Effects of Magnetic Field and Inclined Load in Micropolar Thermoelastic Medium Possessing Cubic Symmetry under Three Theories

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Abstract

A model of the equations of two-dimensional problems is studied in a half space, whose surface in a medium free of micropolar thermoelastic possesses cubic symmetry as a result of inclined load. There acts an initial magnetic field parallel to the plane boundary of the half-space. The inclined load is assumed to be a linear combination of a normal load and a tangential load. The formulation is performed in the context of the Lord-Shulman and Green-Lindsay theories, as well as the classical dynamical coupled theory. Comparisons are made with the results in the presence of a magnetic.

Keywords : Lord-Shulman theory, Green-Lindsay theory, Magneto-thermoelasticity, Cubic symmetry, Microrotation, Micropolar thermoelastic.

1 Introduction

The linear theory of elasticity is of paramount importance in the stress analysis of steel, which is the commonest engineering structural material. To a lesser extent, linear elasticity describes the mechanical behavior of the other common solid materials, e.g., concrete, wood and coal. However, the theory does not apply to the behavior of many of the new synthetic materials of the clastomer and polymer type, e.g., polymethyl-methacrylate (Perspex), polyethylene and polyvinyl chloride. The linear theory of micropolar elasticity is adequate to represent the behavior of such materials. For ultrasonic waves, i.e., for

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the case of elastic vibrations characterized by high frequencies and small wavelengths, the influence of the body microstructure becomes significant. This influence of microstructure results in the development of new types of waves, not in the classical theory of elasticity. Metals, polymers, composites, soils, rocks, and concrete are typical media with microstructures. More generally, most of the natural and man-made materials including engineering, geological and biological media possess a microstructure. Eringen and Suhubi [2] and Eringen [3] developed the linear theory of micropolar elasticity.

Thermoelasticity theories, which admit a finite speed for thermal signals, have been receiving a lot of attention for the past four decades. In contrast to the conventional coupled thermoelasticity theory based on a parabolic heat equation (Biot, [1]), which predicts an infinite speed for the propagation of heat, these theories involve a hyperbolic heat equation and are referred to as generalized thermoelasticity theories.

Two generalizations to the coupled theory were introduced. The first is ascribed to Lord and Shulman [19] who introduced the theory of generalized thermoelasticity with one relaxation time by postulating a new law of heat conduction to replace the classical Fourier's law. Othman [22] constructed the model of generalized thermoelasticity in an isotropic elastic medium under the dependence of the modulus of elasticity on the reference temperature with one relaxation time.

The second generalization to the coupled theory of thermoelasticity is what is known as the theory of thermoelasticity with two relaxation times or the theory of temperature rate dependent thermoelasticity, and was proposed by Green and Lindsay [4]. It is based on a form of the entropy inequality proposed by Green and Laws [5]. Green and Lindsay [4] obtained another version of the constitutive equations. These equations were also obtained independently and more explicitly by Suhubi [25]. This theory contains two constants that act as relaxation times and modifies all the equations of the coupled theory, not only the heat equation. The classical Fourier's law of heat conduction is not violated if the medium under consideration has a center of symmetry. Othman [23] studied the relaxation effects on on thermal shock problems in the elastic half space of generalized magneto- thermoelastic waves under three theories.

Following various methods, the elastic fields of various loadings, inclusion and inhomogeneity problems, and interaction energy of point defects and dislocation arrangement have been discussed extensively in the past. Generally, all materials have elastic anisotropic properties which mean the mechanical behavior of an engineering material is characterized by the direction dependence. However, the three dimensional study for an anisotropic material is much more complicated to obtain than the isotropic one, owing to the large number of elastic constants involved in the calculation. In particular, transversely isotropic and orthotropic materials, which may not be distinguished from each other in plane strain and plane stress, have been more regularly studied. A brief look at the literature on micropolar orthotropic continua shows that Iesan [7]-[9] analyzed the static problems of plane micropolar strain of a homogeneous and orthotropic elastic solid, the torsion problem of homogeneous and orthotropic cylinders in the linear theory of micropolar elasticity and bending of orthotropic micropolar elastic beams by terminal couple. Nakamura et al. [21] applied the finite element method to orthotropic micropolar elasticity. Kumar and Choudhary [10]-[14] have discussed various problems in orthotropic micropolar continua. Singh and Kumar [26] and Singh [27] have also studied the plane waves in micropolar generalized thermoelastic solid.

A wide class of crystals such as W, Si, Cu, Ni, Fe, Au, Al etc., which are some fre-

quently used substances, belong to cubic materials. The cubic materials have nine planes of symmetry whose normals are on the three coordinate axes and on the coordinate planes making an angle with the coordinate axes. With the chosen coordinate system along the crystalline directions, the mechanical behavior of a cubic crystal can be characterized by five independent elastic constants.

Minagawa et. al. [20] discussed the propagation of plane harmonic waves in a cubic micropolar medium. Kumar and Rani [15] studied time harmonic sources in a thermally conducting cubic crystal. Kumar and Ailawalia [16, 17] discussed some source problems in micropolar media with cubic symmetry. Kuo [18] and Garg et al. [6] have discussed the problem of inclined load in the theory of elastic solids. The deformation due to other sources such as strip loads, continuous line loads, etc. can also be similarly obtained. The deformation at any point of the medium is useful to analyze the deformation field around mining tremors and drilling into the crust of the earth. It can also contribute to the theoretical consideration of the seismic and volcanic sources since it can account for the deformation fields in the entire volume surrounding the source region. No attempt has been made so far to study the response of inclined load in micropolar thermoelastic media possessing cubic symmetry.

The purpose of the present paper is to determine the normal displacement, normal force stress, and tangential couple stress in a micropolar elastic solid with cubic symmetry. The normal mode method is used to obtain the exact expressions for the considered variables. The distributions of the considered variables are represented graphically. A comparison is carried out between the temperature, stresses, couple stress, microrotation and displacement components as calculated from the generalized thermoelasticity (L-S), (G-L) and (CD) theories for the propagation of waves in semi-infinite elastic solids with cubic symmetry.

2   Formulation of the problem

We consider a homogeneous, micropolar generalized thermoelastic solid half-space with cubic symmetry. We consider rectangular coordinate system (x, y, z) having the origin on the surface $y = 0$ and the $y-$axis pointing vertically into the medium. A magnetic field with constant intensity $H = (0, 0, H_0)$ acts parallel to the bounding plane (taken as the direction of the $z-$axis). Suppose that an inclined line load is acting along the interface of the $y-$axis and its inclination with the $z-$axis is θ.

Due to the application of initial magnetic field $\mathbf{H}$, there are results of an induced magnetic field $\mathbf{h}$ and an induced electric field $\mathbf{E}$. The simplified linear equations of electrodynamics of a slowly moving medium for a homogeneous, thermally and electrically conducting elastic solid are

$$curl\,\mathbf{h} = \mathbf{J} + \varepsilon_0\,\mathbf{E} \tag{2.1}$$

$$curl\,\mathbf{E} = -\mu_0\,\mathbf{h} \tag{2.2}$$

$$div\,\mathbf{h} = 0 \tag{2.3}$$

$$\mathbf{E} = -\mu_0\,(\dot{\mathbf{u}} \times \mathbf{H}) \tag{2.4}$$

where $\dot{u}$ is the partied velocity of the medium, ε_0 is the dielectric constant, μ_0 is the magnetic permeability, and the small effect of temperature gradient on $\mathbf{J}$ is ignored. The dynamic displacement vector is actually measured from a steady state deformed position

and the deformation is supposed to be small.

