem Formulation for Modified Gauss Elimination Technique

Here we consider the Linear Programming problem (LPP) as:

$$MaxZ = cx + \beta \tag{2.1}$$

$$Ax \leqslant b$$

$$x \geqslant 0$$

To apply modified Gauss elimination technique on LPP, we formulate this LPP again by taking objective function as constraints and all constraints of same sign of inequality. So reduce form of LPP for modified Gauss elimination technique is as

$$MaxZ \tag{2.2}$$

$$Z - (cx + \beta) \leqslant 0$$

$$Ax \leqslant b$$

$$-x \leqslant 0$$

Now variables eliminated by combining inequalities in such a way that the inequalities and variables reduced one by one in each iteration. If at any stage we get an absurd inequality like $0 \leqslant$ d where d is a negative number then we conclude that the given LPP has infeasible solution otherwise LPP has feasible solution. Now we have to consider the constraint, which we had left. Let for the variables x_j , l_j and u_j be the greatest integer less than x_j , l_j and least integer greater than x_j , l_j respectively. Thus for each variable x_j , l_j , we can write $l_j \leqslant x_j \leqslant u_j$. At least one ordered pair (l_j, u_j) gives a feasible solution.

3 The problem

Let us consider the nonlinear programming problem

$$Max.(orMin.) \;\; f_j(x_1, x_2, ..., x_n) \tag{3.3}$$

$$g_{ij}(x_1, x_2, ..., x_n) \leqslant \; b_i$$

$$x_j \geqslant \; 0$$

If the objective function and constraints are separable then it can be written as

$$f_j(x_1, x_2, ..., x_n) = \sum_{j=1}^{n} \; f_j(x_j) \tag{3.4}$$

$$g_{ij}(x_i, x_2, ..., x_n) = \sum_{j=1}^{n} \; g_{ij}(x_j)$$

Thus separable nonlinear programming problem can be written as

$$Max.(orMin.) \quad \sum_{j=1}^{n} f_j(x_j) \tag{3.5}$$

$$\sum_{j=1}^{n} g_{ij}(x_j) \leqslant b_i$$

$$x_j \geqslant 0$$

where some or all $g_{ij}, x_{ij}, f(x_j)$ are non-linear.
Here we can approximate the sub-function g_{ij} by a set of arbitrary break points. Let the number of breaking points for the j^{th} variable be equal to K_j and a_{jk} be its k^{th} breaking value and w_{jk} be the weight associated with the k^{th} breaking point of j^{th} variable. Now the reduce separable nonlinear programming problem is as

$$Max.(orMin.) \quad \sum_{j=1}^{n}\sum_{k=1}^{K_j} f_j(a_{jk})w_{jk} \tag{3.6}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{K_j} g_{ij}(a_{jk})w_{jk} \leqslant b_i$$

$$0 \leqslant w_{j1} \leqslant y_{j1}$$

$$0 \leqslant w_{jk} \leqslant y_{j,k-1} + y_{jk}$$

$$0 \leqslant w_{jK_j} \leqslant y_{j,K_j-1}$$

$$\sum_{k=1}^{K_j} w_{jk} = 1, \sum_{k=1}^{K_j-1} y_{jk} = 1$$

$$y_{jk} = 0 \ or \ 1$$

The technique of nonlinear programming problem break points thus transforms the separable nonlinear programming problem into a linear programming problem with decision variables for the approximating problem is given by w_{jk} and y_{jk} .
To explain the whole procedure, we consider a numerical example of SNPP which having optimal/feasible solution in the next section.

4 Numerical Example

Here we consider an example of SNPP as:

$$Max. \ z = x_1 + {x_2}^4 \tag{4.7}$$

$$3x_1 + 2{x_2}^2 \leqslant 9$$

$$x_1, x_2 \geqslant 0$$

Step-1 Making standard form of the given problem by taking objective function as in maximizing form but here the given problem is already in maximizing form, so proceed for next step.

