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Research Article

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Entropy Generation In an Unsteady MHD Channel Flow With Navier Slip and Asymmetric Convective Cooling

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Abstract

The combined effects of magnetic field, Navier slip and convective heating on the entropy generation in a flow of a viscous incompressible electrically conducting fluid between two infinite horizontal parallel plates under a constant pressure gradient have been examined. Both the lower and upper plates of the channel are subjected to asymmetric convective heat exchange with the ambient fluid. The governing non-linear governing partial differential equations are solved using the MATLAB PDE solver. The entropy generation number and the Bejan number are also obtained. The influences of the pertinent flow parameters on velocity, temperature, entropy generation and Bejan number are discussed graphically. It is observed that the plate surfaces act as a strong source of entropy generation and heat transfer irreversibility. Also, the entropy generation number decreases for increasing values of the magnetic parameter. The slip parameters are found to control the entropy generation. By using asymmetric cooling of the plates, it is possible to operate the system with reduced entropy generation rate.

Keywords : MHD channel flow; Navier slip; Convective cooling; Entropy generation number; Bejan number.

1 Introduction

One of the fundamental flow geometries encountered in engineering processes is the channel between two parallel plates. When a viscous fluid flows in a parallel plate channel, a velocity boundary layer develops along the inner surfaces of the channel. If the plates have different thermal conditions the heat transfer starts from the inlet of the channel and temperature profile develops. Although the analysis of the heat transfer in such a flow system is more complex due to the variation of the velocity distribution in all directions, the investigators have given numerical solutions under various constraints. Magnetohydrodynamics (MHD) has potential in many engineering branches. Lorentz forces produced by an applied magnetic field in a moving electrically conducting fluid are able to push fluids in the mixing process without a need for any mechanical components with the device.

The rapid depletion of energy resources worldwide has prompted almost every country in the world to focus attention on energy conservation and improving existing energy systems to minimize the energy waste. The scientific commu-

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nity has responded to the challenge by developing new techniques of analysis and design so that the available work destruction is either eliminated or minimized. The improvement of thermal systems has gained a growing interest due to the relations with the problems of material processing, energy conversion and environmental effects. Efficient energy utilization during the convection in any fluid flow is one of the fundamental problems of the engineering processes to improve the system. One of the methods used for the prediction of performance of the engineering processes has been the second law analysis. The second law of the thermodynamics is applied to investigate the irreversibilities in terms of the generation of entropy.

Since entropy generation is the measure of the destruction of available work of the system, the determination of the active factors motivating the entropy generation is important in upgrading the system performances. Rapid progress in science and technology has led to the development of an increasing number of flow devices that involve the manipulation of fluid flow in various geometries. Many text books of fluid dynamics fails to mention that the no-slip condition remains an assumption due to unusual agreement with experimental results for a century. Nevertheless, another approach supposed that fluid can slide over a solid surface because the experimental fact was not always accepted in the past. Navier [1] proposed general boundary conditions which include possibility of fluid slip at the solid boundary. He proposed that velocity at a solid surface is proportional to the shear stress at the surface. The phenomenon of slip occurrence has been demonstrated by the recent theoretical and experimental studies such as Sahraoui et al. [2], Buckingham et al. [3], Berh [4], Raoufpanah [5], Chauhan et al. [6], Tripathi et al. [7] and Gupta [8]. Moreover, entropy generation in engineering and industrial flow systems provides insight into the power consumption through thermodynamic losses. Therefore, the entropy minimization provides power optimization for the fluid motion in the porous channel. Efficient energy utilization during the convection in any fluid flow is one of the fundamental problems of the engineering processes to improve the system.