The components of the magnetic intensity vector in the medium are

$$H_x = 0, \quad H_y = 0, \quad H_z = H_0 + h(x, y, z) \tag{2.5}$$

The electric intensity vector is normal to both the magnetic intensity and the velocity vectors. Thus, it has the components

$$E_x = -\mu_0 H_0 \dot{v}, \quad E_y = \mu_0 H_0 \dot{u}, \quad E_z = 0 \tag{2.6}$$

The current density vector J is parallel to E , thus

$$J_x = \frac{\partial h}{\partial y} + \mu_0 H_0 \varepsilon_0 \ddot{v}, \quad J_y = -\frac{\partial h}{\partial x} - \mu_0 H_0 \varepsilon_0 \ddot{u}, \quad J_z = 0 \tag{2.7}$$

$$\mathbf{h} = -H_0(0, 0, e) \tag{2.8}$$

If we restrict our analysis to plane strain parallel to the $xy-$plane with displacement vector $u = (u, v, 0)$ and the microrotation vector is $\varphi = (0, 0, \varphi_3)$, then the field equations and constitutive relations for the micropolar thermoelastic solid with cubic symmetry in the absence of body forces, body couples and heat sources can be written by following the equations given by Minagawa et. al. [20], Green and Lindsay [4] and Othman and Baljeet [24] as,

$$\begin{aligned} A_1 \frac{\partial^2 u}{\partial x^2} \quad &+ A_3 \frac{\partial^2 u}{\partial y^2} + (A_2 + A_4) \frac{\partial^2 v}{\partial x \partial y} + (A_3 - A_4) \frac{\partial \varphi_3}{\partial y} + \mu_0 H_0^2 \frac{\partial e}{\partial x} - \mu_0^2 H_0^2 \varepsilon_0 \frac{\partial^2 u}{\partial t^2} \\ &- v \frac{\partial}{\partial x} \left(T + t_1 \frac{\partial T}{\partial t} \right) = \rho \frac{\partial^2 u}{\partial t^2} \end{aligned} \tag{2.9}$$

$$\begin{aligned} A_3 \frac{\partial^2 v}{\partial x^2} \quad &+ A_1 \frac{\partial^2 u}{\partial y^2} + (A_2 + A_4) \frac{\partial^2 u}{\partial x \partial y} - (A_3 - A_4) \frac{\partial \varphi_3}{\partial x} + \mu_0 H_0^2 \frac{\partial e}{\partial y} - \mu_0^2 H_0^2 \varepsilon_0 \frac{\partial^2 v}{\partial t^2} \\ &- v \frac{\partial}{\partial y} \left(T + t_1 \frac{\partial T}{\partial t} \right) = \rho \frac{\partial^2 v}{\partial t^2} \end{aligned} \tag{2.10}$$

$$B_3 \nabla^2 \varphi_3 + (A_3 - A_4) \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) - 2(A_3 - A_4) \varphi_3 = \rho j \frac{\partial^2 \varphi_3}{\partial t^2} \tag{2.11}$$

$$K^* \nabla^2 T - \rho C^* (n_1 + t_0 \frac{\partial}{\partial t}) \dot{T} = \nu T_0 (n_1 + n_0 t_0 \frac{\partial}{\partial t}) \dot{e} \tag{2.12}$$

$$\sigma_{xx} = A_1 \frac{\partial u}{\partial x} + A_2 \frac{\partial v}{\partial y} - \nu \left(T + t_1 \frac{\partial T}{\partial t} \right) \tag{2.13}$$

$$\sigma_{yy} = A_2 \frac{\partial u}{\partial x} + A_1 \frac{\partial v}{\partial y} - \nu \left(T + t_1 \frac{\partial T}{\partial t} \right) \tag{2.14}$$

$$\sigma_{xy} = A_4 \left(\frac{\partial u}{\partial y} - \varphi_3 \right) + A_3 \left(\frac{\partial v}{\partial x} + \varphi_3 \right) \tag{2.15}$$

$$\sigma_{yx} = A_4 \left(\frac{\partial v}{\partial x} - \varphi_3 \right) + A_3 \left(\frac{\partial u}{\partial y} + \varphi_3 \right) \tag{2.16}$$

$$m_{yz} = B_3 \frac{\partial \varphi_3}{\partial y} \tag{2.17}$$

$$m_{xz} = B_3 \frac{\partial \varphi_3}{\partial x} \tag{2.18}$$

where σ_{ij} and m_{ij} are the components of force stress and coupled stress, respectively. ρ is the density, T is the absolute temperature, $\nu = (A_1 + 2A_2)\alpha_T$, α_T is the coefficient of linear expansion, j is the microinertia, K^* is the coefficient of thermal conductivity, C^* is the specific heat at constant strain; t_0 and t_1 are the thermal relaxation times and

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

For the thermoelastic micropolar isotropic medium, A_1, A_2, A_3, A_4 and B_3 are characteristic constants of the material defined as

$$A_1 = \lambda + 2\mu + k, \quad A_2 = \lambda, \quad A_3 = \mu + k, \quad A_4 = \mu, \quad B_3 = \gamma \quad (2.19)$$

where, λ, μ, k and γ are moduli of the medium.
For simplification, we shall use the following non-dimensional variables:

$$\overline{x}_i = \frac{\omega^*}{C_0}x_i, \quad \overline{u}_i = \frac{\rho C_0 \omega^*}{\nu T_0}u_i, \qquad \overline{t} = \omega^* t, \qquad \overline{t}_0 = \omega^* t_0, \quad \overline{t}_1 = \omega^* t_1, \quad \overline{T} = \frac{T}{T_0}$$

$$\overline{\sigma}_{ij} = \frac{\sigma_{ij}}{\nu T_0}, \quad \overline{m}_{ij} = \frac{\omega^*}{C_0 \nu T_0}m_{ij}, \quad \overline{\varphi}_3 = \frac{\rho C_0^2}{\nu T_0}\varphi_3, \quad \overline{g} = \frac{C_0}{\omega^*}g, \quad \overline{h} = \frac{h}{H_0}, \quad \{\overline{F}_1, \overline{F}_2\} = \frac{\{F_1, F_2\}}{\nu T_0}$$
$$(2.20)$$

where $\omega^* = \frac{\rho C^* C_0^2}{K^*}$ and $C_0^2 = \frac{A_1}{\rho}$.

