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Triangular approximations of fuzzy number with value and ambiguity functions

M. Adabitabarfirozja * † , Z. Eslampia ‡

Abstract

For any fuzzy number value and ambiguity are very important, hence we propose a method for finding the nearest triangular approximations of fuzzy number by definitions of value and ambiguity functions. Initially, we define the value and ambiguity functions for each fuzzy number. This evident that, two fuzzy numbers will be close if functions of value and ambiguity is close, therefor for finding nearest triangular fuzzy number of any fuzzy number we find the nearest value and ambiguity functions by least square approach, such that the computational complexity is less than other methods. At last, we show some examples of fuzzy numbers and its triangular approximation obtained by this method.

Keywords : General fuzzy number; Triangular approximations; Least square approach.

1 Introduction

In some applications of fuzzy logic, it is difficult to use general fuzzy numbers therefore, it may be better to use fuzzy numbers with the same type. Some researchers introduce the defuzzification method which replaces a general fuzzy number by obtaining of typical value from a fuzzy number according to some specified characteristics, such as center of gravity, median, etc by a single number (See [11, 18]) but it is evident that we lose many important information. Some of the researchers considered a parametric approximation of a fuzzy number such as Nasibov and Peker [13], Ban [7], which respect to the average Euclidean distance. One of the other methods interval approximation of fuzzy numbers which, Chanas [9], Grzegorzewski [14], Grzegorzewski [15], Roventa and Spircu [19], which a fuzzy area is converted into one in the interval area. Furthermore, there are so many methods for triangular or trapezoidal approximation of fuzzy numbers such as, Saneifard [20] suggests a new approach to the problem of defuzzification using the weighted metric (weighted distance) between two fuzzy numbers. Saneifard [21] solved the problem of defuzzification in computing with regular weighted function. Abbasbandy and Asady [3] introduced a fuzzy trapezoidal approximation using a metric between two fuzzy numbers. Grzegorzewski and Mrowka [16], discussed a bout the problem of the trapezoidal approximation of a fuzzy number with expected interval but, the result of approximation is not always a trapezoidal fuzzy number where it shows in Allahviranloo and Adabitabar Firozja [6], which then resolved by Grzegorzewski and Mrowka [17] and Ban [8]. Also Abbasbandy and Amirfakhrian [1] have used the value and ambiguity of fuzzy number and have solved an optimization problem to obtain the nearest triangular or trapezoidal fuzzy number. Abbasbandy, E. Ahmady and N. Ahmady [4] obtained a nearest triangular fuzzy number by using a weighted valuations for an arbitrary fuzzy number. Now, in this paper, we use a new definitions of value and ambiguity functions to approximate a fuzzy number, and we will obtain the nearest triangular fuzzy number using value and ambiguity functions. In Section 2, we recall some basic definition and results on fuzzy numbers. In Section 3, we will introduce the value and ambiguity functions and then nearest triangular fuzzy number with some reasonable properties are given. Examples are provided in Section 4 and the conclusions are given in the final Section 5.

*Corresponding author. mohamadsadega@yahoo.com

†Department of Mathematics, Qaemshar Branch, Islamic Azad University, Qaemshahr, Iran.

‡Department of Mathematics, Qaemshar Branch, Islamic Azad University, Qaemshahr, Iran.

2 Preliminaries

Definition 2.1 *A fuzzy number $\tilde{u}$ in parametric form is a pair of functions $(\underline{u}(\alpha), \bar{u}(\alpha))$; $0 \leq \alpha \leq 1$, which satisfy the following requirements:*

(1) $\underline{u}(\alpha)$ *is a bounded monotonic increasing left continuous function,*

(2) $\bar{u}(\alpha))$ *is a bounded monotonic decreasing left continuous function,*

(3) $\underline{u}(\alpha) \leq \bar{u}(\alpha),\ 0 \leq \alpha \leq 1,$

(4) $\underline{u}(1) = \bar{u}(1).$

In this definition, if $\underline{u}(1) < \bar{u}(1)$, then we will have the fuzzy interval.
A fuzzy set $\tilde{u}$ is a generalized left right fuzzy number (GLRFN) and denoted as $\tilde{u} = (a, b, c)_{LR}$, if its membership function satisfy the following:

$$\tilde{u}(x) = \begin{cases} L(\frac{b-x}{b-a}), & a \leq x \leq b, \\ R(\frac{x-b}{c-b}), & b \leq x \leq c, \\ 0, & otherwise \end{cases} \tag{2.1}$$