The problem of the slip flow regime is very important in this era of modern science, technology and vast ranging industrialization. In many practical applications, the fluid adjacent to a solid surface no longer takes the velocity of the surface. The fluid at the surface has a finite tangential velocity; it slips along the surface. The flow regime is called the slip flow regime and its effect cannot be neglected. The effects of slip conditions on the hydromagnetic steady flow in a channel with permeable boundaries were discussed by Makinde and Osalusi [9]. Khalid and Vafai [10] obtained the closed form solutions for steady periodic and transient velocity field under slip condition. Watanebe et al. [11] studied the effect of Navier Slip on Newtonian fluids at solid boundary. Chen and Tian [12] investigated entropy generation in a micro annulus flow and discussed the influence of velocity slip on entropy generation. Use of an external magnetic field is of considerable importance in many industrial applications, particularly as a control mechanism in material manufacturing. Homogeneity and quality of single crystals grown from doped semiconductor melts is of interest to manufacturers of electronic chips. One of the main purposes of electromagnetic control is to stabilize the flow and suppress oscillatory instabilities, which degrades the resulting crystal. The magnetic field strength is one of the most important factors for crystal formation. The scientific treatment of the problems of irrigation, soil erosion and tile drainage are the present focus of the development of porous channel flow. The magnetohydrodynamic channel flow with heat transfer has attracted the attention of many researchers due to its numerous engineering and industrial applications. Such flows finds applications in thermofluid transport modeling in magnetic geosystems, meteorology, turbo machinery, solidification process in metallurgy and in some astrophysical problems. One of the methods used for predicting the performance of the engineering processes is the second law analysis. The second law of thermodynamics is applied to investigate the irreversibilities in terms of the entropy generation rate. Since entropy generation is the measure of the destruction of the available work of the system, the determination of the factors responsible for the entropy generation is also important in upgrading the system performances. The method is introduced by Bejan [13, 14]. The entropy generation is encountered in many energy-related applications, such as solar power collectors, geothermal energy systems and the cooling of modern electronic systems. Efficient utilization of energy is the primary objective in the design of any thermodynamic system. This can be achieved by minimizing entropy generation in processes. The theoretical method of entropy generation has been used in the specialized literature to treat external and internal irreversibilities. The irreversibility phenomena, which are expressed by entropy generation in a given system, are related to heat and mass transfers, viscous dissipation, magnetic field etc. Several researchers have discussed the irreversibility in a system under various flow configurations [15-26]. They showed that the pertinent flow parameters might be chosen in order to minimize entropy generation inside the system. Salas et al. [27] analytically showed a way of applying entropy generation analysis for modelling and optimization of magnetohydrodynamic induction devices. They restricted their analysis to only Hartmann model flow in a channel. Thermodynamics analysis of mixed convection in a channel with transverse hydromagnetic effect has been investigated by Mahmud et al.[28].

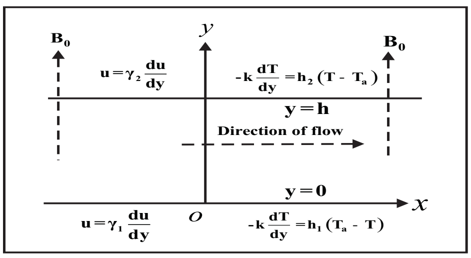

Figure 1: Geometry of the problem

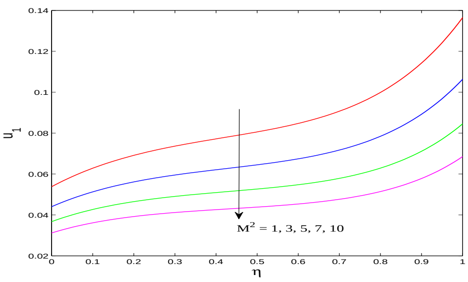

Figure 2: Velocity profiles for different M^2 when $\tau = 0.2$, $\beta_1 = 0.1$, $\beta_2 = 0.1$

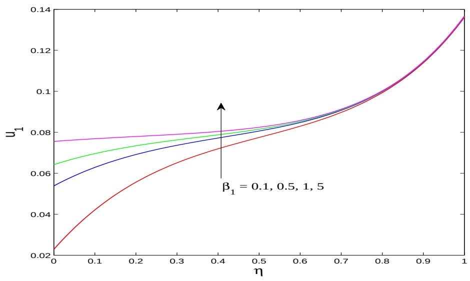

Figure 3: Velocity profiles for different β_1 when $M^2 = 5$, $\beta_2 = 0.1$, $\tau = 0.2$

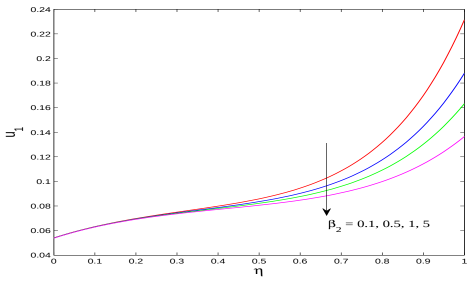

Figure 4: Velocity profiles for different β_2 when $M^2 = 5$, $\beta_1 = 0.1$, $\tau = 0.2$

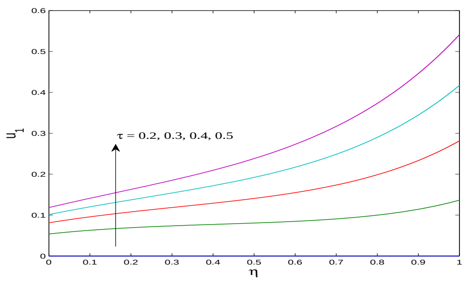

Figure 5: Velocity profiles for different τ when $M^2 = 5$, $\beta_1 = 0.1$, $\beta_2 = 0.1$