Eqs. (2.9)-(2.12) take the following form (dropping the dashed for convenience)

$$\alpha\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \frac{A_3}{A_1} \cdot \frac{\partial^2 u}{\partial y^2} + \frac{(A_2 + A_4)}{A_1} \cdot \frac{\partial^2 v}{\partial x \partial y} + \frac{(A_3 - A_4)}{A_1} \cdot \frac{\partial\varphi_3}{\partial y} + R_H\frac{\partial e}{\partial x} - \frac{\partial}{\partial x}\left(T + t_1\frac{\partial T}{\partial t}\right)$$
$$(2.21)$$

$$\alpha\frac{\partial^2 v}{\partial t^2} = \frac{\partial^2 v}{\partial y^2} + \frac{A_3}{A_1} \cdot \frac{\partial^2 v}{\partial x^2} + \frac{(A_2 + A_4)}{A_1} \cdot \frac{\partial^2 u}{\partial x \partial y} + \frac{(A_3 - A_4)}{A_1} \cdot \frac{\partial\varphi_3}{\partial x} + R_H\frac{\partial e}{\partial y} - \frac{\partial}{\partial y}\left(T + t_1\frac{\partial T}{\partial t}\right)$$
$$(2.22)$$

$$\nabla^2\varphi_3 + \frac{(A_3 - A_4)C_0^2}{B_3\omega^{*2}}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - 2\frac{2(A_3 - A_4)C_0^2}{B_3\omega^{*2}}\varphi_3 = \frac{\rho j C_0^2}{B_3} \cdot \frac{\partial^2\varphi_3}{\partial t^2} \quad (2.23)$$

$$\nabla^2 T = \left(n_1 + t_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t} + \varepsilon(n_1 + n_0 t_0\frac{\partial}{\partial t})\frac{\partial}{\partial T}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \quad (2.24)$$

Introducing potential functions defined by

$$u = \frac{\partial q}{\partial x} + \frac{\partial\psi}{\partial y}, \quad v = \frac{\partial q}{\partial y} + \frac{\partial\psi}{\partial x} \quad (2.25)$$

in Eqs. (2.21)-(2.24), where $q(x, y, t)$ and $\psi(x, y, t)$ are scalar potential functions, we obtain

$$\left(\beta^2\nabla^2 - \alpha\frac{\partial^2}{\partial t^2}\right)q = \left(1 + t_1\frac{\partial}{\partial t}\right)T \quad (2.26)$$

$$\left(a_{11}\nabla^2 - \alpha\frac{\partial^2}{\partial t^2}\right)\psi = a_{12}\varphi_3 \quad (2.27)$$

$$a_{13}\nabla^2\psi + \left(\nabla^2 + 2a_{13} - a_{14}\frac{\partial^2}{\partial t^2}\right)\varphi_3 = 0 \tag{2.28}$$

$$\left[\nabla^2 - \left(n_1\frac{\partial}{\partial t} + t_0\frac{\partial^2}{\partial t^2}\right)\right]T = \varepsilon\left(n_1 + n_0 t_0\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\nabla^2 q \tag{2.29}$$

From Eqs. (2.8) and (2.25), we can obtain

$$h = -\nabla^2 q \tag{2.30}$$

where

$$a_{11} = \frac{A_3}{A_1}, \quad a_{12} = \frac{A_3 - A_4}{A_1}, \quad a_{13} = \frac{(A_4 - A_3)C_0^2}{B_3\omega^{*2}}, \quad a_{14} = \frac{\rho j C_0^2}{B_3},$$

$$\varepsilon = \frac{\nu^2 T_0}{\rho\omega^* K^*}, \quad R_H = \frac{\mu_0 H_0^2}{\rho C_0^2}, \quad \beta^2 = 1 + R_H \tag{2.31}$$

3   Normal mode analysis

The solution of the considered physical variables can be decomposed in terms of normal modes in the following form:

$$[\varphi_3, \psi, e, \sigma_{ij}, q, m_{ij}, T](x, y, t) = \left(\varphi_3^*(y), \psi^*(y), e^*(y), \sigma_{ij}^*(y), q^*(y), m_{ij}^*(y), T^*(y)\right)\exp(\omega t + iax) \tag{3.32}$$

where ω is a complex constant and a is the wave number in the $x-$direction. Using Eqs. (2.26)-(2.30) and (3.32),

$$\left(D^2 - a^2 - b^2\omega^2\right)q^* - n_3 T^* = 0 \tag{3.33}$$

$$\left(D^2 - a^2 - a_1\omega^2\right)\psi^* - a_2\varphi_3^* = 0 \tag{3.34}$$

$$a_{13}\left(D^2 - a^2\right)\psi^* + \left(D^2 - a^2 - 2a_{14} - a_{15}\omega^2\right)\varphi_3^* = 0 \tag{3.35}$$

$$\left(D^2 - a^2 - n_4\right)T^* - \varepsilon^*\left(D^2 - a^2\right)q^* = 0 \tag{3.36}$$

where

$$b^2 = \frac{\alpha}{\beta^2}, \qquad a_1 = \frac{\alpha}{a_{11}}, \qquad a_2 = \frac{a_{12}}{a_{11}},$$

$$n_3 = \frac{1 + t_1\omega}{\beta^2}, \quad n_4 = \omega(n_1 + t_0\omega), \quad \varepsilon^* = \varepsilon\omega(n_1 + n_0\omega t_0)$$

Eliminating ψ^* and T^* in Eqs. (3.33)-(3.36), we obtain

$$\left[D^4 - B_1 D^2 + B_2\right](q^*, T^*) = 0 \tag{3.37}$$

and

$$\left[D^4 - B_3 D^2 + B_4\right](\varphi^*, \psi^*) = 0 \tag{3.38}$$

where

$$B_1 = 2a^2 + n_4 + b^2\omega^2 + n_3\varepsilon^* \tag{3.39}$$

$$B_2 = a^4 + \left(n_4 + b^2\omega^2 + n_3\varepsilon^*\right)a^2 + n_4 b^2\omega^2 \tag{3.40}$$

$$B_3 = 2a^2 + (a_1 + a_{15})\,\omega^2 - 2a_{13} - a_2 a_{13} \tag{3.41}$$

$$B_4 = a^4 + \left(a_1\omega^2 - 2a_{13} + a_{14}\omega^2 - a_2 a_{13}\right) + a_1\omega^2\left(-2a_{13} + a_{14}\omega^2\right) \tag{3.42}$$

The solutions of Eqs. (3.37) and (3.38), which are bounded for $y > 0$, are given by

$$q^* = \sum_{j=1}^{2} M_j(a,\omega)e^{-k_j y} \tag{3.43}$$

$$T^* = \sum_{j=1}^{2} M_j^{'}(a,\omega)e^{-k_j y} \tag{3.44}$$

$$\psi^* = \sum_{n=3}^{4} M_n(a,\omega)e^{-k_n y} \tag{3.45}$$

$$\varphi_3^* = \sum_{n=3}^{4} M_n^{'}(a,\omega)e^{-k_n y} \tag{3.46}$$

where $M_j(a,\omega)$, $M_j^{'}(a,\omega)$, $M_n(a,\omega)$ and $M_n^{'}(a,\omega)$ are some parameters depending on a and ω. $k_j^2, (j = 1, 2)$ are the roots of the characteristic equation of Eq. (3.37) and $k_n^2, (n = 3, 4)$ are the roots of the characteristic equation of Eq. (3.38). Setting Eqs. (3.43)-(3.46) into Eqs. (3.37) and (3.38), we get the following relations