Step-2 Collect the separable functions of the given problem.

$$f_1(x_1) = x_1 \ , f_2(x_2) = {x_2}^4.$$

$$g_{11}(x_1) = 3x_1 \ , g_{12}(x_2) = 2{x_2}^4$$

Here $f_1(x_1)$ and $g_{11}(x_1)$ are in the linear form, so we can left them in their present form and consider the breaking points for $f_2(x_2)$ and $g_{12}(x_2)$ only. Moreover, we observe that the above separable functions satisfy the concavity-convexity conditions for the maximization problem.

Step-3 It is observed from the constraints set that

$$x_1 \leqslant \frac{9}{3} = 3, \ x_2 \leqslant \sqrt{\frac{9}{2}} = 2.13 \ and \ x_1, x_2 \geqslant 0$$

$$\Rightarrow \quad 0 \leqslant \ x_1 \leqslant \ 3$$

$$0 \leqslant \ x_2 \leqslant \ 3$$

The upper limit for the variables x_1 and x_2 are 3 and lower limit for the variables x_1 and x_2 are 0. Therefore, we divide the close interval [0 , 3] into four equal parts for breaking points.

Step-4 Now considering non-linear $f_2(x_2)$ and $g_{12}(x_2)$ into linear form by using (k=4) breaking points as

K	a_{2k}	$f_2(a_{2k}) = {x_2}^4$	$g_{12}(a_{2k}) = 2{x_2}^2$
1	0	0	0
2	1	1	2
3	2	16	8
4	3	81	18

From the above breaking points, we have

$$f_2(x_2) \cong w_{21}f_2(a_{21}) + w_{22}f_2(a_{22}) + w_{23}f_2(a_{23}) + w_{24}f_2(a_{24})$$

$$= w_{21}0 + w_{22}1 + w_{23}16 + w_{24}81$$

$$= w_{22} + 16w_{23} + 81w_{24}$$

$$g_{12}(x_2) \cong w_{21}g_{12}(a_{21}) + w_{22}g_{12}(a_{22}) + w_{23}g_{12}(a_{23}) + w_{24}g_{12}(a_{24})$$

$$= w_{21}0 + w_{22}2 + w_{23}8 + w_{24}18$$

$$= 2w_{22} + 8w_{23} + 18w_{24}$$

Step-5 Thus, the reduced LPP is

$$Max. \ z = \ x_1 + w_{22} + 16w_{23} + 81w_{24} \quad (4.8)$$

$$3x_1 + 2w_{22} + 8w_{23} + 18w_{24} \leqslant \ 9$$

$$w_{21} + w_{22} + w_{23} + w_{24} = 1$$

$$w_{21}, w_{22}, w_{23}, w_{24} \geqslant 0$$

With the additional restrictions that
(i) for each j =1, 2, more than two w_{jk} are positive, and
(ii) if two w_{jk} are positive, they must correspond to adjacent points.
To solve the above approximate LPP by modified Gauss elimination technique:
Making standard form by treating objective function as constraints and all inequality of same sign for modified Gauss elimination technique, we have

$$Max z \tag{4.9}$$

$$z - x_1 - w_{22} - 16w_{23} - 81w_{24} \leqslant 0$$
$$3x_1 + 2w_{22} + 8w_{23} + 18w_{24} \leqslant 9$$
$$w_{21} + w_{22} + w_{23} + w_{24} \leqslant 1$$
$$-w_{21} \leqslant 0$$
$$-w_{22} \leqslant 0$$
$$-w_{23} \leqslant 0$$
$$-w_{24} \leqslant 0$$
$$-x_1 \leqslant 0$$