The entropy generation during fluid flow between two parallel plates with moving bottom plate has been analyzed by Latife et al. [29]. Ibanez et al. [30] have examined the minimization of entropy generation by asymmetric convective cooling. The entropy generation inside a porous channel with viscous dissipation have been investigated by Mahmud [31]. The heat transfer and entropy generation during compressible fluid flow in a channel partially filled with porous medium have been analyzed by Chauhan and Kumar [32]. Tasnim et al. [33] have studied the entropy generation in a porous channel with hydromagnetic effects. Eegunjobi and Makinde [34] have studied the combined effect of buoyancy force and

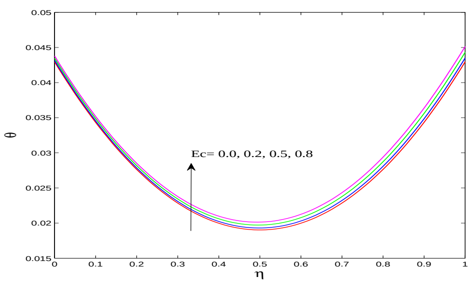

Figure 6: Temperature profiles for different Ec when $Pr = 0.72$, $Bi_1 = 0.1$, $Bi_2 = 0.1$, $\tau = 0.2$

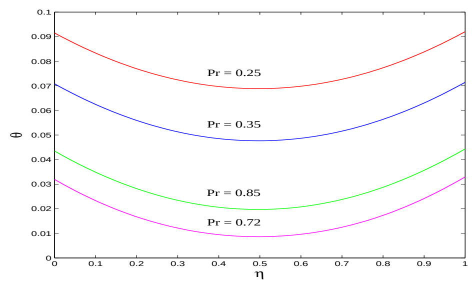

Figure 7: Temperature profiles for different Pr when $Ec = 0.5$, $Bi_1 = 0.1$, $Bi_2 = 0.1$, $\tau = 0.2$

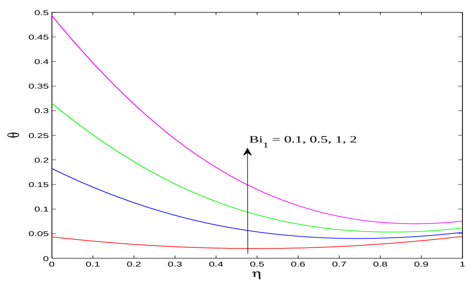

Figure 8: Temperature profiles for different β_1 when $Pr = 0.72$, $Bi_1 = 0.1$, $Bi_2 = 0.1$, $\tau = 0.2$

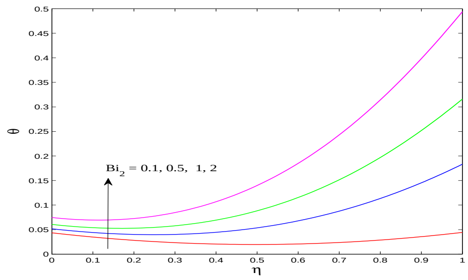

Figure 9: Temperature profiles for different β_2 when $Ec = 0.5$, $Bi_1 = 0.1$, $Bi_1 = 0.1$, $\tau = 0.2$

Navier slip on entropy generation in a vertical porous channel. The second law analysis of laminar flow in a channel filled with saturated porous media has been studied by Makinde and Osalusi [35]. Makinde and Maserumule [36] has presented the thermal criticality and entropy analysis for a variable viscosity Couette flow. Makinde and Osalusi [37] have investigated the entropy generation in a liquid film falling along an incline porous heated plate. Second law analysis for a variable viscosity plane Poiseuille flow with asymmetric convective cooling has been presented by Makinde and Aziz [38]. Cimpean and Pop [39, 40] have presented the parametric analysis of entropy generation in a channel. The effect of an external oriented magnetic field on entropy generation in natural convection has been investigated by Jery et al. [41]. Dwivedi et al. [42] have made an analysis on the incompressible viscous laminar flow through a channel filled with porous media. Makinde and Chinyoka [43] have presented the unsteady hydromagnetic generalized Couette flow of a reactive third-grade fluid with asymmetric convective cooling. Analysis of entropy generation rate in an unsteady porous channel flow with Navier slip and convective cooling has been presented by Chinyoka and Makinde [44].

In this paper, our objective is to examine the combined effects of magnetic field, asymmetric convective cooling and Navier slips on entropy generation in an MHD flow through a channel under a constant pressure gradient. A parametric study is carried out to see how the pertinent parameters of the problem affect the flow field, temperature field and the entropy generation. Such a flow model has great significance not only of its theoretical interest but also for applications to engineering.