$$T^* = \sum_{j=1}^{2} R_j M_j(a,\omega)e^{-k_j y} \tag{3.47}$$

$$\varphi_3^* = \sum_{n=3}^{4} R_n M_n(a,\omega)e^{-k_n y} \tag{3.48}$$

where

$$R_{1,2} = \frac{1}{n_1}\left(K_{1,2}^2 - a^2 - b^2\omega^2\right) \tag{3.49}$$

$$R_{3,4} = \frac{1}{a_1}\left(K_{3,4}^2 - a^2 - a_1\omega^2\right) \tag{3.50}$$

The roots $K_{1,2}^2$ and $K_{3,4}^2$ of Eqs. (3.37) and (3.38), respectively, are given by

$$K_{1,2}^2 = \frac{1}{2}\left(B_1 \pm \sqrt{B_1^2 - 4B_2}\right) \tag{3.51}$$

$$K_{3,4}^2 = \frac{1}{2}\left(B_3 \pm \sqrt{B_3^2 - 4B_4}\right) \tag{3.52}$$

4   Application

We consider a normal line load F_1 acting in the positive $y-$direction on the interface $y = 0$ along the $z-$axis and the tangential load F_2 acting at the origin in the positive $x-$direction, then the boundary conditions at the horizontal plane $y = 0$ are

$$\sigma_{yy} = -F_1, \quad \sigma_{yx} = -F_2, \quad m_{yz} = 0, \quad T = f(x, t) \tag{4.53}$$

Using (2.20), (2.25), (2.26)-(2.29) in the non-dimensional boundary conditions and using (3.43), (3.45), (3.47)-(3.48), we obtain the expressions of displacements, force stress, coupled stress and temperature distribution for the micropolar generalized thermoelastic medium with magnetic field as follows:

$$u^*(y) = ia\left(M_1 e^{-k_1 y} + M_2 e^{-k_2 y}\right) - k_3 M_3 e^{-k_3 y} - k_4 M_4 e^{-k_4 y} \tag{4.54}$$

$$v^*(y) = -k_1 M_1 e^{-k_1 y} - k_2 M_2 e^{-k_2 y} - ia\left(M_3 e^{-k_3 y} + k_4 M_4 e^{-k_4 y}\right) \tag{4.55}$$

$$\sigma_{yy}^*(y) = s_1 M_1 e^{-k_1 y} + s_2 M_2 e^{-k_2 y} + s_3 M_3 e^{-k_3 y} + s_4 M_4 e^{-k_4 y} \tag{4.56}$$

$$\sigma_{yx}^*(y) = r_1 M_1 e^{-k_1 y} + r_2 M_2 e^{-k_2 y} + r_3 M_3 e^{-k_3 y} + r_4 M_4 e^{-k_4 y} \tag{4.57}$$

$$m_{yz}^*(y) = -B_3\left(k_3 R_3 M_3 e^{-k_3 y} + k_4 R_4 M_4 e^{-k_4 y}\right)\omega^{*2}/\rho C_0^4 \tag{4.58}$$

$$T^*(y) = R_1 M_1 e^{-k_1 y} + R_2 M_2 e^{-k_2 y} \tag{4.59}$$

where

$$\begin{aligned}
s_1 &= -a^2 A_2/\rho C_0^2 + A_1 k_1^2/\rho C_0^2 - R_1(1 + \omega t_1) \\
s_2 &= -a^2 A_2/\rho C_0^2 + A_1 k_2^2/\rho C_0^2 - R_2(1 + \omega t_1) \\
s_3 &= iak_3(A_1 - A_2)/\rho C_0^2 \\
s_4 &= iak_4(A_1 - A_2)/\rho C_0^2 \\
r_1 &= -iak_1(A_3 + A_4)/\rho C_0^2 \\
r_2 &= -iak_2(A_3 + A_4)/\rho C_0^2 \\
r_3 &= \left(a^2 A_4 + R_3(A_3 - A_4) + A_3 k_3^2\right)/\rho C_0^2 \\
r_4 &= \left(a^2 A_4 + R_4(A_3 - A_4) + A_3 k_4^2\right)/\rho C_0^2
\end{aligned} \tag{4.60}$$

Invoking the boundary conditions (4.53) at the surface $y = 0$ of the plate, we obtain a system of four equations. After applying the inverse of matrix method, we have the values of the four constants $M_j, (j = 1, 2)$ and $M_n, (n = 3, 4)$. Hence, we obtain the expressions of displacements, force stress, coupled stress and temperature distribution for the micropolar generalized thermoelastic medium.

5   Numerical results and discussions

In order to illustrate the theoretical results obtained in the preceding section and to compare these in the context of various theories of thermoelasticity, we now present some numerical results. In the calculation process, we take the case of magnesium crystal (Eringen, 1984) as the material subjected to mechanical and thermal disturbances for numerical calculations considering the material medium as that of copper. Since, ω is complex, then we take $\omega = \omega_0 + i\zeta$. The other constants of the problem are taken as $\omega_0 = -2; \zeta = 1;$ The physical constants used are:

$$\rho = 1.74 \; gm/cm^3, \qquad j = 0.2 \times 10^{-15} \; cm^3, \qquad \lambda = 9.4 \times 10^{11} \; dyne/cm^2,$$

$$T_0 = 23 \; {}^\circ C, \qquad \qquad \mu = 4.0 \times 10^{11} \; dyne/cm^2, \quad K = 1.0 \times 10^{11} \; dyne/cm^2,$$

$$\gamma = 0.779 \times 10^{-4} \; dyne, \quad C^* = 0.23 \; cal/gm{}^\circ C, \qquad K^* = 0.6 \times 10^{-2} \; cal/cm \; sec{}^\circ C$$

The variation of the normal component of displacement u, normal force stress σ_{zz}, tangential couple stress m_{zy} and temperature distribution with distance y at the plane $x = 2$, $a = 2$, $t = 0.1$, $F_1 = F\cos\theta$, $F_2 = F\sin\theta$, $F = 1$, $\theta = 45^\circ$ and $f = 2$.

The numerical values for the normal displacement component u, the temperature T, the force stress component σ_{yx}, σ_{yy} and the couple stress m_{yz} are shown in Fig. 1-Fig. 6. These figures represent the solution obtained using the coupled theory (CD theory: $n_0 = 0, n_1 = 1, n_2 = 1, t_0 = 0, t_1 = 0$), the generalized theory with one relaxation time (Lord-Shulman (L-S) theory: $n_0 = 1, n_1 = 1, n_2 = 1, t_0 = 0.02, t_1 = 0$), and generalized theory with two relaxation times (Green-Lindsay (G-L) theory: $n_0 = 0, n_1 = 1, n_2 = 1, t_0 = 0.02, t_1 = 0.03$).
Fig. 1. shows that temperature satisfies the boundary condition at $y = 0$. For the three theories, temperature decreases exponentially as distance y increases. Fig. 2. shows that the displacement component u increases as y increases. Fig. 3. shows that the displacement component v decreases at the beginning and starts increasing at $y = 0.5$ (minimum) and then converges to zero as y increases. Fig. 4. shows that the stress component σ_{yy} satisfies the boundary condition at $y = 0$. It increases at the beginning and starts decreasing at $y = 0.5$ (maximum) in the context of the three theories. Fig. 5. shows that the stress component σ_{yx} satisfies the boundary condition at $y = 0$. It increases at the beginning and starts decreasing at $y = 0.2$ (maximum) in the context of the three theories. Fig. 6. explains that the tangential coupled stress m_{yz} satisfies the boundary condition at $y = 0$. It increases at the beginning and starts decreasing around $y = 0.2$ (maximum) and increases at $y = 0.7$ (minimum) and then converges to zero as y increasing. Fig. 7. shows that the microrotation component φ_3 increases at the beginning and starts decreasing at $y = 0.4$ (maximum) in the context of the three theories. Fig. 8-Fig. 13 show the comparison between the temperature T, displacement components u, v, the force stress component σ_{yy}, the couple stress m_{yz} the microrotation component φ_3 in the case of different inclined load (variation of angles at $\theta = 5, \theta = 10$ and $\theta = 15$) under GL theory.
By comparing figures of solutions obtained under the three thermoelastic theories, important phenomena are observed: the curves in the context of the (L-S), (G-L) and CD theories decrease exponentially as distance y and inclined load increase. The value of all the physical quantities converges to zero with an increase in distance y. The amplitude