Where L and R are strictly decreasing functions defined on $[0, 1]$ and satisfying the conditions:

$$\begin{array}{lll} L(t) = R(t) = 1 & if & t \leq 0 \\ L(t) = R(t) = 0 & if & t \geq 1 \end{array} \tag{2.2}$$

Triangular fuzzy numbers (TFN) are special cases of GLRFN with $L(t) = R(t) = 1 - t$ where the membership function is:

$$\tilde{u}(x) = \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b, \\ \frac{c-x}{c-b}, & b \leq x \leq c, \\ 0 & otherwise \end{cases} \tag{2.3}$$

Its parametric form is $\underline{u}(\alpha) = (b-a)\alpha + a$ and $\bar{u}(\alpha) = c - (c-b)\alpha$.

3 Triangular Approximate of fuzzy Numbers

Delgado, Ma and Voxman [10] define two parameters of fuzzy number $\tilde{u}$, the value and ambiguity with respect to a reducing function $s(\alpha)$ by $V_s(\tilde{u})$ and $A_s(\tilde{u})$ as follows:

$$V_s(\tilde{u}) = \int_0^1 s(\alpha)(\underline{u}(\alpha) + \overline{u}(\alpha))d\alpha, \tag{3.4}$$

$$A_s(\tilde{u}) = \int_0^1 s(\alpha)(\overline{u}(\alpha) - \underline{u}(\alpha))d\alpha$$

In this section, we use a new definition of value and ambiguity functions to approximate the nearest triangular fuzzy number of a general fuzzy number.

Definition 3.1 *We show the value function of $\tilde{u}$ with function $s(\alpha)$ as $V_s f_{\tilde{u}}(.)$ and define it as follows;*

$$\begin{cases} V_s f_{\tilde{u}}(.) : [0, 1] \mapsto \mathbb{R} \\ V_s f_{\tilde{u}}(\alpha) = s(\alpha)(\underline{u}(\alpha) + \overline{u}(\alpha)) \end{cases} \tag{3.5}$$

Where $s(\alpha)$, is a weight function such that continuous positive function defined on $[0, 1]$.

Definition 3.2 *we show the ambiguity function of $\tilde{u}$ with function $s(\alpha)$ as $A_s f_{\tilde{u}}$ and define it as follows;*

$$\begin{cases} A_s f_{\tilde{u}}(.) : [0, 1] \mapsto [0, +\infty) \\ A_s f_{\tilde{u}}(\alpha) = s(\alpha)(\overline{u}(\alpha) - \underline{u}(\alpha)) \end{cases} \tag{3.6}$$

Where $s(\alpha)$, is a weight function such that continuous positive function defined on $[0, 1]$.

Theorem 3.1 *If $\tilde{u} = (a, b, c)$ is a triangular fuzzy number then*

$$V_s f_{\tilde{u}}(\alpha) = s(\alpha)((2b - a - c)\alpha + a + c), \tag{3.7}$$

$$A_s f_{\tilde{u}}(\alpha) = s(\alpha)(c - a - (c - a)\alpha).$$

In this paper, we found the nearest triangular fuzzy number to $\tilde{u}$ with a triangular fuzzy number $T(\tilde{u}) = (t_1(u), t_2(u), t_3(u))$ where $V_s f_{\tilde{u}}(\alpha)$ and $A_s f_{\tilde{u}}(\alpha)$ is nearest to $V_s f_{T(\tilde{u})}(\alpha)$ and $A_s f_{T(\tilde{u})}(\alpha)$ and because, $Core(\tilde{u})$ is a very important for decision maker, hence we want that $Core(\tilde{u}) = Core(T(\tilde{u}))$, therefor $t_2(u) = \underline{u}(1) = \bar{u}(1)$.