2 Mathematical formulation and its solution

Consider the viscous incompressible electrically conducting fluid bounded by two infinite horizontal parallel porous plates separated by a distance h. Choose a Cartesian co-ordinate system with x-axis along the lower stationary plate in the direction of the flow, the y-axis is perpendicular to the plates. The flow is driven by a constant pressure gradient P. The top and bottom plates are cooled by convection. The coolant temperature T_a is the same for both plates but the convection heat transfer coefficients h_1 and h_2 are different, thus providing an asymmetric cooling

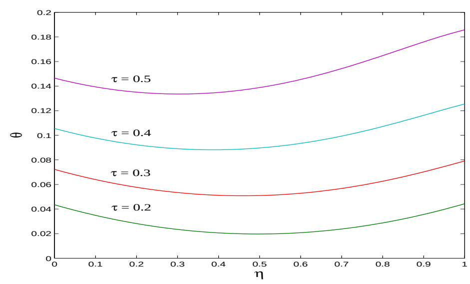

Figure 10: Temperature profiles for time τ when $Pr = 0.72$, $Bi_1 = 0.1$, $Bi_2 = 0.1$, $Ec = 0.5$

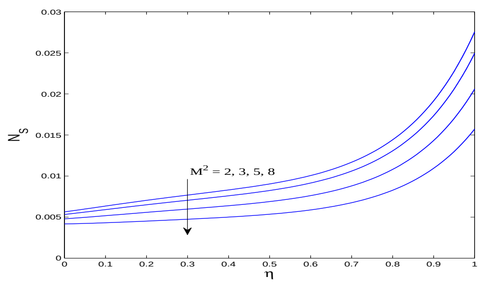

Figure 11: $\mathrm{N_S}$ for M^2 when $Pr = 0.71$, $\beta_1 = 0.1$, $\beta_2 = 0.1$, $Ec = 0.5$, $Bi_1 = 0.1$, $Bi_2 = 0.1$, $Br\Omega^{-1} = 1$

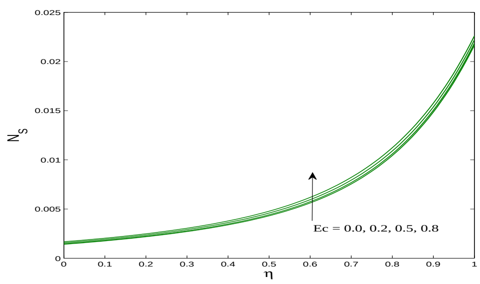

Figure 12: $\mathrm{N_S}$ for Ec when $Pr = 0.71$, $\beta_1 = 0.1$, $\beta_2 = 0.1$, $M^2 = 5$, $Bi_1 = 0.1$, $Bi_2 = 0.1$, $Br\Omega^{-1} = 1$

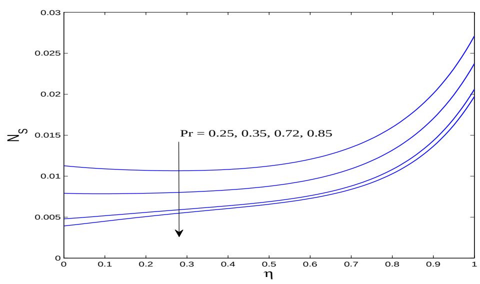

Figure 13: $\mathrm{N_S}$ for Pe when $M^2 = 5$, $\beta_1 = 0.1$, $\beta_2 = 0.1$, $Ec = 0.5$, $Bi_1 = 0.1$, $Bi_2 = 0.1$, $Br\Omega^{-1} = 1$

effect. A uniform transverse magnetic field B_0 is applied perpendicular to the channel plates. Since the magnetic Reynolds number is very small for most fluid used in industrial applications, we assume that the induced magnetic field is negligible. Since the plates are infinitely long, all physical variables, except pressure, depend on y and t only.
The equations of motion along x-direction is

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u, \quad (2.1)$$

where u is the fluid velocity in the x-axis, ρ is the fluid density, ν the kinematic viscosity, σ the electrical conductivity of the fluid and p is the fluid pressure.

The energy equation is

$$\rho c_p\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma B_0^2 u^2, \quad (2.2)$$

where T is the fluid temperature, μ the coefficient of viscosity, k the thermal conductivity, c_p the specific heat at constant pressure. Equation 2.1 states that heat can be transported in fluid by convection (the left-hand side), by conduction (first term on the righthand side), by viscous dissipation (second term on the right-hand side) and also by Joule heating (third term on the right-hand side).

The initial and boundary conditions for velocity and temperature distributions are

$$\begin{aligned} u &= 0,\ T = T_0 \text{ for } 0 \le y \le h,\ t \le 0,\\ u &= \gamma_1\frac{\partial u}{\partial y},\\ -k\frac{\partial T}{\partial y} &= h_1(T_a - T) \text{ at } y = 0,\ t > 0,\\ u &= \gamma_2\frac{\partial u}{\partial y},\\ -k\frac{\partial T}{\partial y} &= h_2(T - T_a) \text{ at } y = h,\ t > 0, \quad (2.3)\end{aligned}$$

where T_0 is the fluid initial temperature, T_a the ambient temperature, γ_1 the slip coefficient at the lower plate and γ_2 the slip coefficient at the upper plate, h_1 the heat transfer coefficient at the lower plate and h_2 the heat transfer coefficient at the upper plate.