of displacement component u, force stress component σ_{yy}, σ_{yx} the couple stress m_{yz} and the microrotation component φ_3 increase and then decrease with an increase in distance y, but the temperature component T and displacement components v decrease with an increase in distance y.
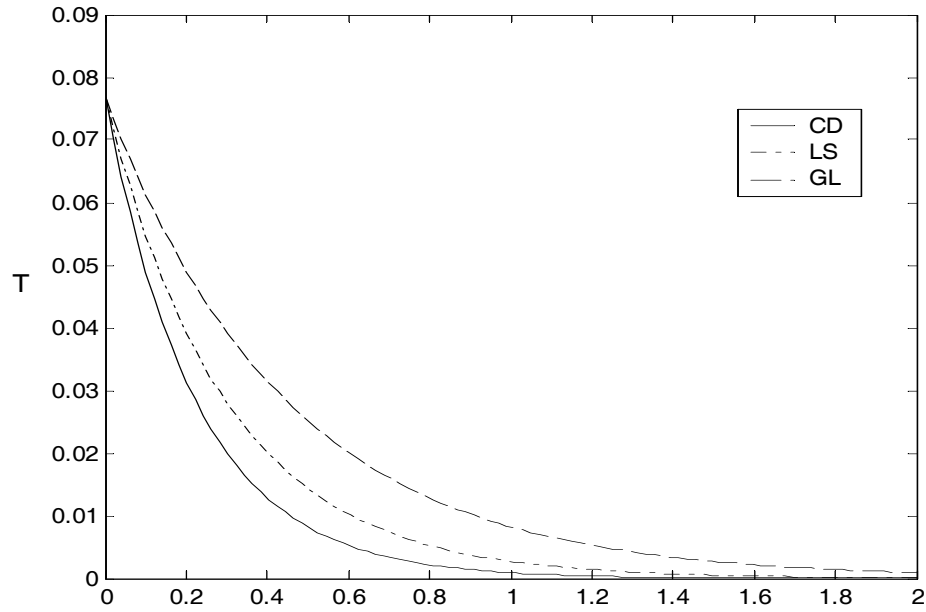
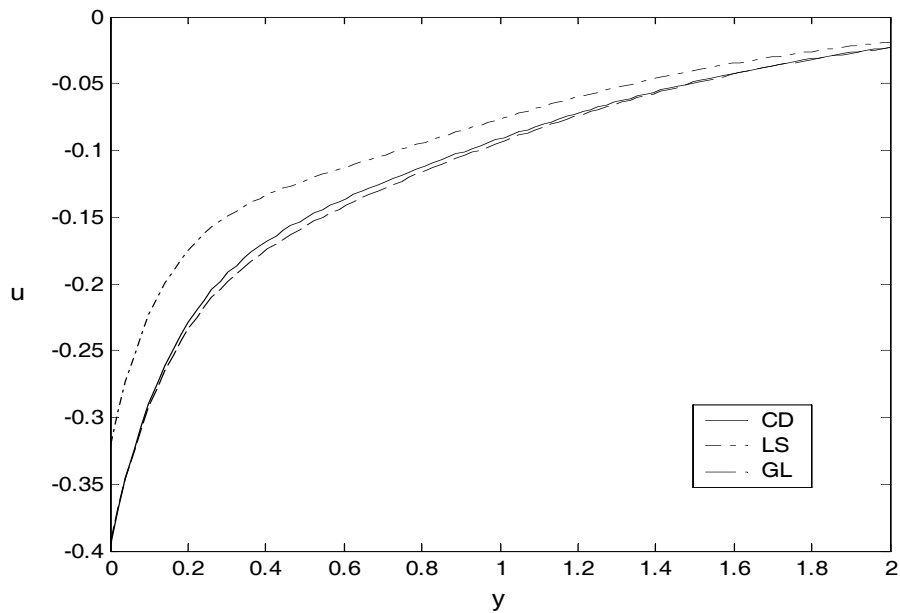Fig. 1. Variation of temperature distribution T at $\theta = 45°$

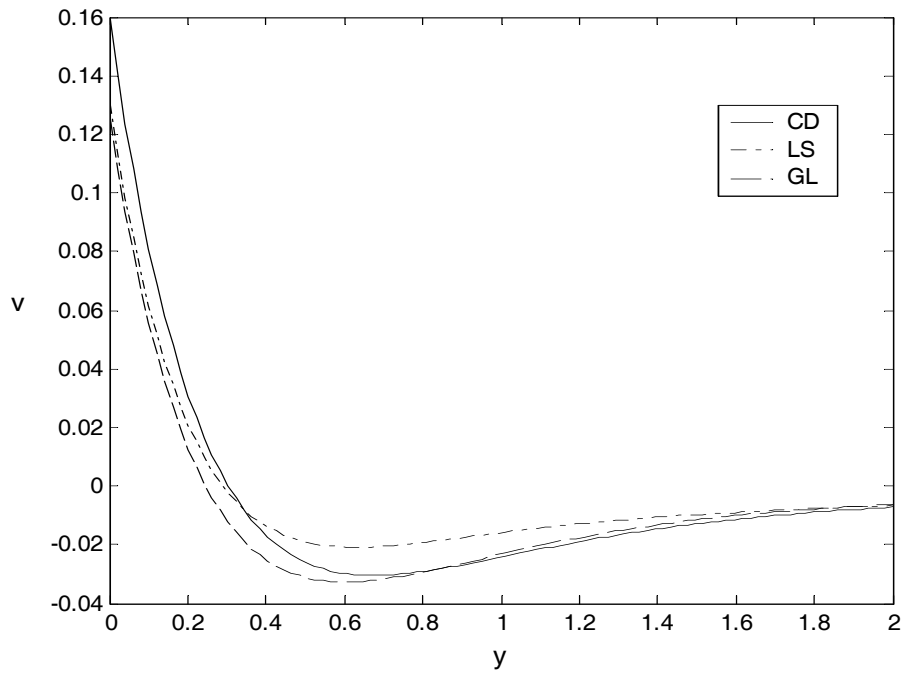Fig. 2. Variation of displacement distribution u at $\theta = 45°$