Theorem 3.2 *Let, $T(\tilde{u}) = (t_1(u), t_2(u), t_3(u))$ is the nearest TFN to $\tilde{u}$ then $V_s f_{T(\tilde{u})}(\alpha)$ is the nearest to the $V_s f_{\tilde{u}}(\alpha)$ if*

$$t_1(u) + t_3(u) =$$

$$\frac{2t_2(\tilde{u}) \int_0^1 (\alpha^2 - \alpha)s^2(\alpha)d\alpha - \int_0^1 (\alpha - 1)s(\alpha)V_s f_{\tilde{u}}(\alpha)d\alpha}{\int_0^1 (1 - \alpha)^2 s^2(\alpha)d\alpha} \tag{3.8}$$

Proof. Regarding to Theorem 3.1

$$V_s f_{T(\tilde{u})}(\alpha) = s(\alpha)((2t_2(u) - t_1(u) - t_3(u))\alpha + t_1(u) + t_3(u)) \tag{3.9}$$

We solve the following optimization with least square approach for obtaining nearest $V_s f_{\tilde{u}}(\alpha)$ to $V_s f_{T(\tilde{u})}(\alpha)$

$$min\ E = min \int_0^1 (V_s f_{\tilde{u}}(\alpha) - V_s f_{T(\tilde{u})}(\alpha))^2 d\alpha \tag{3.10}$$
$$=$$

$$min \int_0^1 [V_s f_{\tilde{u}}(\alpha) - \{s(\alpha)((2t_2(u) - t_1(u) - t_3(u))\alpha + t_1(u) + t_3(u))\}]^2 d\alpha.$$

Where should

$$\begin{cases} \frac{\partial E}{\partial t_1(u)} = 0 \\ \frac{\partial E}{\partial t_3(u)} = 0 \end{cases} \Rightarrow t_1(u) + t_3(u) = \tag{3.11}$$

$$\frac{2t_2(\tilde{u}) \int_0^1 (\alpha^2 - \alpha)s^2(\alpha)d\alpha - \int_0^1 (\alpha - 1)s(\alpha)V_s f_{\tilde{u}}(\alpha)d\alpha}{\int_0^1 (1-\alpha)^2 s^2(\alpha)d\alpha}$$

Theorem 3.3 *Let, $T(\tilde{u}) = (t_1(u), t_2(u), t_3(u))$ is the nearest TFN to $\tilde{u}$ then $A_s f_{T(\tilde{u})}(\alpha)$ is the nearest to the $A_s f_{\tilde{u}}(\alpha)$ if*

$$t_3(u) - t_1(u) = \frac{\int_0^1 s(\alpha)(1-\alpha)A_s f_{\tilde{u}}(\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} \tag{3.12}$$

Proof. Regarding to Theorem 3.1.

$$A_s f_{T(\tilde{u})}(\alpha) = s(\alpha)(t_3(u) - t_1(u) - (t_3(u) - t_1(u))\alpha). \tag{3.13}$$

We solve the following optimization with least square approach for obtaining nearest $A_s f_{\tilde{u}}(\alpha)$ to $A_s f_{T(\tilde{u})}(\alpha)$

$$min\ E = min \int_0^1 (A_s f_{\tilde{u}}(\alpha) - A_s f_{T(\tilde{u})}(\alpha))^2 d\alpha = \tag{3.14}$$

$$min \int_0^1 [A_s f_{\tilde{u}}(\alpha) - \{s(\alpha)(t_3(u) - t_1(u) - (t_3(u) - t_1(u))\alpha)\}]^2 d\alpha,$$

Where should

$$\begin{cases} \frac{\partial E}{\partial t_1(u)} = 0 \\ \frac{\partial E}{\partial t_3(u)} = 0 \end{cases} \Rightarrow t_3(u) - t_1(u) = \tag{3.15}$$

$$\frac{\int_0^1 s(\alpha)(1-\alpha)A_s f_{\tilde{u}}(\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha}. \qquad \blacksquare$$