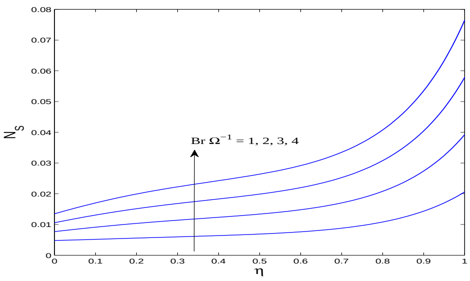

Figure 14: $\mathrm{N_S}$ for $Br\Omega^{-1}$ when $Pr = 0.71$, $\beta_1 = 0.1$, $\beta_2 = 0.1$, $Ec = 0.5$, $Bi_1 = 0.1$, $Bi_2 = 0.1$, $M^2 = 5$

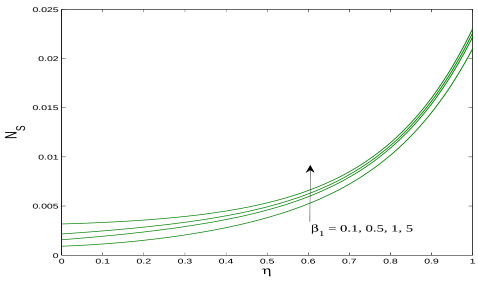

Figure 15: $\mathrm{N_S}$ for β_1 when $Pr = 0.71$, $M^2 = 5$, $\beta_2 = 0.1$, $Ec = 0.5$, $Bi_1 = 0.1$, $Bi_2 = 0.1$, $Br\Omega^{-1} = 1$

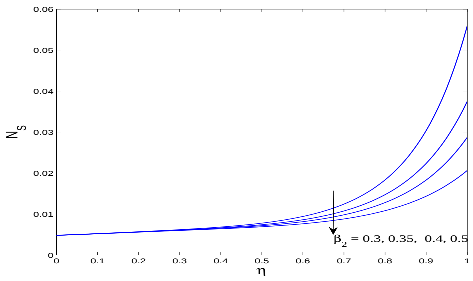

Figure 16: $\mathrm{N_S}$ for β_2 when $Pr = 0.71$, $\beta_1 = 0.1$, $M^2 = 5$, $Ec = 0.5$, $Bi_1 = 0.1$, $Bi_2 = 0.1$, $Br\Omega^{-1} = 1$

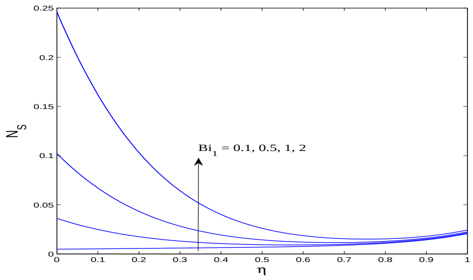

Figure 17: $\mathrm{N_S}$ for Bi_1 when $Pr = 0.71$, $\beta_1 = 0.1$, $\beta_2 = 0.1$, $Ec = 0.5$, $M^2 = 5$, $Bi_2 = 0.1$, $Br\Omega^{-1} = 1$

Introducing the non-dimensional variables

$$\eta = \frac{y}{h},\ \tau = \frac{\nu t}{h^2},\ u_1 = \frac{h u}{\nu},$$
$$\theta = \frac{T - T_0}{T_a - T_0}, \tag{2.4}$$

equations 2.1 and 2.2 become

$$\frac{\partial u_1}{\partial \tau} = P + \frac{\partial^2 u_1}{\partial \eta^2} - M^2 u_1, \tag{2.5}$$
$$Pr\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \eta^2} + Ec\,Pr\left[\left(\frac{\partial u_1}{\partial \eta}\right)^2 + M^2 u_1^2\right], \tag{2.6}$$

where $M^2 = \frac{\sigma B^2 h^2}{\rho \nu}$ is the magnetic parameter, $Ec = \frac{\nu^2}{c_p (T_a - T_0) h^2}$ the Eckert number, $Pr = \frac{\rho \nu c_p}{k}$ the Prandlt number and $P = \frac{h^3}{\rho \nu^2}\left(-\frac{\partial p}{\partial x}\right)$ the non-dimensional pressure gradient.