Fig. 3. Variation of displacement distribution v at $\theta = 45°$
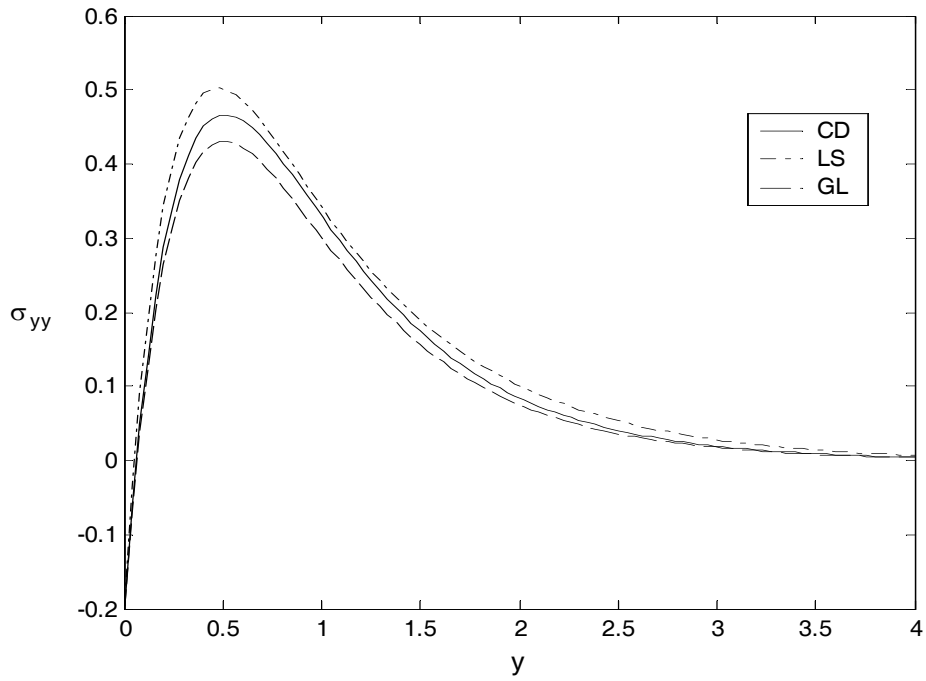
Fig. 4. Variation of stress distribution σ_{yy} at $\theta = 45°$
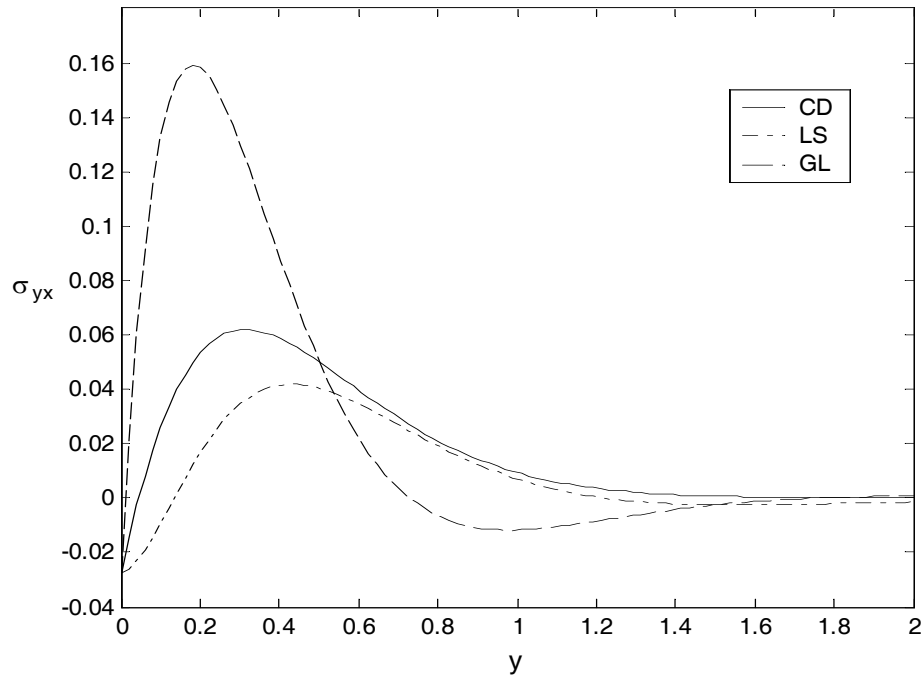
Fig. 5. Variation of stress distribution σ_{yx} at $\theta = 45°$
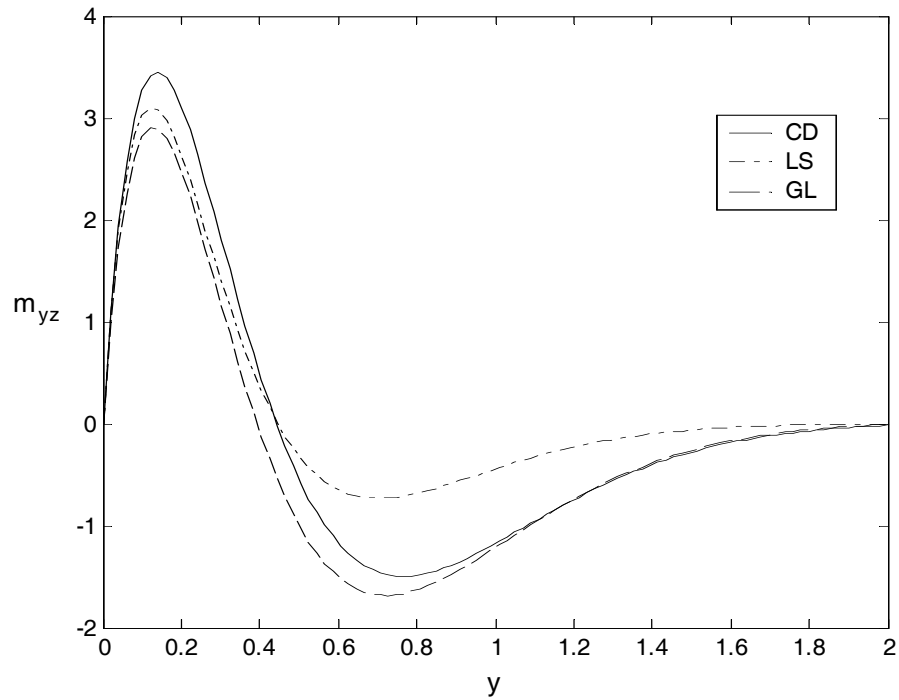