In the first stage eliminating w_{21} by using modified Gauss elimination, we have

$$Max z \tag{4.10}$$

$$z - x_1 - w_{22} - 16w_{23} - 81w_{24} \leqslant 0$$
$$3x_1 + 2w_{22} + 8w_{23} + 18w_{24} \leqslant 9$$
$$w_{22} + w_{23} + w_{24} \leqslant 1$$
$$-w_{22} \leqslant 0$$
$$-w_{23} \leqslant 0$$
$$-w_{24} \leqslant 0$$
$$-x_1 \leqslant 0$$

In the second stage eliminating w_{23} by using modified Gauss elimination, we have

$$Max z \tag{4.11}$$

$$z - x_1 - 16w_{23} - 81w_{24} \leqslant 0$$
$$3x_1 + 8w_{23} + 18w_{24} \leqslant 9$$
$$w_{23} + w_{24} \leqslant 1$$
$$-w_{23} \leqslant 0$$
$$-w_{24} \leqslant 0$$
$$-x_1 \leqslant 0$$

In the third stage eliminating x_1 by using modified Gauss elimination, we have

$$Maxz \tag{4.12}$$

$$z - 16w_{23} - 81w_{24} \leqslant 0$$
$$8w_{23} + 18w_{24} \leqslant 9$$
$$w_{23} + w_{24} \leqslant 1$$
$$-w_{23} \leqslant 0$$
$$-w_{24} \leqslant 0$$

In the fourth stage eliminating w_{23} by using modified Gauss elimination, we have

$$Maxz \tag{4.13}$$

$$z - 65w_{24} \leqslant 16$$
$$10w_{24} \leqslant 1$$
$$-w_{24} \leqslant 1$$
$$-w_{24} \leqslant 0$$

Rewritten as

$$Maxz \tag{4.14}$$

$$z - 65w_{24} \leqslant 16$$
$$w_{24} \leqslant \frac{1}{10}$$
$$-w_{24} \leqslant 0$$

In the fifth stage eliminating w_{24} by using modified Gauss elimination, we have

$$Maxz \tag{4.15}$$

$$z \leqslant \frac{45}{2} = 22.5$$
$$0 \leqslant \frac{1}{10}$$

Now Max $z = \frac{45}{2}$ and using back substituting by putting $z = \frac{45}{2}$ in the above inequalities, we get $w_{24} = \frac{1}{10}$. Now putting $z = \frac{45}{2}$ and $w_{24} = \frac{1}{10}$ in the above inequalities. we get $w_{23} = \frac{9}{10}$. Using back substituting by putting $z = \frac{45}{2}$, $w_{24} = \frac{1}{10}$, and $w_{23} = \frac{9}{10}$ in the inequalities, we have $w_{22} = x_1 = w_{21} = 0$. Hence the solution of above LPP by modified Gauss elimination technique is $z = \frac{45}{2}$, $w_{23} = \frac{9}{10}$, $w_{24} = \frac{1}{10}$ and $w_{22} = x_1 = w_{21} = 0$. . Now, to obtain the solution of original SNPP in terms of original variables x_1 and x_2 , we consider $w_{24} = \frac{1}{10}$, $w_{23} = \frac{9}{10}$, and $w_{22} = x_1 = w_{21} = 0$. Therefore, $x_2 = 2w_{23} + 3w_{24} = 2(\frac{9}{10}) + 3(\frac{1}{10}) = \frac{21}{10} = 2.1$ and $x_1 = 0$. Hence the optimal solution of SNPP is

$$x_1 = 0,\ x_2 = 2.1;$$
$$Maxz = x_1 + {x_2}^4 = 0 + (2.1)^4 = 19.45$$

5 Conclusion

The paper gives an approach to find the solution of SNPP by using modified Gauss elimination method. Sometimes modified Gauss elimination method to solve SNPP contains small difference in the value of optimal solution, which may be further improved by increasing the number of breaking points. The proposed modified Gauss elimination method is better than earlier existing simplex method due to simple and less calculations. By this modified Gauss elimination approach an optimal goal can be attained.

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