The initial and boundary conditions for velocity and temperature distributions are

$$u_1 = 0,\ \theta = 0 \ \text{for}\ 0 \le \eta \le 1,\ \tau \le 0 \tag{2.7}$$
$$u_1 = \beta_1 \frac{\partial u_1}{\partial \eta},$$
$$\frac{\partial \theta}{\partial \eta} = \mathrm{Bi}_1(\theta - 1)\ \text{at}\ \eta = 0,\ \tau > 0,$$
$$u_1 = \beta_2 \frac{\partial u_1}{\partial \eta},$$
$$\frac{\partial \theta}{\partial \eta} = \mathrm{Bi}_2(1 - \theta)\ \text{at}\ \eta = 1,\ \tau > 0,$$

where $\beta_1 = \frac{\gamma_1}{h}$ is the lower plate slip parameter, $\beta_2 = \frac{\gamma_2}{h}$ the upper plate slip parameter, $Bi_1 = \frac{h h_1}{k}$ and $Bi_2 = \frac{h h_2}{k}$ are the Biot numbers at both the lower and upper plates, respectively.

3 Results and discussion

The numerical computations are done by a written program which used a symbolic and computational computer language MATLAB. The entire procedure is implemented on MATLAB. To study the effects of magnetic field and Reynolds number on the velocity field we have presented the

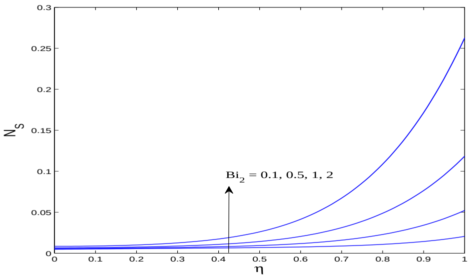

Figure 18: N_S for Bi_2 when $Pr = 0.71$, $\beta_1 = 0.1$, $\beta_2 = 0.1$, $Ec = 0.5$, $M^{=}2 = 5$, $Bi_2 = 0.1$, $Br\Omega^{-1} = 1$

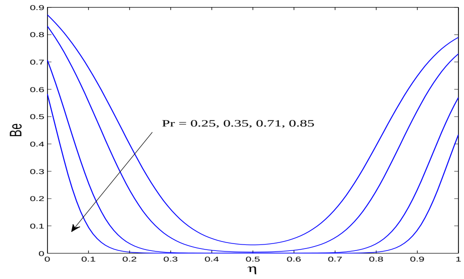

Figure 20: Bejan number for Pr when $M^2 = 5$, $Bi_1 = 0.1$, $Bi_2 = 0.1$, $Br\Omega^{-1} = 1$

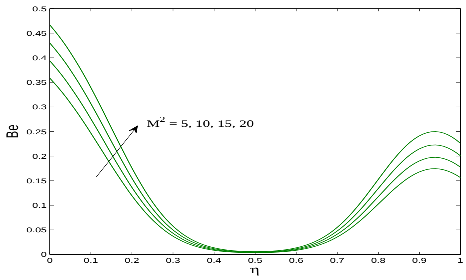

Figure 19: Bejan number for M^2 when $Pr = 0.71$, $Bi_1 = 0.1$, $Bi_2 = 0.1$, $Br\Omega^{-1} = 1$

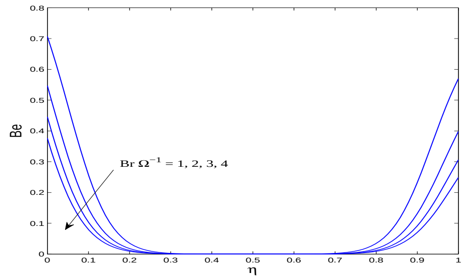

Figure 21: Bejan number for $Br\Omega^{-1}$ when $M^2 = 5$, $Bi_1 = 0.1$, $Bi_2 = 0.1$, $Pr = 0.71$

non-dimensional velocity u_1 against η in Figs. 2-5 for several values of magnetic parameter M^2, slip parameters β_1 and β_2 and time τ with $P = 1$. It is seen from Fig. 2 that the fluid velocity u_1 decreases with an increase in magnetic parameter M^2. An increase in the value of the magnetic parameter M^2 has the effect of imposing more viscous forces that dominate electromagnetic forces in retarding flow in the channel. Figs. 3-4 show that fluid velocity u_1 decreases with an increase in slip parameter β_1 whereas it increases with an increase in slip parameter β_2.

Fig. 5 reveals that the fluid velocity u_1 increases with an increase in time τ. We have plotted the temperature distribution θ against η in Figs. 6-10 for several values of Eckert number Ec, Prandtl number Pr, Biot numbers Bi_1, Bi_2 and time τ. By analyzing Fig. 6 it is revealed that the effect of Eckert number Ec is to significantly increase the fluid temperature in the flow region. This is because heat energy is stored in the liquid due to frictional heating. Thus, the effect of increasing Ec, is to enhance the temperature at any point. Fig. 7 shows that the fluid temperature θ decreases with an increase in Prandtl number Pr. The effect of symmetrical as well as asymmetrical cooling of the channel is depicted in Figs. 8-9 The fluid temperature θ increases with an increase in Biot numbers Bi_1 and Bi_2. Biot number is the ratio of the hot fluid side convection resistance to the cold fluid side convection resistance on a surface. Higher Biot numbers mean the higher degrees of convective heating at the channel plates, thus leading to rise temperature at the channel walls and hence also in the bulk fluid. The overall temperature profiles thus increase with increasing Biot number. When the channel is cooled symmetrically ($Bi_1 = Bi_2$), the temperature distribution is symmetrical about the centreline ($\eta = 0.5$), and the heat dissipations from the lower and the upper plates are equal. The temperature profiles maintain their symmetry about the centreline of the channel in view of the symmetrical cooling conditions imposed at the plates. Fig. 10 reveals that the fluid temperature θ increases when time progresses.