Fig. 6. Variation of tangential couple stress m_{yz} at $\theta = 45°$
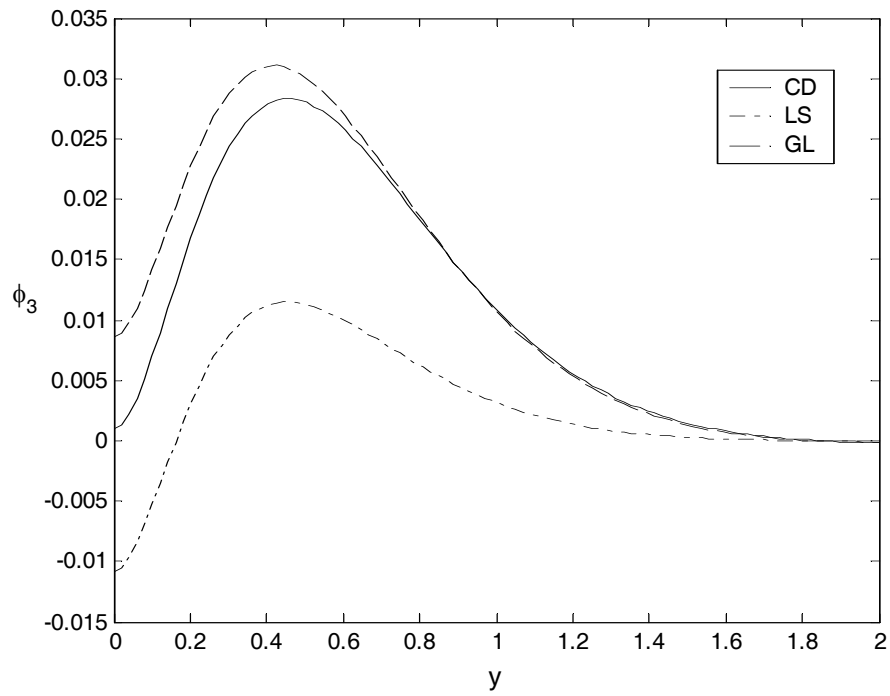
Fig. 7. Variation of the microrotation component φ_3 at $\theta = 45°$
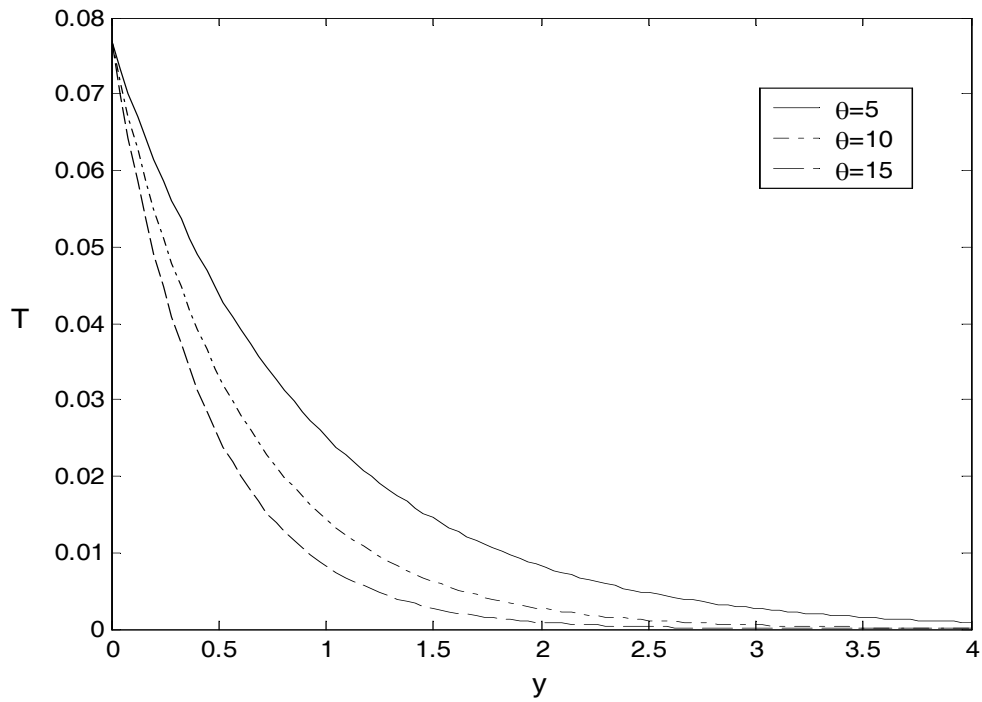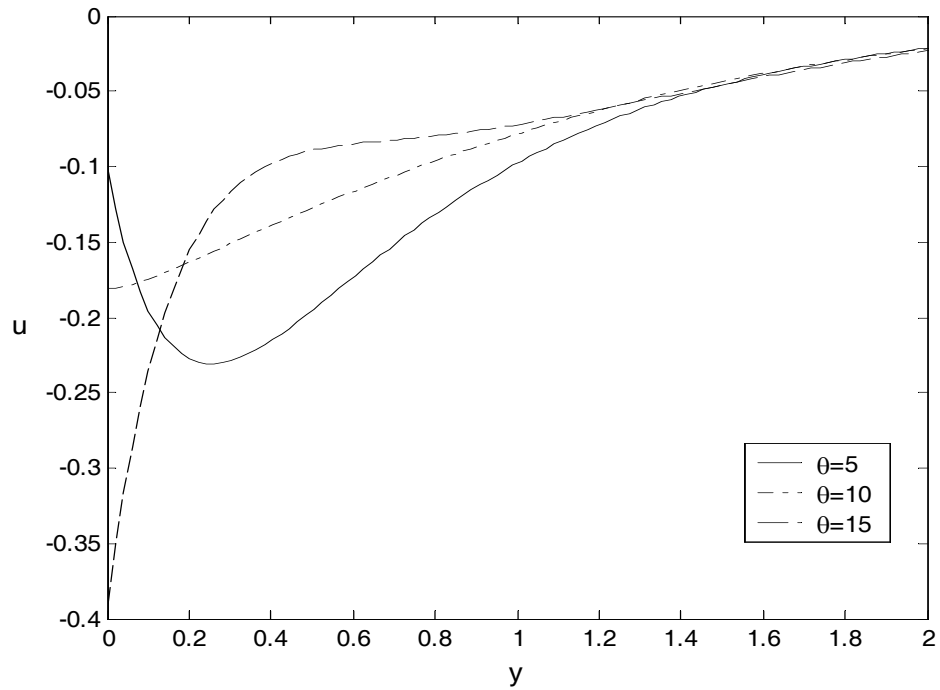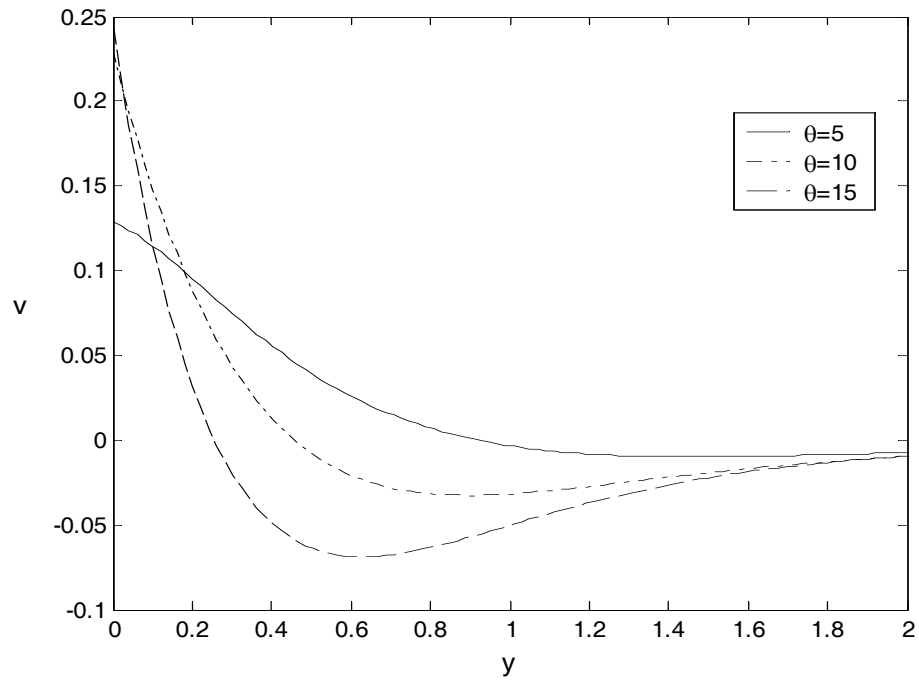