Theorem 3.4 *If $T(\tilde{u}) = (t_1(u), t_2(u), t_3(u))$ is a nearest triangular approximation of $\tilde{u} = (\underline{u}(\alpha), \overline{u}(\alpha))$ based on the nearest $V_s f_{\tilde{u}}(.)$ and $A_s f_{\tilde{u}}(.)$, then*

$$\begin{cases} t_2(u) = \underline{u}(1) = \overline{u}(1) \\ t_1(u) = \frac{\int_0^1 s^2(\alpha)(1-\alpha)(\underline{u}(\alpha) - t_2(u)\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} \\ t_3(u) = \frac{\int_0^1 s^2(\alpha)(1-\alpha)(\overline{u}(\alpha) - t_2(u)\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} \end{cases} \tag{3.16}$$

Proof. Regarding to assumptions in this work where, $Core(\tilde{u}) = Core(T(\tilde{u}))$ and (3.8) and (3.12) proof is evident.■

3.1 Properties

In this section we consider some reasonable properties of the approximation.

Theorem 3.5 *If $\tilde{u}$ is a triangular fuzzy number, then $T(\tilde{u}) = \tilde{u}$.*

Proof. Let $\tilde{u} = (a, b, c)$ is a triangular fuzzy number, then by $\underline{u}(\alpha) = (b-a)\alpha + a$ and $\overline{u}(\alpha) = c - (c-b)\alpha$ and Theorem 3.4

$$\begin{cases} t_2(u) = \underline{u}(1) = \overline{u}(1) = b \\ t_1(u) = \frac{\int_0^1 s^2(\alpha)(1-\alpha)(\underline{u}(\alpha) - t_2(u)\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} = \frac{\int_0^1 s^2(\alpha)(1-\alpha)((b-a)\alpha + a - b\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} = a \qquad \blacksquare \\ t_3(u) = \frac{\int_0^1 s^2(\alpha)(1-\alpha)(\overline{u}(\alpha) - t_2(u)\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} = \frac{\int_0^1 s^2(\alpha)(1-\alpha)(c - (c-b)\alpha - b\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} = c \end{cases} \tag{3.17}$$

Theorem 3.6 *Nearest triangular approximation is invariant to translation, $T(\tilde{u} + k) = T(\tilde{u}) + k$ for all $k \in \mathbb{R}$.*

Proof. By Theorem 3.4

$$\begin{cases} t_2(u+k) = \underline{u+k}(1) = \overline{u+k}(1) = \underline{u}(1) + k = \overline{u}(1) + k = t_2(u) + k \\ t_1(u+k) = \frac{\int_0^1 s^2(\alpha)(1-\alpha)(\underline{u+k}(\alpha) - t_2(u+k)\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} = \frac{\int_0^1 s^2(\alpha)(1-\alpha)(\underline{u}(\alpha) + k - t_2(u)\alpha - k\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} \\ = \frac{\int_0^1 s^2(\alpha)(1-\alpha)(\underline{u}(\alpha) - t_2(u)\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} + \frac{\int_0^1 s^2(\alpha)(1-\alpha)k(1-\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} = t_1(u) + k \qquad \blacksquare \\ t_3(u+k) = \frac{\int_0^1 s^2(\alpha)(1-\alpha)(\overline{u+k}(\alpha) - t_2(u+k)\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} = \frac{\int_0^1 s^2(\alpha)(1-\alpha)(\overline{u}(\alpha) + k - t_2(u)\alpha - k\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} \\ = \frac{\int_0^1 s^2(\alpha)(1-\alpha)(\overline{u}(\alpha) - t_2(u)\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} + \frac{\int_0^1 s^2(\alpha)(1-\alpha)k(1-\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} = t_3(u) + k \end{cases} \tag{3.18}$$

Theorem 3.7 *The nearest triangular approximation is scale invariant,*

$$T(k\tilde{u}) = kT(\tilde{u}) = \begin{cases} k(t_1(u), t_2(u), t_3(u)) = (kt_1(u), kt_2(u), kt_3(u)) & k \geq 0, \\ k(t_1(u), t_2(u), t_3(u)) = (kt_3(u), kt_2(u), kt_1(u)) & k < 0, \end{cases}$$

for all $k \in \mathbb{R} \setminus \{0\}$.