4 Entropy generation

In many engineering and industrial processes, entropy production destroys the available energy

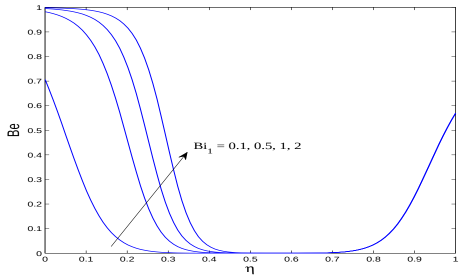

Figure 22: Bejan number for Bi_1 when $M^2 = 5$, $Pr = 0.71$, $Bi_2 = 0.1$, $Br\Omega^{-1} = 1$

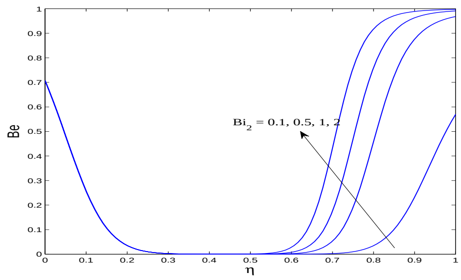

Figure 23: Bejan number for Bi_2 when $M^2 = 5$, $Pr = 0.71$, $Bi_1 = 0.1$, $Br\Omega^{-1} = 1$

in the system. It is therefore imperative to determine the rate of entropy generation in a system, in order to optimize energy in the system for efficient operation in the system. The convection process in a channel is inherently irreversible and this causes continuous entropy generation. Woods [45] gave the local volumetric rate of entropy generation for a viscous incompressible conducting fluid in the presence of magnetic field as follows:

$$E_G = \frac{k}{T_0^2}\left(\frac{\partial T}{\partial y}\right)^2 + \frac{\mu}{T_0}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma B_0^2}{T_0}u^2. \quad (4.8)$$

The entropy generation equation 4.8 consists of three terms, the first term is the irreversibility due to the heat transfer, the second term is entropy generation due to viscous dissipation, while the third term is local entropy generation due to the effect of magnetic field (Joule heating or Ohmic heating).

The dimensionless entropy generation number may be defined by the following relationship:

$$N_S = \frac{T_0^2 h^2 E_G}{k(T_a - T_0)^2}. \quad (4.9)$$

In terms of the dimensionless velocity and temperature, the entropy generation number becomes

$$N_S = \left(\frac{d\theta}{d\eta}\right)^2 + \frac{Br}{\Omega}\left[\left(\frac{du_1}{d\eta}\right)^2 + M^2u_1^2\right], \quad (4.10)$$

where $Br = \frac{\mu\nu^2}{k(T_a - T_0)h^2}$ is the Brinkmann number and $\Omega = \frac{T_a - T_0}{T_0}$ the non-dimensional temperature difference.

The entropy generation number $\mathrm{N_S}$ can be written as a summation of the entropy generation due to heat transfer denoted by N_1 and the entropy generation due to fluid friction with magnetic field denoted by N_2 as

$$N_1 = \left(\frac{\partial\theta}{\partial\eta}\right)^2,$$

$$N_2 = \frac{Br}{\Omega}\left[\left(\frac{\partial u_1}{\partial\eta}\right)^2 + M^2u_1^2\right]. \quad (4.11)$$

An alternative irreversibility distribution parameter is the Bejan number, which gives an idea whether the fluid friction irreversibility dominates over heat transfer irreversibility or the heat transfer irreversibility dominates over fluid friction irreversibility ([46]). It is simply the ratio of entropy generation due to heat transfer to the total entropy generation:

$$Be = \frac{\begin{pmatrix}\text{Entropy generation}\\ \text{due to heat transfer}\end{pmatrix}}{\text{Total entropy generation}} = \frac{N_1}{\mathrm{N_S}} = \frac{1}{1+\Phi}, \quad (4.12)$$

where Φ is the irreversibility distribution ratio which is given by:

$$\Phi = \frac{\begin{pmatrix}\text{Fluid friction irreversibility}\\ +\text{ Magnetic field irreversibility}\end{pmatrix}}{\text{Heat transfer irreversibility}} = \frac{N_2}{N_1}, \quad (4.13)$$

As the Bejan number ranges from 0 to 1, it approaches zero when the entropy generation due to the combined effects of fluid friction and magnetic field is dominant ([47]). Similarly, $Be > 0.5$ indicates that the irreversibility due to heat transfer dominates, with $Be = 1$ as the limit at which the irreversibility is solely due to heat transfer. Consequently, $0 \leq \Phi < 1$ indicates that the irreversibility is primarily due to the heat transfer irreversibility, whereas for $\Phi > 1$ it is due to the sum of the fluid friction and magnetic field irreversibility.