Fig. 8. Temperature distribution T with variation of angles under GL theory

Fig. 9. Displacement distribution u with variation of angles under GL theory

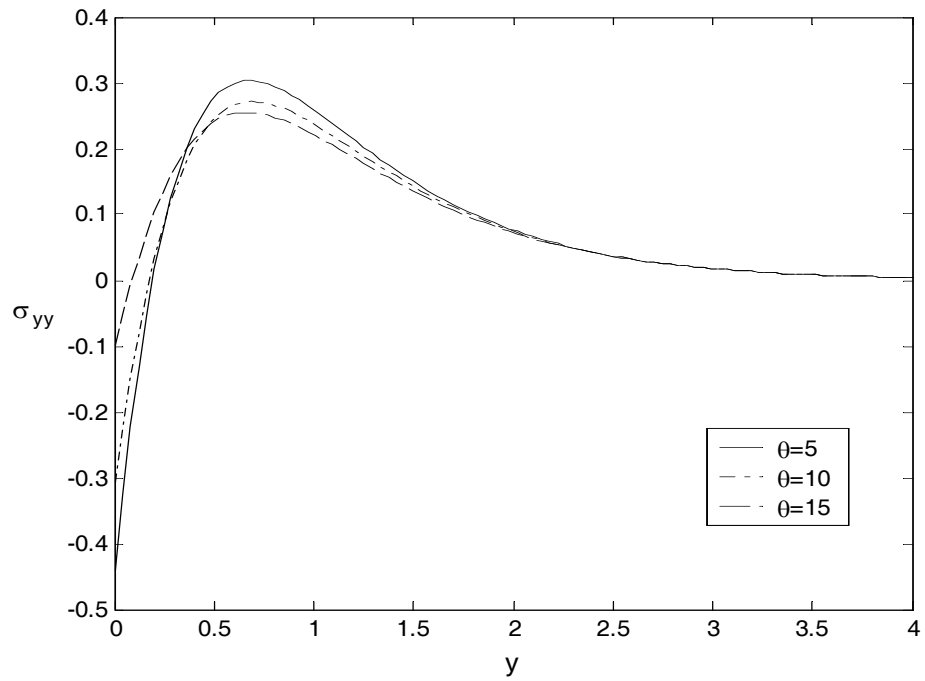Fig. 10. Displacement distribution v with variation of angles under GL theory

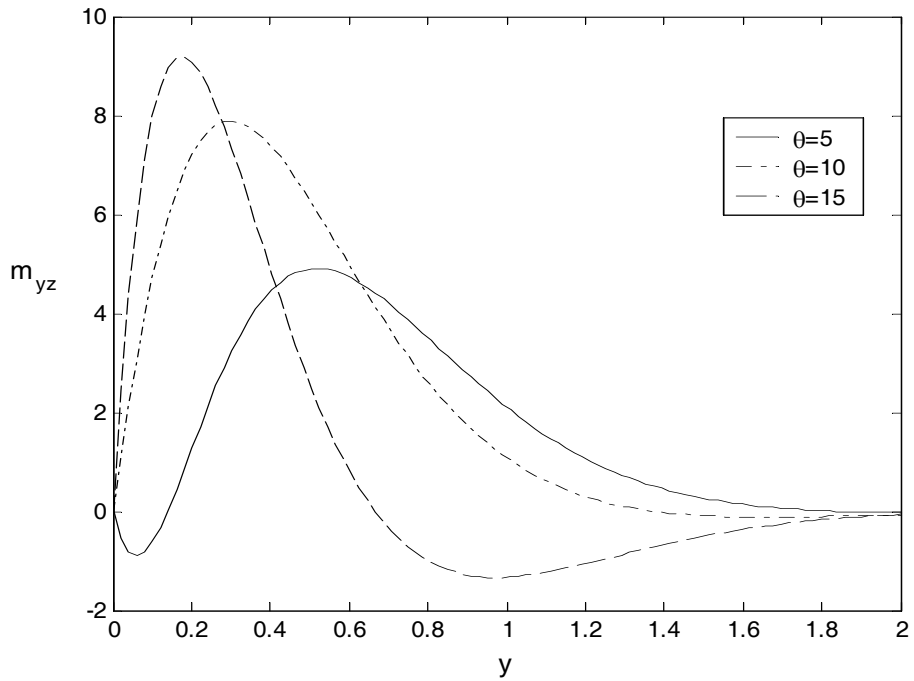Fig. 11. Stress distribution σ_{yy} with variation of angles under GL theory

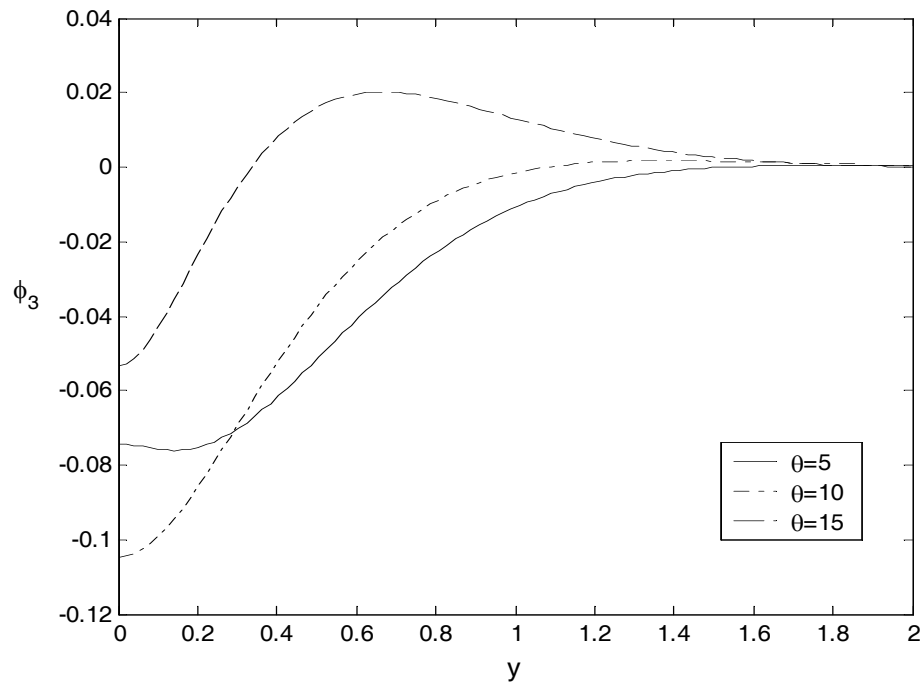Fig. 12. Tangential couple stress m_{yz} with variation of angles under GL theory

Fig. 13. The microrotation component φ_3 with variation of angles under GL theory

6   Conclusion

The properties of a body depend largely on the direction of symmetry and the inclination of the applied source. When concentrated force is applied to the surface of a solid, the values of of normal displacement, tangential force stress and the couple stress decrease as the angle of inclination of the source increase. The value of all the physical quantities converge to zero with an increase in the distance y.

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