Proof. By Theorem 3.4, if $k \geq 0$,

$$\begin{cases} t_2(ku) = \underline{ku}(1) = \overline{ku}(1) = kt_2(u), \\ t_1(ku) = \frac{\int_0^1 s^2(\alpha)(1-\alpha)(\underline{ku}(\alpha)-t_2(ku)\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} \\ = \frac{\int_0^1 s^2(\alpha)(1-\alpha)k(\underline{u}(\alpha)-t_2(u)\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} = kt_1(u) \\ t_3(ku) = \frac{\int_0^1 s^2(\alpha)(1-\alpha)(\overline{ku}(\alpha)-t_2(ku)\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} \\ = \frac{\int_0^1 s^2(\alpha)(1-\alpha)k(\overline{u}(\alpha)-t_2(u)\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} = kt_3(u) \end{cases} \tag{3.19}$$

■

if $k < 0$

$$\begin{cases} t_2(ku) = \underline{ku}(1) = \overline{ku}(1) = kt_2(u) \\ t_1(ku) = \frac{\int_0^1 s^2(\alpha)(1-\alpha)(\underline{ku}(\alpha)-t_2(ku)\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} \\ = \frac{\int_0^1 s^2(\alpha)(1-\alpha)k(\overline{u}(\alpha)-t_2(u)\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} = kt_3(u) \\ t_3(ku) = \frac{\int_0^1 s^2(\alpha)(1-\alpha)(\overline{ku}(\alpha)-t_2(ku)\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} \\ = \frac{\int_0^1 s^2(\alpha)(1-\alpha)k(\underline{u}(\alpha)-t_2(u)\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} = kt_1(u) \end{cases} \tag{3.20}$$

■

Theorem 3.8 *The nearest triangular approximation is linear,*

$$T(\tilde{u} + \tilde{v}) = T(\tilde{u}) + T(\tilde{u}).$$

Proof. By Theorem 3.4

$$\begin{cases} t_2(u+v) = \underline{u+v}(1) = \overline{u+v}(1) = \underline{u}(1) + \underline{v}(1) \\ \quad = \overline{u}(1) + \overline{v}(1) = t_2(u) + t_2(v), \\ t_1(u+v) = \frac{\int_0^1 s^2(\alpha)(1-\alpha)(\underline{u+v}(\alpha)-t_2(u+v)\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} \\ = \frac{\int_0^1 s^2(\alpha)(1-\alpha)(\underline{u}(\alpha)+\underline{v}(\alpha)-t_2(u)\alpha-t_2(v)\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} \\ = \frac{\int_0^1 s^2(\alpha)(1-\alpha)(\underline{u}(\alpha)-t_2(u)\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} \\ + \frac{\int_0^1 s^2(\alpha)(1-\alpha)(\underline{v}(\alpha)-t_2(v)\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} \\ = t_1(u) + t_1(v), \\ t_3(u+v) = \frac{\int_0^1 s^2(\alpha)(1-\alpha)(\overline{u+v}(\alpha)-t_2(u+v)\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} \\ = \frac{\int_0^1 s^2(\alpha)(1-\alpha)(\overline{u}(\alpha)+\overline{v}(\alpha)-t_2(u)\alpha-t_2(v)\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} \\ = \frac{\int_0^1 s^2(\alpha)(1-\alpha)(\overline{u}(\alpha)-t_2(u)\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} \\ + \frac{\int_0^1 s^2(\alpha)(1-\alpha)(\overline{v}(\alpha)-t_2(v)\alpha)d\alpha}{\int_0^1 s^2(\alpha)(1-\alpha)^2 d\alpha} \\ = t_3(u) + t_3(v). \end{cases} \tag{3.21}$$

■

4 Numerical examples

Here, we present examples to illustrate our method for triangular approximate of fuzzy number with parametric form $\tilde{u} = (\underline{u}(\alpha), \overline{u}(\alpha)), \alpha \in (0, 1]$ according to Theorem 3.4 with $s(\alpha) = \frac{1}{2}$ and compared with some of the other methods.