The influences of the different governing parameters on entropy generation within the channel are presented in Figs. 11-23. The effect of magnetic parameter M^2 on the entropy generation number is displayed in Fig. 11. This figure shows that the entropy generation decreases with an increase in magnetic parameter M^2. It is meant that an increase in magnetic parameter M^2 tends to increase the entropy generation number since the magnetic parameter has an increasing effect on all friction, heat transfer and magnetic irreversibilities. Fig. 12 show that the entropy generation number N_S increases with an increase in Eckert number Ec. It is revealed from Fig. 13 that the entropy generation number N_S decreases with an increase in Prandtl number Pr. As the fluid temperature decreases, temperature gradient decreases within the channel and consequently, there is a decrease in entropy generation number in the channel. Fig. 14 shows that the entropy generation number N_S increases with an increase in group parameter $Br\Omega^{-1}$. This is attributed to the increase in fluid friction irreversibility (N_2) with an increase in $Br\Omega^{-1}$. Figs. 15-16 illustrate that the entropy generation number N_S increases with an increase in slip parameter β_1 while it decreases with an increase in slip parameter β_2. As expected, an increase in the slip parameter β_1 correspondingly increases fluid velocity and the fluid velocity decreases for increasing values of slip parameter β_2 . Thus, the slip parameters have a capability to control the entropy generation in the channel.

The influence of the Biot numbers on the entropy generation number N_S is shown in Figs. 17-18. As the Biot numbers Bi_1 and Bi_2, the entropy generation number increases. In Figs. 11-18, the entropy generation is expectedly maximum at the channel plates where fluid velocity and temperature gradients are highest and minimum around the channel centerline. It is seen from Fig. 19 that the Bejan number Be increases with an increase in magnetic parameter M^2. It is observed that the Bejan number decreases with an increase in magnetic parameter, due to its significant effect on friction irreversibilities. The effect of Prandtl number Pr on the Bejan number Be is displayed in Fig. 20. It is seen that the Bejan number Be decreases for increasing values of Pr. Fig. 21 reveals that the Bejan number Be increases with an increase in group parameter $Br\Omega^{-1}$. An increase in the values of the group parameter $Br\Omega^{-1}$ due to the combined effects of viscous heating and temperature difference yields a higher entropy generation number. The group parameter is an important dimensionless number for irreversibility analysis. It determines the relative importance of viscous effects to that of temperature gradient entropy generation. Generally, it is observed that an increase in group parameter strengthens the effect of fluid friction irreversibility, but heat transfer irreversibility dominates over fluid friction irreversibility. Figs. 22-23 show that the Bejan number Be increases with an increase in Biot numbers Bi_1 and Bi_2. An increase in the values of the Biot numbers result in an increase in the dominant effect of heat transfer irreversibility at the plate surfaces. This means that the plate surfaces act as a strong source of irreversibility. The Bejan number is asymmetric about the centerline of the channel due to the asymmetric temperature distribution. From Figs. 19-23, it is noted that the fluid friction irreversibility is zero at centerline($\eta = 0.5$) of the channel due to zero velocity gradient.

5 Conclusion

The combined effects of magnetic field, Navier slip and convective cooling on the entropy generation in an unsteady MHD flow through a channel have been investigated. The velocity and temperature profiles are used to evaluate the entropy generation profiles in the flow field. The study leads to the following conclusions:

- The magnetic field retards the fluid velocity.
- The fluid temperature decreases for increasing values of Prandtl number.
- The fluid temperature increases for increasing values of Biot numbers.

- The entropy generation number decreases for increasing values of the magnetic parameter.

- Slip parameters controls the entropy generation.

- With the use of asymmetric cooling of the plates, it is possible to operate the system with reduced entropy generation rates.

- The plate surfaces act as strong source of entropy and heat transfer irreversibility.

- The results show that heat transfer irreversibility dominates over fluid friction irreversibility and viscous dissipation has no effect on the entropy generation rate at the centerline of the channel.

- The results of the study provide valuable fundamental information on the physics of the simultaneously developing transient laminar convection in a parallel plate channel with moving bottom plate to improve the corresponding engineering applications. The designers under the responsibilities for the design and optimization of corresponding thermal systems can employ the results given about the entropy generation to reduce the loss of available work.

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