Example 4.1 *let* $\tilde{u} = (-1 + \sqrt{\alpha}, 1 - \sqrt{\alpha})$, *we show compar in Table 4.1.*

Table 4.1.

method	$T(\tilde{u}) = (t_1(u), t_2(u), t_3(u))$
Yeh [23]	(-0.7,0,0.7)
Yeh [22]	(-0.62857,0,0.62857)
Ban [8]	(-0.66667,0,0.66667)
Grzegorzewski [17]	(-0.66667,0,0.66667)
Abbasbandy et al. $(p = 0)$[4]	(-0.7,0,0.7)
Abbasbandy et al. $(p = \frac{1}{2})$[4]	(-0.7,0,0.7)
Abbasbandy et al. [5]	(-0.62857,0,0.62857)
our method	(-0.7,0,0.7)

Example 4.2 *let* $\tilde{u} = (-1+\sqrt{\alpha}, 0)$, *we show compar in Table 4.2.*

Table 4.2.

method	$T(\tilde{u}) = (t_1(u), t_2(u), t_3(u))$
Abbasbandy et al.[5]	(-0.62857,0,0)
Yeh [22]	(-0.63571,0.00714,0.00714)
our method	(-0.7,0,0)

Example 4.3 *let* $\tilde{u} = (\alpha^2 - 1, 1 - \sqrt{\alpha})$, *we show compar in Table 4.3.*

Table 4.3.

method	$T(\tilde{u}) = (t_1(u), t_2(u), t_3(u))$
Yeh [23]	(-1.19167,-0.11667,0.75833)
our method	(-1.25,0,0.7)

Example 4.4 *let* $\tilde{u} = (-1+\sqrt{\alpha}, 1-\alpha^2)$, *we show compar in Table 4.4.*

Table 4.4.

method	$T(\tilde{u}) = (t_1(u), t_2(u), t_3(u))$
Yeh [22]	(-0.76072,0.1107,1.28928)
Abbasbandy et al. [5]	(-0.62857,0,1.25)
our method	(-0.7,0,1.25)

Example 4.5 *let* $\tilde{u} = (0, 1 - \sqrt{\alpha})$, *we show compar in Table 4.5.*

Table 4.5.

method	$T(\tilde{u}) = (t_1(u), t_2(u), t_3(u))$
Yeh [23]	(0.01333,-0.01333,0.70667)
our method	(0,0,0.7)

Example 4.6 *let* $\tilde{u} = (2\alpha - 2, 1 - \sqrt{\alpha})$, *we show compar in Table 4.6.*

Table 4.6.

method	$T(\tilde{u}) = (t_1(u), t_2(u), t_3(u))$
Ban [8]	(-1.96667,-0.36667,0.7)
Abbasbandy et al.$(p = \frac{1}{2})$ [4]	(-2.01265,0,068735)
Abbasbandy et al. (p=0)[4]	(-1.98043,0,0.71957)
our method	(-2,0,0.7)

Example 4.7 *let* $\tilde{u} = (\alpha + 1, 5 - 3^{\alpha})$, *we show compar in Table 4.7.*

Table 4.7.

method	$T(\tilde{u}) = (t_1(u), t_2(u), t_3(u))$
Delgado et al. [10]	(1,2,2.5)
Grzegorzewski et al. [16]	(1,2,5)
Abbasbandy et al. [2]	(1,2,5)
Abbasbandy et al. [3]	(1,2,5)
M. Ma et al. [12]	(-0.5,2,4.5)
our method	(1,2,4.25949)

5 Conclusion

In this paper first, we define the value and ambiguity functions and then propose a method for nearest triangular approximations of fuzzy number by value and ambiguity functions for each fuzzy number such that the computational complexity is less than other methods.

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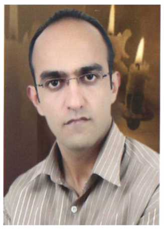

Mohammad adabitabar firozja, he has born in the mazandaran, babol in 1975. Got B.Sc and M.Sc degrees in applied mathematics, operation research field from Teacher Training University, Tehran Iran and PHD degree in applied mathematics, fuzzy numerical analysis field from Science and Research Branch, Islamic Azad University. Main research interest include ranking, fuzzy metric distance, ordinary differential equation, partial differential equation, fuzzy system and similarity measure.

Zahra Eslampia she has born in nowshahr-Iran in 1983. Got B. Sc degree in applied mathematics, from Pure Mathematics, Payam Noor University of Ramsar, and , M.Sc degrees in Applied Mathematics , numerical analysis field from Azad University of Lahijan. Main research interest includ distance of fuzzy numbers, the similarity measure of fuzzy numbers and numerical method of differential equation .